TABLE VI.—Period of Conjunction of Mercury and Jupiter.
(0° denoting Conjunction—63 sets for Kew—43 sets for Trevandrum.)

				Kew.	Trevandrum.
Between	0°	and	30°	+633	+453
"	30	"	60	+759	+270
"	60	"	90	+652	+129
"	90	"	120	+328	−118
"	120	"	150	−119	−384
"	150	"	180	−504	−467
"	180	"	210	−678	−487
"	210	"	240	−677	−407
"	240	"	270	−548	−122
"	270	"	300	−322	+223
"	300	"	330	− 10	+415
"	330	"	360	+343	+503

I desire, in conclusion, to thank Mr. William Dodgson, who has given me much assistance in the calculations and diagrams of this paper.

III. "Note on the Value of Euler's Constant; likewise on the Values of the Napierian Logarithms of 2, 3, 5, 7, and 10, and of the Modulus of common Logarithms, all carried to 260 places of Decimals." By Professor J. C. ADAMS, M.A., F.R.S. Received December 6, 1877.

In the "Proceedings of the Royal Society," vol. xix, pp. 521, 522, Mr. Glaisher has given the values of the logarithms of 2, 3, 5, and 10, and of Euler's constant to 100 places of decimals, in correction of some previous results given by Mr. Shanks.

In vol. xx, pp. 28 and 31, Mr. Shanks gives the results of his re-calculation of the above-mentioned logarithms and of the modulus of common logarithms to 205 places, and of Euler's constant to 110 places of decimals.

Having calculated the value of 31 Bernoulli's numbers, in addition to the 31 previously known, I was induced to carry the approximation to Euler's constant to a much greater extent than had been before practicable. For this purpose I likewise re-calculated the values of the above-mentioned logarithms, and found the sum of the reciprocals of the first 500 and of the first 1000 integers, all to upwards of 260 places of decimals. I also found two independent relations between the logarithms just mentioned and the logarithm of 7, which furnished a test of the accuracy of the work.

On comparing my results with those of Mr. Shanks, I found that the latter were all affected by an error in the 103rd and 104th places of decimals, in consequence of an error in the 104th place in the determination of $\log \frac{81}{80}$. With this exception, the logarithms given by Mr. Shanks were found to be correct to 202 places of decimals.

The error in the determination of $\log_\epsilon 10$, of course entirely vitiated Mr. Shanks' value of the modulus from the 103rd place onwards. As he gives the complete remainder, however, after the division by his value of $\log_\epsilon 10$, I was enabled readily to find the correction to be applied to the erroneous value of the modulus. Afterwards I tested the accuracy of the entire work by multiplying the corrected modulus by my value of $\log_\epsilon 10$.

Mr. Shanks' values of the sum of the reciprocals of the first 500 and of the first 1000 integers, as well as his value of Euler's constant, were found to be incorrect from the 102nd place onwards.

Let S_n, or S simply, when we are concerned with a given value of n, denote the sum of the harmonic series,

$$1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots+\frac{1}{n}$$

Also let R_n, or R simply, denote the value of the semi-convergent series,

$$\frac{B_1}{2n^2}-\frac{B_2}{4n^4}+\frac{B_3}{6n^6}-\ \ldots\ldots$$

where B_1, B_2, B_3, &c., are the successive Bernoulli's numbers.

Then if Euler's constant be denoted by E, we shall have

$$E=S_n+R_n-\frac{1}{2n}-\log_\epsilon n,$$

and the error committed by stopping at any term in the convergent part of R_n will be less than the value of the next term of the series.

I have calculated accurately the values of the Bernoulli's numbers as far as B_{62}, and approximately as far as B_{100}, retaining a number of significant figures varying from 35 to 20.

When $n=1000$, the employment of the numbers up to B_{61} suffices to give the value of R_{1000} to 265 places of decimals. When $n=500$, it is necessary to employ the approximate values up to B_{74}, in order to determine R_{500} with an equal degree of exactness.

