In working out the result it will be most convenient to use consistently the C.G.S. system. On this system of measurement the pressure employed was $9\frac{1}{2}\times 981$ dynes per square centimetre, and therefore the work expended per second in generating the waves was $196\times 9\frac{1}{2}\times 981$ ergs. Now the mechanical value of a series of progressive waves is the same as the kinetic energy of the whole mass of air concerned, supposed to be moving with the maximum velocity of vibration (v); so that, if S denotes the area of the wave-front considered, a be the velocity of sound, and ρ be the density of air, the mechanical value of the waves passing in a unit of time is expressed by $\frac{1}{2}S\,.\,a\,.\,\rho\,.\,v^2$, in which the numerical value of a is about 34100, and that of ρ about ·0013. In the present application S is the area of the surface of a hemisphere whose radius is 82000 centimetres; and thus, if the whole energy of the escaping air were converted into sound, and there were no dissipation on the way, the value of v at the distance of 82000 centimetres would be given by the equation

$$v^2=\frac{2\times 196\times 9\frac{1}{2}\times 981}{2\pi(82000)^2\times 34100\times \cdot 0013},$$

whence $v=\cdot 0014$ centimetre per second.

This result does not require a knowledge of the pitch of the sound. If the period be τ, the relation between the maximum excursion x and the maximum velocity v is

$$x=\frac{v\,\tau}{2\pi}.$$

In the present case the note of the whistle was f^{iv}, with a frequency of about 2730. Hence

$$x=\frac{\cdot 0014}{2\pi\times 2730}=10^{-8}\times 8\cdot 1,$$

or the amplitude of the aerial particles was less than a ten-millionth of a centimetre.

I am inclined to think that on a still night a sound of this pitch, whose amplitude is only a hundred-millionth of a centimetre, would still be audible.

III. "On the alleged Correspondence of the Rainfall at Madras with the Sun-spot Period, and on the True Criterion of Periodicity in a series of Variable Quantities." By General Strachey, R.E., C.S.I., F.R.S. Received May 3, 1877.

A paper has recently been printed by Dr. Hunter, the Director-General of Statistics to the Government of India, having for its object to show that the records of the rainfall at Madras, for a period extending over sixty-four years, establish a cycle of rainfall at that place which has a marked coincidence with a corresponding cycle of sun-spots—the rainfall and sun-spots attaining a minimum in the eleventh, first, and second years, and a maximum in the fifth year.

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Irrespective of its scientific interest, the conclusion thus adopted would, if sound, be of no little practical importance, as it would supply a means of indicating the probable recurrence of those seasons of excessive drought which produce such terrible results in India, and from one of which the Madras provinces are now suffering in an extreme degree.

It is probably generally known that the conclusion which it has thus been stated is to be drawn from the Madras observations had been considered to be established some years ago, in a more general manner, by Mr. Meldrum, the Director of the Meteorological Observatory at Mauritius, a paper by whom on this subject was read before the Royal Society in 1873, and may be found in vol. xxi. of the 'Proceedings,' p. 297.

As the numerical results of the method of treating the rainfall observation which Mr. Meldrum and Dr. Hunter have followed at first sight may appear to support the conclusions they have adopted, it has seemed to me desirable to examine the facts, with a view to arriving at an independent opinion as to the trustworthiness or otherwise of those conclusions. I shall first refer to the Madras observations and Dr. Hunter's results.

The Madras register extends over sixty-four years, beginning with 1813. The mean rainfall for the whole period is 48·5 inches. The deviations from the mean vary from 30·1 ins. in defect to 39·9 ins. in excess. The arithmetical mean of these deviations (disregarding the signs) is 12·4 ins. The greatest difference between two consecutive years is 50·5 ins., and the average difference 15·8 ins.

Dr. Hunter, in order to test the point which he proposes to investigate, divides the sixty-four years' observations into six cycles of eleven years, and calculates the arithmetical mean of the successive years of the whole series of cycles. The results are shown in the following Table, the figures in which represent the differences of the several years' observations from the mean of the whole :—

TABLE I.

