of the above range, more especially for any of which the period is between 24 and 25 days. We find strong evidence of an inequality of considerable magnitude of which the period is 24·00 days, very nearly. We have also found preliminary evidence of the existence of two considerable inequalites of periods not very far from 24·65 and 24·80 days. These two appear to come together in about 11 years, but we cannot yet give the exact time of this.

We have not found a trace of any inequality with a period of 24·25 days.

V. "Some Experiments on Metallic Reflexion." By Sir JOHN CONROY, Bart., M.A. Communicated by Professor G. G. STOKES, Sec. R.S. Received November 18, 1878.

In the experiments made by Sir David Brewster, M. Jamin, Professor Haughton, and others, on the light reflected by polished metallic surfaces, the reflecting surfaces were in contact with air; and, as far as I am aware, the only observations which have been made when the reflecting surfaces were in contact with other media are those by Quincke, an account of which is given in "Poggendorff's Annalen," vol. cxxviii, p. 541, and in the "Jubelband," p. 336. He found that he obtained different values for the principal incidence and principal azimuth, according as the reflecting surface of a film of silver was in contact with air, crown glass, flint glass, water, or turpentine, and that the only connexion between the values of these angles and the refractive index of the medium in which the reflexion took place was, that in general with the same metal, the principal incidence and the principal azimuth became less as the refractive index of the medium increased.

I therefore hope that a short account of some attempts that I have recently made to determine the principal incidence for, and the principal azimuth of, the light reflected by polished surfaces of gold and copper in contact with different media, may be of interest.

The experiments are, I regret to say, incomplete, as, finding that my eyes were beginning to suffer, I thought it best, for the present at least, to discontinue them.

I used a Babinet's goniometer, to the arms of which two tubes containing nicols were attached, a vertical divided circle being fixed at one end of each, so that the position of the nicols could be read by a vernier to 5′. The goniometer had, in addition to the horizontal stage, a vertical one, so arranged that the reflecting surface could be placed in the axis of the instrument; toothed wheels, working into a pinion rotating on an axis fixed in one of the arms of the divided circle, were attached to the vertical stage, the position of which could

be read by a vernier to 15″, and to the telescope arm; the ratio of the wheels to the pinion being such that, on moving the telescope arm, the vertical stage also moved in the same direction, but with half the angular velocity; so that when the reflecting surface had been properly adjusted, the light which passed along the axis of the tube fixed to the collimator arm, was reflected along the axis of that fixed to the telescope arm, at all angles of incidence.

A quarter undulation plate was placed at the inner end of the tube fixed to the collimator arm, and a small direct vision spectroscope, with a photographic scale, could be attached to the other tube. The lower edge of the vertical stage being some distance above the graduated circle of the goniometer, a cylindrical vessel of thin glass, about 6 centims. in diameter, could be placed on the horizontal stage, so as to surround the lower part of the reflecting surface; this being filled with the liquid, and a narrow vertical slit placed so as to limit the incident light, fairly good observations could be made when the reflecting surface was in contact with various liquids.

When the principal section of the first or polarizing nicol was inclined at an angle of 45° to the plane of incidence, and one of the neutral axes of the quarter undulation plate placed in that plane, the transmitted light was elliptically polarized; and at a particular incidence, varying with its refrangibility, it was reflected by the metallic surface as plane polarized light; the plane of polarization being determined by the second nicol.

Had the retarding plate really been "a quarter undulation plate" for light of any given wave-length, the angle of incidence at which it was reflected as plane polarized light, and the azimuth of its plane of polarization, would have been the principal incidence and principal azimuth for light of that refrangibility.

The retardation of a given plate varies so much for different portions of the spectrum, that even had it been possible to obtain one producing a retardation of exactly 90° for light of any definite refrangibility, it would have differed greatly from a quarter plate for other portions of the spectrum.

Both the neutral axes of the plate were successively placed in the plane of incidence, and the mean of the two values of the angle of incidence taken as the principal incidence.