In order to reduce as much as possible the number of quantities which must be added together to find S_{500} and S_{1000}, I have resolved the reciprocal of every integer up to 1000 into fractions whose denominators are primes or powers of primes.

Thus S_{500} and S_{1000} may be expressed by means of such fractions, and by adding or subtracting one or more integers, each of these

fractions may be reduced to a positive proper fraction, the value of which in decimals may be taken from Gauss' Table, in the second volume of his collected works, or calculated independently.

Thus I have found that:—

$$S_{500} = \frac{249}{256} + \frac{2}{81} + \frac{3}{5} + \frac{120}{343} + \frac{3}{121} + \frac{86}{169} + \frac{205}{289} + \frac{58}{361} + \frac{1}{23} + \frac{3}{29}$$

$$+ \frac{21}{31} + \frac{30}{37} + \frac{11}{41} + \frac{15}{43} + \frac{26}{47} + \frac{32}{53} + \frac{24}{59} + \frac{33}{61} + \frac{27}{67} + \frac{67}{71} + \frac{28}{73} + \frac{38}{79}$$

$$+ \frac{73}{83} + \frac{72}{89} + \frac{33}{97} + \frac{61}{101} + \frac{45}{103} + \frac{11}{107} + \frac{102}{109} + \frac{68}{113} + \frac{23}{127} + \frac{111}{131}$$

$$+ \frac{116}{137} + \frac{25}{139} + \frac{126}{149} + \frac{27}{151} + \frac{28}{157} + \frac{29}{163} + \frac{85}{167} + \frac{88}{173} + \frac{91}{179} + \frac{92}{181}$$

$$+ \frac{97}{191} + \frac{98}{193} + \frac{100}{197} + \frac{101}{199} + \frac{107}{211} + \frac{113}{223} + \frac{115}{227} + \frac{116}{229} + \frac{118}{233} + \frac{121}{239}$$

$$+ \frac{122}{241}$$

+ (the sum of the reciprocals of the primes from 251 to 499) — 19.

Similarly I have found that:—

$$S_{1000} = \frac{249}{512} + \frac{310}{729} + \frac{181}{625} + \frac{75}{343} + \frac{62}{121} + \frac{35}{169} + \frac{220}{289} + \frac{11}{361} + \frac{300}{529} + \frac{726}{841}$$

$$+ \frac{32}{961} + \frac{34}{37} + \frac{21}{41} + \frac{10}{43} + \frac{40}{47} + \frac{48}{53} + \frac{28}{59} + \frac{56}{61} + \frac{7}{67} + \frac{31}{71} + \frac{40}{73} + \frac{45}{79}$$

$$+ \frac{25}{83} + \frac{49}{89} + \frac{44}{97} + \frac{69}{101} + \frac{82}{103} + \frac{90}{107} + \frac{104}{109} + \frac{12}{113} + \frac{67}{127} + \frac{84}{131} + \frac{121}{137}$$

$$+ \frac{85}{139} + \frac{144}{149} + \frac{10}{151} + \frac{26}{157} + \frac{141}{163} + \frac{83}{167} + \frac{34}{173} + \frac{53}{179} + \frac{132}{181} + \frac{171}{191}$$

$$+ \frac{102}{193} + \frac{196}{197} + \frac{125}{199} + \frac{90}{211} + \frac{95}{223} + \frac{21}{227} + \frac{212}{229} + \frac{138}{233} + \frac{22}{239} + \frac{223}{241}$$

$$+ \frac{211}{251} + \frac{216}{257} + \frac{221}{263} + \frac{226}{269} + \frac{47}{271} + \frac{48}{277} + \frac{236}{281} + \frac{49}{283} + \frac{246}{293} + \frac{53}{307}$$

$$+ \frac{261}{311} + \frac{54}{313} + \frac{266}{317} + \frac{57}{331} + \frac{170}{337} + \frac{175}{347} + \frac{176}{349} + \frac{178}{353} + \frac{181}{359} + \frac{185}{367}$$

$$+ \frac{188}{373} + \frac{191}{379} + \frac{193}{383} + \frac{196}{389} + \frac{200}{397} + \frac{202}{401} + \frac{206}{409} + \frac{211}{419} + \frac{212}{421} + \frac{217}{431}$$

$$+ \frac{218}{433} + \frac{221}{439} + \frac{223}{443} + \frac{226}{449} + \frac{230}{457} + \frac{232}{461} + \frac{233}{463} + \frac{235}{467} + \frac{241}{479} + \frac{245}{487}$$

$$+ \frac{247}{491} + \frac{251}{499}$$

+ (the sum of the reciprocals of the primes from 503 to 997) — 43.