Number of cycle.		Years of cycle.										
		1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	10th.	11th.
		in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
Original observations.	1st cycle	− 3·4	−16·1	+ 7·5	− 7·4	+15·1	+27·7	−12·2	+21·5	− 1·4	+11·1	−21·9
	2nd ,,	−14·8	+ 7·5	+12·2	+39·9	−10·6	−11·6	−16·1	− 4·2	−30·1	−11·4	− 9·5
	3rd ,,	− 7·0	− 3·8	+ 0·8	+ 3·8	+ 4·6	+10·1	+ 9·8	−12·0	+ 1·8	+16·9	−10·5
	4th ,,	+31·3	+32·5	+ 6·3	− 8·7	−11·6	+15·8	+24·2	−12·7	− 5·3	−16·2	− 1·5
	5th ,,	+ 4·4	0	+ 6·6	−20·9	−11·3	−10·3	+ 6·1	− 1·3	− 6·9	+ 2·9	−24·1
	6th ,,	− 7·1	−16·2	+25·6	+ 7·8	+25·2	+ 3·3	+14·4	−11·4	−27·0		
Mean difference from the mean of 64 years ...		+ 0·6	+ 0·7	+ 9·8	+ 2·4	+ 1·9	+ 5·8	+ 4·4	− 3·4	−11·5	+ 0·7	−13·5

In the above calculation the first year of the cycle of eleven is 1813, so that the average period of maximum sun-spots will be about the third or fourth year of the cycle, and the period of minimum will be about the tenth or eleventh of the cycle. This Table certainly seems to indicate a period of maximum between the third and the seventh years, and of minimum between the eighth and the second years.

But to estimate the true weight of these results we must look a little deeper. Now the only signification of the arithmetical mean of a series of observed quantities is that it is a quantity above and below which there is an equal amount of deviation in the aggregate of individual observations. Further, conformity to a law of any sort in a series of observations in relation to quantity is obviously to be tested by the extent of deviation of the observed quantities from the results that such law requires. Consequently, in the present case, the question whether or not the mean values shown in Table I. can be accepted as showing a definite law of variation from year to year in the cycle must be determined by examining the differences between those means and the individual observations on which they are based.

Treating the observations in this manner, the following results are obtained :—

TABLE II.

Number of cycle.		Years of cycle.										
		1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	10th.	11th.
		in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
Differences of mean values in Table I. from individual observations.	1.	− 4·0	−16·8	− 2·3	− 9·8	+13·2	+21·9	−16·6	+24·9	+10·1	+10·4	− 8·4
	2.	−15·4	+ 6·8	+ 2·4	+37·5	−12·5	−17·4	−20·5	− 0·8	−18·6	−12·1	+ 4·0
	3.	− 7·6	− 4·5	− 9·0	+ 1·4	+ 2·7	+ 4·3	+ 5·4	− 8·6	+13·3	+16·2	+ 3·0
	4.	+30·7	+31·8	− 3·5	−11·1	−13·5	+10·0	+19·8	− 9·3	+ 6·2	−16·9	+12·0
	5.	+ 3·8	− 0·7	− 3·2	−23·3	−13·4	−16·1	+ 1·7	+ 2·1	+ 4·6	+ 2·2	−10·6
	6.	− 7·7	−16·9	+15·8	+ 5·4	+23·3	− 2·5	+10·0	− 8·0	−15·5		
Mean difference, irrespective of sign		11·5	12·9	6·0	14·8	13·1	12·0	12·3	9·0	11·4	11·6	7·6
Mean of mean differences 11·2 inches.												

Thus it appears that the mean difference of the individual observations from the calculated means shown in the last line of Table I. differs but little from the mean difference of the individual observations from the arithmetical mean of the whole series. In other words, the supposed law of variation obtained from the means of the six 11-year cycles hardly

gives a closer approximation to the actual observations than is got by taking the simple arithmetical mean as the most probable value for any year.

In order to obtain a practical test of the probable physical reality of the cycle of eleven years, I have calculated a series of mean values corresponding to those given in Table I. for a series of cycles of five, six, seven, eight, nine, ten, twelve, and fourteen years. I find that the mean differences between these means and the observed quantities, and therefore corresponding to the mean differences shown in the last line of Table II., are all within a very small fraction of one another and of the mean obtained from the 11-year cycle—in short, that one cycle is in this respect almost as good or as bad as another.

The mean differences for the several cycles are given in the following Table:—

TABLE III.