This arrangement is very similar to the one used by Dr. Eilhard Wiedemann in his observations on the light reflected by surfaces of fuchsine and copper, and described in "Pogg. Ann.," vol. cli, p. 6. In Dr. Wiedemann's experiments the angle of incidence remained constant, the position of the quarter undulation plate and of the nicol being varied; whilst in mine, the position of the quarter undulation plate was constant, and the angle of incidence and the position of the nicol were altered. By this means the principal incidence and azimuth

were determined directly, but less accurately than by Dr. Wiedemann's arrangement, which, however, involves a good deal of calculation.

The analysing portion of Dr. Wiedemann's instrument appears to differ merely by the addition of a small direct vision spectroscope from the Elliptic Analyser of Professor Stokes, described in the "Report of the British Association for 1851," Part II, p. 14.

The experiments were made with sun-light and with lamp-light; with the former, the angle of incidence and the azimuth of the analysing nicol were altered till the dark band in the spectrum was most intense at certain definite positions, as measured by the scale of the spectroscope; with the latter, till the light which had passed through a piece of red glass was reduced to a minimum.

Numerous measurements were made of both these angles with a plate of gold in air, water, carbon bisulphide, and carbon tetrachloride; and of copper, in air, water, and carbon tetrachloride; but the results were not very satisfactory. In addition to the difficulty of determining accurately the zero of the nicols, and of placing the neutral axis of the quarter undulation plate in the plane of incidence, I found that very different values were obtained for the principal incidence, according as one or other of the neutral axes of the quarter undulation plate I was using was in the plane of incidence.

In all cases, however, the principal incidence which, as is well known, is less for the more refrangible rays, diminishes, and the principal azimuth increases with the increase of the refractive index of the medium in contact with the metallic surface; and further, the diminution in the value of the principal incidence appears to be nearly in proportion to the increase of the refractive index of the surrounding medium.

The decrease of the principal incidence, with an increase in the refractive index of the surrounding medium, is exactly what might be expected to take place if the principal incidence for a metal were the same as the angle of polarization of a transparent substance; that is, the angle whose tangent is equal to the refractive index.

If such is the case, the metals must all have very high refractive indices; but some experiments of Quincke's ("Pogg. Ann.," vol. cxix, p. 379, and vol. cxx, p. 602) appear to show that their refractive indices are less than 1.

The following are some results I obtained with a gold plate (formed by soldering a slip of thin sheet gold to a brass plate), in air, with lamp-light, a deep red glass being interposed; the position of the quarter undulation plate in which the ray polarized perpendicularly to the plane of incidence was retarded relatively to the other, being called A, and that in which the retarded ray was the one polarized in the plane of incidence, B. The signs of the azimuth of the plane of

polarization of the reflected light show which ray is retarded by the plate; and, to confirm this, the light transmitted by the nicol and plate was examined with an Iceland spar, cut to show the rings.

The azimuths are reckoned as positive when measured from the plane of incidence in the direction in which the hands of a watch move, to a person supposed to be so placed as to receive the light, whether incident or reflected, into his eye.

Quarter Undulation Plate at A.

Plane of polarization of incident light.	Principal incidence.	Principal azimuth.
°	° ′	° ′
+45	80 10	+36 05
	80 21	36 35
	80 14	35 25
	80 23	36 05
−45	80 14	−36 50
	79 50	34 55
	80 05	35 40
	80 27	35 25
Mean	80 13	35 52

Quarter Undulation Plate at B.

Plane of polarization of incident light.	Principal incidence.	Principal azimuth.
°	° ′	° ′
+45	71 12	−34 40
	71 03	35 25
	70 51	34 25
	71 02	34 30
−45	71 0	+37 30
	70 22	36 35
	70 07	36 35
	70 22	37 55
Mean	70 45	35 57

Similar measurements were made when the gold plate was in water and carbon bisulphide. The values of the incidences differed greatly according as one or other of the neutral axes of the quarter undulation plate was in the plane of incidence, the measurements being about as concordant as those made with the gold plate in air; the means of these determinations were taken as nearest the truth.

Since the retardation of the ray polarized perpendicularly to the

plane of incidence probably varies more for each degree when the light is incident at an angle greater than that of the principal incidence, than when it falls on the surface at a less angle, the mean of these two sets of determinations can only be considered as an approximation to the truth, especially when, as in this instance, the difference between the two values is a considerable one.