This mode of finding S_{500} and S_{1000} is attended with the advantage that if an error were made in the calculation of the former of these quantities, it would not affect the latter.

The logarithms required have been found in the following manner:—

$$\text{Let } \log\frac{10}{9}=a,\ \log\frac{25}{24}=b,\ \log\frac{81}{80}=c,\ \log\frac{50}{49}=d, \text{ and } \log\frac{126}{125}=e.$$

Then we have

$$\log 2=7a-2b+3c,\ \log 3=11a-3b+5c,\ \log 5=16a-4b+7c.$$

Also $$\log 7=\tfrac{1}{2}\,(39a-10b+17c-d)\,;$$

or again, $$\log 7=19a-4b+8c+e,$$

and we have the equation of condition,

$$a-2b+c=d+2e,$$

which supplies a sufficient test of the accuracy of the calculations by which a, b, c, d, and e have been found.

$$\text{Since } \log\frac{10}{9}=-\log\left(1-\frac{1}{10}\right)$$

$$\log\frac{25}{24}=-\log\left(1-\frac{4}{100}\right)$$

$$\log\frac{81}{80}=\log\left(1+\frac{1}{80}\right)$$

$$\log\frac{50}{49}=-\log\left(1-\frac{2}{100}\right)$$

$$\log\frac{126}{125}=\log\left(1+\frac{8}{1000}\right)$$

If we have settled beforehand on the number of decimal places which we wish to retain, and have already formed the decimal values of the reciprocals of the successive integers to the extent required, then the formation of the values of a, b, c, d, and e, will only involve operations which, though numerous, are of extreme simplicity.

In this way have been found the following results:—

Log 10÷9= ·10536 05156 57826 30122 75009 80839 31279 83061 20372 98327
40725 63939 23369 25840 23240 13454 64887 65695 46213 41207
66027 72591 03705 17148 67351 70132 21767 11456 06836 27564
22686 82765 81669 95879 19464 85052 49713 75112 78720 90836
46753 73554 69033 76623 27864 87959 35883 39553 19538 32230
68063 73738 05700 33668 65

Log 25÷24= ·04082 19945 20255 12955 45770 65155 31987 01772 11747 63352
02297 28561 42083 06828 16287 62241 55690 62020 38337 10701
85958 13391 57612 02856 02344 55254 44440 90711 64191 09254
90615 87090 13793 32587 08185 56690 89768 86470 69797 42768
97243 12354 16791 64980 33118 36535 36811 73829 09383 64151
16223 48133 67972 69296

Log 81 ÷ 80 = ·01242 25199 98557 15331 12931 28631 20890 67623 60339 58145
90685 43409 40510 22236 97287 99924 04408 75833 17607 39941
83907 88915 98331 57135 00593 07313 64880 85644 69078 59065
10006 71375 61155 92285 64823 02773 78467 95356 20673 20672
56121 24774 48623 61600 82118 41837 57253 45313 78157 48027
60627 91715 42041 36587 2

Log 50 ÷ 49 = ·02020 27073 17519 44840 80453 01024 19238 78525 33383 73356
83210 27195 49256 65918 71880 87170 92908 14086 00703 48551
55810 69865 22995 29709 68602 61790 51909 27000 19877 96234
68586 52194 37909 61418 83597 32774 05301 16399 74760 65371
30928 59153 97434 74168 79079 46094 49807 56880 62620 29129
95963 65850 08854 45

Log 126 ÷ 125 = ·00796 81696 49176 87351 07973 39067 84478 84307 61916 78206
21803 11515 15228 34251 08036 00862 32503 51700 93221 55597
11104 32429 31908 69430 97326 52573 22928 44338 63827 35942
41437 63883 38664 80785 92159 70835 21671 40563 92519 30299
88730 07233 43319 67047 32333 55315 84852 90164 08154 11413
00140 51668 01463 4832

All these are Napierian logarithms.