Cycles of	Years of the cycle.														Mean.
	1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th	9th.	10th.	11th.	12th.	13th.	14th.	
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
5 years ...	9·0	9·5	12·8	10·6	18·3	...	...	...	...	...	...	...	...	...	11·9
6 years ...	7·7	13·2	11·5	14·7	13·2	13·3	...	...	...	...	...	...	...	...	12·2
7 years ...	15·9	8·9	6·7	12·4	8·5	19·2	12·5	...	...	...	...	...	...	...	12·1
8 years ...	5·2	15·0	12·1	11·8	8·0	11·3	14·9	16·3	...	...	...	...	...	...	11·8
9 years ...	10·6	11·2	10·9	12·6	11·3	16·7	10·7	13·2	7·0	...	...	...	...	...	11·6
10 years ...	8·5	10·5	7·8	12·4	17·7	9·3	7·9	17·5	8·4	18·8	...	...	...	...	11·7
11 years ...	11·5	12·9	6·0	14·7	13·1	12·0	12·3	9·0	11·4	11·6	7·6	...	...	...	11·2
12 years ...	5·8	11·4	11·6	13·0	11·0	12·7	7·8	15·4	6·8	13·4	14·4	13·9	...	...	11·4
14 years ...	17·7	12·0	4·5	13·9	7·5	24·0	14·1	14·2	5·1	6·3	11·3	10·1	12·2	8·7	11·9

Now if in any series of quantities, such as the rainfall observations at Madras, there be a law of periodicity, each observed quantity may be supposed to be compounded of a periodical and a non-periodical element. If we take the sum of a large number of cycles each of which coincides with the cycle of periodicity, the non-periodical elements will tend to occur in equal amount in excess and defect, and thus to be eliminated, and the means for the successive years of the cycle, or whatever the intervals be (which I will term cyclical means), will tend to indicate the periodical elements for the successive intervals. At the same time, the differences of these cyclical means from the several original quantities from which they were obtained will approximate to the several non-periodical elements. These differences I will call cyclical differences.

In proportion as the periodical elements are small or large in relation

to the corresponding non-periodical elements, so the cyclical differences will be inversely less or more different from the differences between the individual observations and the mean of the whole of them; and if there be no periodicity, the cyclical means will tend to disappear, and the two sets of differences would, in a sufficiently long series, be identical.

Hence it may be inferred that when the cyclical differences closely approximate in magnitude to the mean difference of the original observations from the arithmetical mean of all of them, the periodical elements in those observations must be correspondingly small; and this applies manifestly to the whole of the cycles for which the differences are shown in Table III.

Further to test the reality of the supposed periodicity shown in Table I., I have rearranged the series of 64 years' observations, in a purely arbitrary manner, in cycles of eleven years, by drawing the actual observations at random one after another, and setting them down in succession till the whole were exhausted. From three arbitrary sets of six cycles thus prepared the mean cyclical difference averaged 10·9, 11·2, and 11·6—results which again indicate that, by adopting the actual sequence of the observed quantities of rain instead of taking them at random, we produce no material effect on the mean cyclical differences, nor any such tendency to a diminution in their numerical value as necessarily accompanies a true periodical element.

It is, moreover, important to bear in mind that the mere circumstance of any series of cyclical means showing a single maximum and single minimum gives no more real indication that such a result is a truly periodical feature than would be supplied by the appearance of two or more maxima and minima. The law of periodicity, if it exist at all, can only be inferred by the facts indicated by observation; and it is obviously to argue in a circle, first to assume a cycle on which to work, which shall give a single maximum and minimum, and then to infer that there is true periodicity because of the single maximum and minimum. The test of the periodicity is in truth to be sought altogether outside of the particular values of the successive cyclical means.

It is, of course, manifest that a complication of periodical elements may so mask one another as to prevent positive results being obtained by such an examination of the cyclical means and differences as I have made in the case before us. But the whole scope of my present argument is negative, and it necessarily leads, I think, to the conclusion that the cyclical variations shown in Table II. from the mean values in Table I. are so great as to show that any apparent regularity or tendency to a maximum in one part of the 11-year cycle, and a minimum in another, has no real weight, and that there is no proof of greater tendency to periodicity in the 11-year means than in the original isolated observations.

This might perhaps be considered all that need be said on this subject

but an objection may possibly be made to the effect that the sun-spot period is not exactly a cycle of eleven years, and that a better result may be obtained by a comparison of the observations corresponding to the known periods of maximum and minimum sun-spots, without reference to any special length of cycle. The following Tables show the results thus arrived at, the figures representing differences from the mean rainfall for the whole sixty-four years.