Mean value, from eight observations, four with the P. S. of the polarising nicol inclined to the right and four with it to the left of the plane of incidence, of the principal incidence and principal azimuth, for red light, with Quarter Undulation Plate 1.

Gold in air	Plate at A..	80° 13′	....	35° 52′
	„ B..	70 45	....	35 57
	Mean value.	75 29	....	35 54
Gold in water	Plate at A..	76 46	....	37 13
	„ B..	66 46	....	35 50
	Mean value.	71 46	....	36 31
Gold in carbon bisulphide.	Plate at A..	76 10	....	37 48
	„ B..	62 44	....	37 40
	Mean value.	69 27	....	37 44

The principal incidence and principal azimuth for gold in air, with red light, were determined with six other quarter undulation plates with the following results; the numbers for Plate 1 being the mean of eight observations, whilst those of the remainder are the mean of two only, made with the polarizing nicol on either side of the plane of incidence:—

		Plate 1.	Plate 2.	Plate 3.	Plate 4.
Principal incidence	Plate at A...	80° 13′	78° 45′	80° 01′	79° 57′
	„ B...	70 45	70 12	69 41	70 46
	Mean value..	75 29	74 28	74 51	75 21
Principal azimuth	Plate at A...	35 52	36 30	36 12	36 22
	„ B...	35 57	35 40	36 02	35 10
	Mean value..	35 54	36 05	36 07	35 46

		Plate 5.	Plate 6.	Plate 7.
Principal incidence	Plate at A....	79° 57′	77° 43′	77° 50′
	„ B....	69 56	73 26	73 23
	Mean value...	74 56	75 34	75 36
Principal azimuth	Plate at A....	36 0	34 55	34 52
	„ B....	35 22	35 10	34 42
	Mean value...	35 41	35 02	34 47

In order to ascertain the probable error of the mean principal incidence and azimuth as determined with Plate 1, the measurements were repeated with Plate 6; the difference between the two values of the principal incidence, according as one or other of the neutral axes of the plate was in the plane of incidence, being least, and therefore the retardation for red light differing least from 90° for Plates 6 and 7.

Quarter Undulation Plate at A.

Plane of polarization of incident light.		Principal incidence.		Principal azimuth.
+45°	....	78° 25′	....	+35° 0′
		77 58	....	35 35
		77 42	....	35 05
		78 26	....	35 40
−45	....	78 24	....	−36 45
		77 47	....	35 30
		77 47	....	36 15
		77 59	....	36 0
Mean..........		78 03	....	35 43

Quarter Undulation Plate at B.

+45	....	73 57	....	−34 40
		73 17	....	36 10
		73 52	....	35 40
		73 35	....	35 20
−45	....	74 30	....	+35 20
		73 05	....	36 05
		73 37	....	34 35
		74 07	....	35 0
Mean..........		73 45		35 21

Similar measurements, which were about as concordant, were made with the gold plate in water and carbon bisulphide. The numbers in the table being the means of eight observations, four with the principal section of the polarizing nicol inclined to the right, and four with it to the left of the plane of incidence.

Gold in air.............	Plate at A..	78° 03′	....	35° 43′
	„ B..	73 45	....	35 21
	Mean value.	75 54	....	35 32

Gold in water	Plate at A..	74° 46′	....	36° 0′
	„ B..	70 25	....	36 49
	Mean value.	72 35	....	36 24
Gold in carbon bisulphide	Plate at A..	71 37	....	36 59
	„ B..	68 26	....	36 43
	Mean value.	70 01	....	36 51

The mean values of the principal incidence and principal azimuth obtained with the two quarter undulation plates being different, it was assumed that the errors of the means are as the squares of the small errors of the plates, and that the errors of the incidences in either position of the plate, and therefore the algebraical differences or numerical sums of the errors in the two positions, that is, the differences of the apparent principal incidence in the two positions, as the first powers; and therefore that the errors of the means are as the squares of the difference of incidences in the two positions.

Gold in Air.

	Plate 1.	Plate 6.	Correction.
Principal incidence	75° 29′	75° 54′	+25′
„ azimuth	35 54	35 32	−22
Difference of principal incidence in two positions	9° 28′ or 568′,	4° 18′ or 258′.	