The above-mentioned equation of condition is satisfied to 263 places of decimals.

Whence have been deduced the following :—

Log_ϵ 2 = ·69314 71805 59945 30941 72321 21458 17656 80755 00134 36025
52541 20680 00949 33936 21969 69471 56058 63326 99641 86875
42001 48102 05706 85733 68552 02357 58130 55703 26707 51635
07596 19307 27570 82837 14351 90307 03862 38916 73471 12335
01153 64497 95523 91204 75172 68157 49320 65155 52473 41395
25882 95045 30081 06850 15

Log_ϵ 3 = 1·09861 22886 68109 69139 52452 36922 52570 46474 90557 82274
94517 34694 33363 74942 93218 60896 68736 15754 81373 20887
87970 02906 59578 65742 36800 42259 30519 82105 28018 70767
27741 06031 62769 18338 13671 79373 69884 43609 59903 74257
03167 95911 52114 55919 17750 67134 70549 40166 77558 02222
03170 25294 68992 45403 15

Log_ϵ 5 = 1·60943 79124 34100 37460 07593 33226 18763 95256 01354 26851
77219 12647 89147 41789 87707 65776 46301 33878 09317 96107
99966 30302 17155 62899 72400 52293 24676 19963 36166 17463
70572 75521 79637 49718 32456 53492 85620 23415 25057 27015
51936 00879 77738 97256 88193 54071 27661 54731 22180 95279
48521 29282 13604 17624 80

Log_ϵ 7 = 1·94591 01490 55313 30510 53527 43443 17972 96370 84729 58186
11884 59390 14993 75798 62752 06926 77876 58498 58787 15269
93061 69420 58511 40911 72375 22576 77786 84314 89580 95163
90077 59078 24468 10427 47833 82259 34900 84673 74412 50497
37048 53551 76783 55774 86240 15102 77418 08868 67107 51412
13480 93879 74210 03537 95

Log_ϵ 10 = 2·30258 50929 94045 68401 79914 54684 36420 76011 01488 62877
29760 33327 90096 75726 09677 35248 02359 97205 08959 82983
41967 78404 22862 48633 40952 54650 82806 75666 62873 69098
78168 94829 07208 32555 46808 43799 89482 62331 98528 39350
53089 65377 73262 88461 63366 22228 76982 19886 74654 36674
74404 24327 43685 24474 95

M = ·43429 44819 03251 82765 11289 18916 60508 22943 97005 80366
65661 14453 78316 58646 49208 87077 47292 24949 33843 17483
18706 10674 47663 03733 64167 92871 58963 90656 92210 64662
81226 58521 27086 56867 03295 93370 86965 88266 88331 16360
77384 90514 28443 48666 76864 65860 85135 56148 21234 87653
43543 43573 17247 48049 05993 55353 05

where M denotes the modulus of common logarithms.

In these calculations the value of $\log \frac{50}{49}$ has been determined with less accuracy than that of $\log \frac{126}{125}$, and therefore the value of log 7 found by means of the latter quantity has been preferred.

If now in the formula which gives Euler's constant we take $n = 500$, we find the following results:—

$$\frac{1}{2n} = 0{\cdot}001$$

R_{500} = ·00000 03333 33200 00025 39671 87309 34479 09501 49853 06920
81561 41982 03143 98353 10049 47690 35814 25947 82825 73530
80967 33251 23444 83365 27221 32891 79715 39888 78668 70158
11997 43277 84264 18919 84678 56672 58294 26067 37401 94207
08483 64907 04495 03811 66583 11699 18899 16275 81704 82573
08004 99446 91635