Table IV.

Rainfall near periods of maximum sun-spots.						Rainfall near periods of minimum sun-spots.					
Years.	Two years before.	One year before.	Maximum.	One year after.	Two years after.	Years.	Two years before.	One year before.	Minimum.	One year after.	Two years after.
	in.	in.	in.	in.	in.		in.	in.	in.	in.	in.
1817.	+ 7·5	− 7·4	+15·1	+27·7	−12·2	1823.	− 1·4	+11·1	−21·9	−14·8	+ 7·5
1830.	−10·6	−11·6	−16·1	− 4·2	−30·1	1833.	− 4·2	−30·1	−11·4	− 9·5	− 7·0
1837.	− 7·0	− 3·8	− 0·8	+ 3·8	+ 4·6	1843.	+ 9·8	−12·0	+ 1·8	+16·9	−10·5
1849.	+32·5	+ 6·3	− 8·7	−11·6	+15·8	1856.	− 5·3	−16·2	− 1·5	+ 4·4	− 0·0
1860.	− 0·0	+ 6·6	−20·9	−11·3	−10·3	1867.	− 6·9	+ 2·9	−24·1	− 7·1	−16·2
1871.	−16·2	+25·6	+ 7·8	+25·2	+ 3·3						
Mean...	+ 1·0	+ 2·6	− 4·0	+ 4·9	− 4·8	Mean...	− 1·6	− 8·9	−11·4	− 2·0	− 5·2
Mean difference from mean	12·6	10·2	10·0	14·3	12·7	Mean difference from mean	4·6	12·7	9·3	10·1	7·2
Mean of mean differences 12·0 inches.						Mean of mean differences 8·8 inches.					

These results are also negative. The maximum period cannot be said to be indicated at all. The minimum shows a diminished mean cyclical deviation, but this is due to the years furthest from the supposed minimum epoch; and if the three central years alone are reckoned, the result is much the same as obtained from the 11-year cycles.

I have sought for a further test of the character of the conclusions that I have been discussing in the rainfall observations at Bombay and Calcutta, which have been made for the greater part of the period over which those at Madras extend. It is hardly conceivable that there should be a coincidence with the sun-spot period, such as is supposed to have been found at Madras, based on any physical cause, which should not in some way be discernible in the rainfall at Bombay and Calcutta. Adopting the same 11-year cycle for the observations at these two places as was used in the case of Madras, the mean results for the three localities are exhibited in the following Table:—

TABLE V.

	No. of years' observations.	Mean rainfall of whole period.	Average difference between mean rainfall of whole period of observation and means of each year of cycle of eleven years.										
			1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	10th.	11th.
		in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
Madras ...	64	48·5	+ 0·6	+0·7	+9·8	+2·4	+ 1·9	+5·8	+4·4	−3·4	−11·5	+ 0·7	−13·5
Bombay ...	52	76·9	0·0	+4·1	−1·1	−2·4	+11·2	+2·9	+1·7	+5·4	− 5·8	−10·8	− 9·4
Calcutta ...	47	65·8	+14·8	−5·2	−8·0	+1·5	+ 4·6	−5·3	−0·3	+0·0	− 5·0	+ 1·8	+ 1·8

These results are entirely negative, and indicate no concordance among the means of the several years of the cycle. The Bombay and Calcutta observations, treated as those of Madras were, to ascertain the deviations of individual observations from the successive means of the cycle, give quite similar results. The deviations obtained for Bombay and Calcutta, in the manner shown in Table II., are as follows:—

TABLE VI.

	Average differences irrespective of sign between separate observations and means of each year of cycle of eleven years.											Mean of mean differences.
	1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	10th.	11th.	
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
Bombay	7·1	16·6	5·5	20·4	20·8	13·6	6·7	15·3	9·6	15·8	6·4	12·7
Calcutta	7·9	6·6	7·1	14·6	12·3	8·4	7·9	14·3	5·0	4·5	5·1	8·0

In these cases, as in that of Madras, the mean deviation for the whole eleven years of the cycle differs very little from the mean variation of the single observations from the arithmetical mean of all of them, these quantities being for Bombay 13·4 in., and for Calcutta 9·0 in.