Thus the residual corrections to the results got with Plate 6 will be to the difference on the results got by Plate 1 and Plate 6, as 258^2 to 568^2-258^2, or as No. log $\bar{1}\cdot41491$ to 1; this gives $+6'$ and $-5'$ making the corrected principal incidence and principal azimuth 76° and 35° 27′.

In a similar manner the means of the results got with the gold plate in water and carbon bisulphide were corrected, the final results being with red light.

	Principal incidence.		Principal azimuth.
Gold in air	76° 0′	....	35° 27′
„ water	72 46	....	36 23
„ carbon bisulphide	70 03	....	36 48

In order to determine the principal incidence and azimuth for gold by an independent method, the one originally used by Sir David Brewster was adopted; the quarter undulation plate was removed, and a second gold plate attached to the vertical stage in such a manner that, whilst the plates remained parallel to each other, the distance between them could be altered. The plates were so adjusted that

when the light was incident upon the surface of the first at angle of about 70°, it was reflected once by either plate.

The incident light being polarized in a plane inclined at an angle of 45° to the plane of incidence, the position of the stage and of the analysing nicol were altered till the reflected light was reduced to a minimum.

Plane of polarization of incident light.		Principal incidence.		Azimuth.
°		° ′		° ′
+45	....	75 45	....	−30 20
		75 52	....	31 30
		75 23	....	31 05
		76 02	....	31 55
−45	....	75 54	....	+29 30
		76 0	....	29 50
		75 57	....	29 05
		76 03	....	28 45
Mean..............		75 52	....	30 15

A rectangular glass trough was placed on the horizontal stage of the goniometer so as to surround the gold plates; the trough filled with water, and the principal incidence and the azimuth observed.

The ray of light which had been twice reflected by the plates being parallel to the incident ray, and the trough having been placed with its front perpendicular to the direction of the incident light, the polarization of the ray could not be altered in any way by the glass, as indeed was verified by experiment.

The light having been twice reflected, the square root of the tangent of the angle which the plane of polarization of the reflected ray makes with the plane of incidence, is equal to the tangent of the principal azimuth.

The principal incidence and principal azimuth determined by this method from eight observations, four with the plane of polarization of the incident light on either side of the plane of incidence are—

	° ′		° ′
Gold in air.........	75 52	....	37 22
,, water......	72 28	....	37 48

The principal incidences agree fairly well with those obtained by the other method; but the azimuths are somewhat higher.

The following table contains the values of these constants for gold in air, as previously determined by—

Sir David Brewster.	° ′		° ′	
("Optics," ed. 1853, p. 309, 311)..	70 45	..	33 0	for jewellers' gold.
Professor Haughton.				
("Phil. Trans.," 1863, p. 81).....	75 37	..	47 47	
G. Quincke.				
("Pogg. Jubelband," p. 336).....	72 47	..	43 12	for C line.

Assuming that the tangent of the angle of principal incidence is the index of refraction of the metal for red light, the value of that angle in air, as deduced from the measurements made in water and carbon bisulphide with the quarter undulation plates, is 76·53 and 77·22 instead of 76°.

The numbers given by Quincke ("Pogg. Ann.," vol. cxxviii, p. 541) for silver are—

	Principal incidence.		Principal azimuth.
	° ′		° ′
In air.............	74 19	....	43 48
In water..........	71 28	....	44 03
In turpentine......	69 16	....	43 21

The value for the principal incidence in air calculated according to the same assumption, by multiplying the tangent of the principal incidences in water and turpentine by the refractive indices of these substances, is 75° 55′ and 75° 36′ instead of 74° 19′; in all four cases the value is too high.

Although more experiments are required to decide this point, it seems probable that this relationship between these numbers is not merely an accidental one; and if so that there is additional reason for adhering to Sir David Brewster's opinion that the value of the angle of principal incidence may be taken as indicating the refractive power of a metal.

In conclusion, I must express my thanks to Professor Stokes for much advice and assistance, and specially for pointing out the method for determining the residual corrections to the results obtained with the quarter undulation plates.

January 16, 1879.

W. SPOTTISWOODE, M.A., D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—