S_{500} = 6·79282 34299 90524 60298 92871 45367 97369 48198 13814 39677
91166 43088 89685 43566 23790 55049 24576 49403 73586 56039
17565 98584 37506 59282 23134 68847 97117 15030 24984 83148
07266 84437 10123 70203 14772 22094 00570 47964 42959 21001
09719 01932 14586 27077 01576 02007 28842 06850 09735 01135
74118 52998 6631

$\log_e 500$ = 6·21460 80984 22191 74263 67422 42594 91605 47278 04331 52606
36739 79303 69340 93242 07062 36272 51021 28288 27237 62074
83901 87110 62880 60166 54305 61594 90289 71296 61913 55661
26910 65179 94054 14829 26073 41092 64585 48079 22114 05716
58115 31635 24264 74180 14925 98528 81625 94504 71489 68628
97329 77937 00975

E = ·57721 56649 01532 86060 65120 90082 40243 10421 59335 93992
35988 05767 23488 48677 26777 66467 09369 47063 29174 67495
14631 44724 98070 82480 96050 40144 86542 83622 41739 97644
92353 62535 00333 74293 73377 37673 94279 25952 58247 09491
60087 35203 94816 56708 53233 15177 66115 28621 19950 15079
84793 74508 5697

Again, if in the same formula we take $n = 1000$, we find the following:—

$$\frac{1}{2n} = 0{\cdot}0005$$

R_{1000} = ·00000 00833 33325 00000 39682 49801 59487 73237 84632 11743
88611 32124 18782 98862 06644 51967 06850 04241 14869 65631
43736 78499 44114 24665 37423 82138 50259 70190 89962 61572
33894 07843 88131 36054 55889 69002 08034 44545 27898 47738
31546 74821 27649 54293 18527 10448 88349 55931 43201 82238
86978 52223 81562

S_{1000}= 7·48547 08605 50344 91265 65182 04333 90017 65216 79169 70880
36657 73626 74995 76993 49165 20244 09599 34437 41184 50813
96798 01438 22544 03715 81484 21958 84703 40431 40398 43368
92966 39178 33827 35905 57913 00071 54692 68403 25933 79804
87809 56515 86955 67800 24804 71415 08712 32350 00711 42865
21027 95267 06455

$\log_\epsilon 1000$= 6·90775 52789 82137 05205 39743 64053 09262 28033 04465 88631
89280 99983 70290 27178 29032 05744 07079 91615 26879 48950
25903 35212 68587 45900 22857 63952 48420 26999 88621 07296
34506 84487 21624 97666 40425 31399 68447 86995 95585 18051
59268 96133 19788 65384 90098 66686 30946 59660 23963 10024
23212 72982 31056

E= ·57721 56649 01532 86060 65120 90082 40243 10421 59335 93992
35988 05767 23488 48677 26777 66467 09369 47063 29174 67495
14631 44724 98070 82480 96050 40144 86542 83622 41739 97644
92353 62535 00333 74293 73377 37673 94279 25952 58247 09491
60087 35203 94816 56708 53233 15177 66115 28621 19950 15079
84793 74508 56961

It will be seen that the two values found for E agree to 263 places of decimals, which supplies another independent verification of the value obtained for $\log_\epsilon 2$.

February 14, 1878.

Sir JOSEPH HOOKER, K.C.S.I., President, in the Chair.

The Right Hon. William Henry Smith and the Right Hon. Sir William Henry Gregory, whose certificates had been suspended as required by the Statutes, were balloted for and elected Fellows of the Society.

The Presents received were laid on the table and thanks ordered for them.

The following Papers were read:—

I. "Concerning the Effects on the Heart of Alternate Stimulation of the Vagi." By ARTHUR GAMGEE, M.D., F.R.S., Brackenbury Professor of Physiology in Owens College, and JOHN PRIESTLEY, Assistant Lecturer in Physiology in Owens College. Received December 15, 1877.

In 1869, A. B. Meyer* observed that it was impossible to stop the heart in dogs and rabbits continuously by stimulation of the vagus

* A. B. Meyer "Das Hemmungsnervensystem des Herzens." (Abstract by Schiffer, "Centralblatt," 1869, No. 14, p. 216.)