Although my special object in the present communication is to deal with the alleged correspondence between the Madras rainfall and the sun-spot periods, I have naturally turned my attention to Mr. Meldrum's speculations of a similar character, and I have tested some of them in the manner that I have just explained.

Taking the Greenwich observations for fifty-five years, which will be found at page 307 of vol. xxi. of the 'Proceedings of the Royal Society,' in Mr. Meldrum's paper before noticed, and arranging them in the manner shown in Table IV., the following results are obtained. The mean rainfall for the whole period is 24·9 inches, and the entries are the differences from this mean.

TABLE VII.

Rainfall near periods of maximum sun-spots.						Rainfall near periods of minimum sun-spots.					
Year.	Two years before.	One year before.	Maxi-mum.	One year after.	Two years after.	Year.	Two years before.	One year before.	Mini-mum.	One year after.	Two years after.
	in.	in.	in.	in.	in.		in.	in.	in.	in.	in.
1817	− 4·4	+2·6	+1·7	−1·5	+3·3	1823	+6·6	+0·1	−0·5	+8·1	−2·6
1830	+ 3·5	−2·3	+6·2	+9·6	+4·8	1834	+4·8	+4·0	−1·1	+5·7	+6·1
1837	+ 5·7	+6·1	−5·8	−3·3	+2·2	1844	−4·9	−2·4	−3·0	−6·2	−1·4
1849	−12·1	+5·3	−1·0	−5·2	−3·3	1856	−5·9	−1·1	−3·0	−3·5	−7·1
1860	− 7·1	+1·0	+7·1	−4·5	+1·6	1867	+3·8	+5·8	+3·5	+0·2	−0·9
Mean...	− 2·9	+2·5	+1·6	−1·0	+1·7	Mean ...	+0·9	+1·3	−0·8	+4·3	−1·2
Mean difference from mean	6·0	2·5	4·0	4·2	2·1	Mean difference from mean	5·0	2·9	1·9	4·8	3·0
Mean of mean differences 3·8 inches.						Mean of mean differences 3·5 inches.					

In this case the mean deviation of the original observations from the arithmetical mean of the whole series (24·9 inches) is 4·1 inches. This result therefore is quite analogous to that obtained from the Indian observations.

I have not attempted to make detailed calculations in other cases, but, so far as I can judge, the evidence of the alleged periodicity will be generally found to fail when it is tested by comparison with the individual observations on which it has been made to rest.

It will serve to illustrate the argument on which this paper is based if we consider what would be the consequence of applying it to a case in which a well-ascertained periodicity exists, as that of the diurnal barometric oscillation. The following Table gives an example, taken at random from an old Madras register, the intervals being made two-hourly, so as to reduce the number of calculations. The entries are the differences of the observed barometric heights from the mean of the whole in thousandths of an inch.

TABLE VIII.

Cycles of one day.	Intervals of cycle.—Two hours.											
	0.	2.	4.	6.	8.	10.	12.	14.	16.	18.	20.	22.
1st cycle............	−38	− 5	+53	+83	+45	+10	+32	+52	+78	+54	− 6	−59
2nd ,, ...	−87	−58	−21	+13	−11	−12	− 9	+ 1	+16	+ 5	−32	−72
3rd ,, ...	−72	−43	+ 2	+19	− 2	−16	−13	+21	+57	+42	− 2	−45
4th ,, ...	−63	−28	+18	+47	+22	− 7	+ 4	+20	+64	+37	−13	−46
5th ,, ...	−52	−29	+ 9	+24	+ 7	−21	− 3	+35	+56	+35	−13	−74
Mean difference from mean of 60 obs..........	−62	−33	+12	+37	+12	− 9	+ 2	+26	+54	+35	−13	−59

In this case it will be found that the mean deviation (disregarding sign) of the whole of the observations from the arithmetical mean of the whole (which is here zero) will be 30·3.

The differences between the mean values, given in the last line of Table VIII., for the two-hourly periods and the individual observations are as follows:—

TABLE IX.

