the yellow being the least marked. The red, green, and blue are however particularly well rendered by reflected light, and the plate shows the colours as seen when a dull light is thrown on the slit of the spectroscope, a simile which was suggested to me by Mr. Norman Lockyer.

From the evidence obtained by these experiments it appears that two or three molecular groupings are sufficient to give the necessary colours, a subject which I only allude to, since the more general question of molecular groupings is being considered by others.

III. "A Tenth Memoir on Quantics." By A. Cayley, Sadlerian Professor of Pure Mathematics in the University of Cambridge. Received June 12, 1878.

(Abstract.)

The present memoir, which relates to the binary quintic $(*)(x, y)^5$, has been in hand for a considerable time; the chief subject-matter was intended to be the theory of a canonical form discovered by myself, and which is briefly noticed in "Salmon's Higher Algebra," 3rd Ed (1876), pp. 217, 218; writing $a, b, c, d, e, f, g \ldots u, v, w$, to denote the 23 *covariants* of the quintic, then a, b, c, d, f are connected by the relation $f^2 = -a^3d + a^2bc - 4c^3$; and the form contains these covariants thus connected together, and also e; it in fact is ... $(1, 0, c, f, a^2b - 3c^2, a^2e - 2cf)(x, y)^5$.

But the whole plan of the memoir was changed by Sylvester's discovery of what I term the Numerical Generating Function (N.G.F.) of the covariants of the quintic, and my own subsequent establishment of the Real Generating Function (R.G.F.) of the same covariants. The effect of this was to enable me to establish for any given degree in the coefficients and order in the variables, or, as it is convenient to express it, for any given deg-order whatever, a selected system of powers and products of the covariants, say a system of "segregates;" these are asyzygetic, that is, not connected together by any linear equation with numerical coefficients; and they are also such that every other combination of covariants of the same deg-order, say, every "congregate" of the same deg-order, can be expressed (and that, obviously, in one way only) as a linear function, with numerical coefficients, of the segregates of that deg-order. The number of congregates of a given deg-order is precisely equal to the number of the independent syzygies of the same deg-order, so that these syzygies give in effect the congregates in terms of the segregates: and the proper form in which to exhibit the syzygies is then to make each of them give a single congregate in terms of the segregates, viz., the left hand side can always be taken to be a monomial congregate $a^{\alpha} b^{\beta} \ldots$, or, to avoid fractions, a numerical multiple of such form, and the right hand

side will then be a linear function, with numerical coefficients, of the segregates of the same deg-order. Supposing such a system of syzygies obtained for a given deg-order, any covariant function (rational and integral function of covariants) is at once expressible as a linear function of the segregates of that deg-order; it is in fact only necessary to substitute therein, for every monomial congregate, its value as a linear function of the segregates. Using the word covariant in its most general sense, the general conclusion thus is that every covariant can be expressed, and that in one way only, as a linear function of segregates, or, say, in the segregate form.

Reverting to the theory of the canonical form, and attending to the relation $f^2 = -a^3d + a^2bc - 4c^3$, it thereby appears that every covariant multiplied by a power of the quintic itself, a, can be expressed, and that in one way only, as a rational and integral function of the covariants, a, b, c, d, e, f, linear as regards f; say, every covariant, multiplied by a power of a, can be expressed, and that in one way only, in the "standard" form. As an illustration, we may take $a^2h = 6acd + 4bc^2 + ef$. Conversely, an expression of the standard form, that is, a rational and integral function of a, b, c, d, e, f, linear as regards f, not explicitly divisible by a, may very well be really divisible by a power of a (the expression of the quotient, of course containing one or more of the higher covariants, g, h, &c.), and we say that, in this case, the expression is "divisible," and has for its "divided" form the quotient expressed as a rational and integral function of covariants. Observe that, in general, the divided form is not perfectly definite, only becoming so when expressed in the before-mentioned segregate form, and that this further reduction ought to be made. There is occasion, however, to consider these divided forms, whether or not thus further reduced, and moreover it sometimes happens that the form presents itself or can be obtained with integer numerical coefficients, while the coefficients of the corresponding segregate form are fractional.

The canonical form is peculiarly convenient for obtaining the expressions of the several derivatives (Gordan's "Uebereinanderschiebungen"), $(a, b)^1$, $(a, b)^2$, &c. (or, as I propose to write them, $ab1$, $ab2$, &c.), which can be formed with two covariants the same or different, as rational and integral functions of the several covariants. It will be recollected that, in Gordan's theory, these derivatives are used in order to establish the system of the 23 covariants, but it seems preferable to have the system of covariants and, by means of them, to obtain the theory of the derivatives.

I mention, at the end of the memoir, two expressions (one or both of them due to Sylvester) for the N.G.F. of a binary sextic.

The several points above adverted to are considered in the memoir; the paragraphs are numbered consecutively with those of the former memoirs upon quantics.