Cycles of one day.	Intervals of two hours.											
	0.	2.	4.	6.	8.	10.	12.	14.	16.	18.	20.	22.
1st cycle............	+24	+28	+41	+46	+33	+19	+30	+26	+24	+19	+ 7	0
2nd ,, ...	−25	−25	−33	−24	−23	− 3	−11	−25	−38	−30	−19	−13
3rd ,, ...	−10	−10	−10	−18	−14	− 7	−15	− 5	+ 3	+ 7	+11	+14
4th ,, ...	− 1	+ 5	+ 6	+10	+10	+ 2	+ 2	− 6	+10	+ 2	0	+13
5th ,, ...	+10	+ 4	− 3	−13	− 5	−12	− 5	+ 9	+ 2	0	0	−15
Mean difference, disregarding the signs	14	14	19	22	17	9	13	14	15	12	7	11
Mean of mean differences 14.												

Here a true periodicity existing, the mean of the differences, or, as I before called it, the cyclical deviation, is reduced to 14. Moreover the several differences for the separate observations for the successive hours are well below the mean variation 30, and none of the separate results is so high as that figure.

If we had such a series of sixty observations as is contained in

Table VIII., and had no knowledge of the law of periodicity, and hypothetically applied a period of ten intervals instead of twelve (following the process the results of which for Madras are given in Table III.), the inapplicability of such a period would immediately be made apparent by results such as are found in the rainfall observations. On the hypothesis just mentioned the mean values, corresponding to those given in the last line of Table VIII., for the successive intervals of the six arbitrary cycles that would be formed would be as follows:—

TABLE X.

Cycles of 20 hours.	Intervals of two hours.									
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Mean difference from mean of 60 obs.........	−2	−3	0	−1	+2	+1	+1	+3	+3	−3

From these figures the following differences would be deduced instead of those shown in Table IX.

TABLE XI.

Cycles of 20 hours.	Intervals of two hours.									
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
1st period	36	1	53	84	43	9	31	49	75	57
2nd "	4	55	87	57	23	12	12	15	12	4
3rd "	18	9	32	71	74	44	1	16	5	13
4th "	11	25	57	43	4	46	64	31	15	50
5th "	24	3	4	21	62	36	14	49	55	26
6th "	11	28	7	20	5	34	55	32	16	71
Mean difference, disregarding signs............	17	20	40	49	35	30	30	32	30	37
Mean of mean differences 32.										

The contrast between these results and those given in Table IX. is most striking, and calls for no comment in detail.

In conclusion I would explain that I do not desire to call in question the possible or actual occurrence of periodical terrestrial phenomena

corresponding to the sun-spot period, but to point out in the case of the rainfall not only has no such correspondence been established, but that there has been no sufficient evidence adduced of any periodicity at all.

Appendix.—Received May 17, 1877.

Suppose the numerical values of such a series of observations as that discussed in the preceding paper to be represented by

$A_1, A_2, A_3, \ldots$ for the first year of each cycle of (n) years,
$B_1, B_2, B_3, \ldots$ for the second years,
and so on for (m) cycles.

Also suppose M to represent the arithmetical mean of all the observations, and $p_a, p_b, p_c, \ldots$ to be the periodical variations from the mean, M, for the several series of the A, B, . . . years of the cycles, and the corresponding non-periodical variations to be $a_1, a_2, a_3, \ldots, b_1, b_2, b_3, \ldots$, and so forth—all these quantities being affected by their proper signs.

Then if M_a represents the mean value of any series $A_1, A_2, A_3, \ldots$, we should have for (m) cycles—

$$M_a = M + p_a + \frac{1}{m}\{a_1 + a_2 + \ldots\} = M + p_a + e_a,$$

where e_a with its proper sign represents the non-periodical portion of the mean value of the A series for (m) years.

With a sufficiently prolonged series of cycles the quantities $a_1, a_2, \ldots$ will tend to cancel one another and e_a will disappear; so that M_a will then become equal to $M + p_a$, and $M_a - M = p_a$.

If, therefore, there is a truly periodical element, the difference of the mean of all the observations, and of the mean of any series (A) of the cycle of periodicity, will (in a sufficiently extended series of observations) tend to be identical with the periodical element (p_a) for that cycle.

This holds good of all the series A, B, C, . . .; and therefore the sum of all the differences last mentioned for the several series will tend to equality with the sum of all the periodical elements of the several series. This will be true whether we regard the algebraic sign or not. If we so regard it, the sum of the differences will evidently become equal to zero, as also will the sum of the periodical elements. If we disregard the signs, the sums will have a numerical value, which call S. This, if divided by (n), the number of years in the cycle of periodicity, will indicate the mean deviation, either in excess or defect, of the periodical elements from the arithmetical mean of all the observations.

Hence, when there is a true periodicity, the sum of these differences taken without regard to sign tends to become an invariable quantity, and the numerical magnitude of this quantity indicates the magnitude of the periodical variation.

Let us next consider the case in which there is no truly periodical element. Here $M_a = M + e_a$ and $M_a - M = e_a$. If E_a represents the mean numerical value, irrespective of sign, of all the deviations of an indefi-

nitely prolonged series of observed quantities $A_1, A_2, \ldots$ from their mean M_a, then the probable value of e_a deduced from the mean of (m) cycles, or series of observations of A, also irrespective of sign, will (according to the known laws of the combination of errors) equal $\frac{E_a}{\sqrt{m}}$. The same will hold good of all the series B, C, &c. At the same time, as there is no periodicity, and all the observations are presumably liable to errors or irregularities of the same general character in a positive or negative direction, the quantities E_a, E_b, &c. will, in a sufficiently prolonged series, tend to equality one with another and also with the mean deviation, irrespective of sign, of all the observed quantities from the arithmetical mean of the whole of them, which call E.

Hence, when there is no periodicity, the sum S, as before defined, tends to become $n \times \frac{E}{\sqrt{m}}$.

We are thus led to the conclusion that the consideration of the successive values of the quantity S as the number of cycles of periodicity increases affords a true criterion of the presence or absence of a periodical element. If as (m) increases this quantity is gradually reduced in a ratio approximating to $\frac{E}{\sqrt{m}}$, we may infer that the periodicity is small or does not exist. On the other hand, if the value of S tends to become invariable, and continues to be of considerable numerical magnitude after a prolonged series of cycles, the existence of a true periodical element is apparent.

In the Madras observations the successive values of $\frac{S}{n}$ (obtained by combining one after another the observations of one cycle of 11 years, of two such cycles, of three, and so on, till the whole six are united) are shown below, contrasted with the corresponding calculated values of $\frac{E}{\sqrt{m}}$.

TABLE XII.

Number of cycles.	From observation, $\frac{S}{n}$.	Calculated, $\frac{E}{\sqrt{m}}$.
1 cycle	13·2	13·2
2 cycles	9·5	10·0
3 ,, ...	7·3	6·9
4 ,, ...	4·9	6·4
5 ,, ...	4·5	5·3
6 ,, ...	4·5 ?	5·1
12 ,, ...	3·4	3·8
18 ,, ...	3·2	3·0
24 ,, ...	2·7	2·7

The last cycle of the six is not complete, and the value of $\frac{S}{n}$ is therefore doubtful. As the observations only extend to six cycles, in order to test the process I have continued the calculation to twelve, eighteen, and twenty-four cycles, by combining with the real observations the three series formed at hazard by a chance redistribution of the original observations. This of course gives no support to any conclusions as to the *existence* of periodicity in the case of Madras; but it shows how, in the *absence* of periodicity, the theory is completely verified.

The Bombay, Calcutta, and Greenwich observations, similarly treated, exhibit similar results.

In strong contradistinction to the above results are those that follow on treating in a similar manner the barometric observations given in Tables VIII. and IX.; while these same observations, when their true periodicity is destroyed, as in Tables X. and XI., show results quite similar to those seen in Table XII.

The following Table shows these results, both with the periodicity as it really exists and after it has been destroyed :—

TABLE XIII.

Number of cycles.	Real periodicity.		Periodicity destroyed.	
	From observation, $\frac{S}{n}$.	Calculated, $\frac{E}{\sqrt{m}}$.	From observation, $\frac{S}{n}$.	Calculated, $\frac{E}{\sqrt{m}}$.
1 cycle	37·1	......	29·4	......
2 cycles	31·0	25·2	17·8	26·2
3 „	29·4	19·0	14·7	19·0
4 „	29·5	16·2	10·7	16·8
5 „	29·5	13·0	5·6	14·6
6 „	......	......	1·9	13·0

The fact that the figures in the last column but one are so much less than the calculated values of $\frac{E}{\sqrt{m}}$ evidently arises from the want of conformity of the numerical values of the quantities which are found in Table VIII. with the assumed law of the probable numerical distribution of errors on which the theoretical value $\frac{E}{\sqrt{m}}$ rests; but the mutual destruction of the non-periodical elements earlier than this theory would have required does not affect the general reasoning.