$$\frac{\cos\frac{1}{2}(\widehat{\alpha\beta})}{\sqrt{\alpha\beta}}+\frac{\cos\frac{1}{2}(\widehat{\beta\gamma})}{\sqrt{\beta\gamma}}+\frac{\cos\frac{1}{2}(\widehat{\gamma\delta})}{\sqrt{\gamma\delta}}+\ \&c.,$$

$$+\frac{\cos\frac{1}{2}(\omega\alpha)}{\sqrt{\omega\alpha}}=0."$$

I have examined this equation at length in the special case $n=4$. The tetrazomal curve, which is of the eighth degree, breaks up into two factors. One factor represents the circle and the square of the line gaining the points (γ), $(\beta\delta)$, the other denotes an unicursal quartic whose three double points are $(\alpha\gamma)$ $(\beta\delta)$, and the pole with respect to the circle of the line joining these points.

These results hold for the plane and sphere, and α, β, γ, δ, &c., may denote circles as well as lines. They are also true for conics having double contact with a given conic.

Besides the foregoing, which are manifestly extensions of known theorems, the paper contains some original theorems which are also extended to conics having double contact with a given conic. Thus, "if a circle Σ be touched by any number of circles S_1 S_2 S_3, &c., and if we denote by $P(i)$ the product of the common tangents drawn from any circle S_i of the system to all the remaining circles, then the equations of Σ will be a factor in the polyzomal curve,

$$\frac{\sqrt{S_1^{n-2}}}{P(1)}-\frac{\sqrt{S_2^{n-2}}}{P(2)}+\frac{\sqrt{S_3^{n-2}}}{P(3)}-\ \&c.\ =0.$$

This theorem is, I believe, one of the most fertile in geometry. The paper contains a large number of deductions from it, but I am certain it is far from exhausting them.

V. "On the Bodily Tides of Viscous and Semi-Elastic Spheroids, and on the Ocean Tides on a yielding nucleus." By George H. Darwin, M.A., Fellow of Trinity College, Cambridge. Communicated by J. W. L. Glaisher, M.A., F.R.S. Received May 14, 1878.

(Abstract.)

Sir W. Thomson's investigation of the bodily tides of an elastic sphere* has gone far to overthrow the idea of a semi-fluid interior to the earth, yet geologists are so strongly impressed by the fact that enormous masses of rock have been poured out of volcanic vents in

* Phil. Trans., 1863, p. 573, and Thomson and Tait's Natural Philosophy, edit. of 1867, §§ 733–737 and 834–846.

the earth's surface, that the belief is not yet extinct that we live on a thin shell over a sea of molten lava. It appeared to me, therefore, to be of interest to investigate the consequences which would arise from the supposition that the matter constituting the earth is of a viscous or imperfectly elastic nature. In this paper I follow out these hypotheses, and it will be seen that the results are fully as hostile to the idea of any great mobility of the interior of the earth as are those of Sir W. Thomson.

I begin by showing that the equations of flow of an incompressible viscous fluid have precisely the same form as those of strain of an incompressible elastic solid, at least when inertia is neglected. Hence, every problem about the strains of the latter has its analogue touching the flow of the former. This being so, the solution of Sir W. Thomson's problem of the bodily tides of an elastic sphere may be adapted to give the bodily tides of a viscous spheroid. Sir W. Thomson, however, introduces the effects of the mutual gravitation of the parts of the sphere, by a synthetical method, after he has found the state of internal strain of an elastic sphere devoid of gravitational power The parallel synthetical method becomes, in the case of the viscous spheroid, somewhat complex, and I have preferred to adapt the solution analytically so as to include gravitation.

The solution is only applicable when the disturbing potential is capable of expansion as a series of solid harmonics, and it appears that each harmonic term in the potential acts independently of all others; it is thus only necessary to consider a typical term in the potential.

It is shown finally that if ρ, ϖ, ν, be the velocities of the fluid (at a point whose polar co-ordinates in the sphere are r, θ, ϕ,) radially, and along and perpendicular to the meridian; μ the coefficient of viscosity; $wr^i\mathrm{S}_i$ the disturbing potential, a solid harmonic of degree i; a the mean radius, w the density of the homogeneous spheroid, and g gravity; $r = a + \sigma_i$ the equation to the bounding surface of the spheroid; then

$$\rho=\frac{i^2(i+2)a^2-i(i^2-1)r^2}{2(i-1)[2(i+1)^2+1]\mu}r^{i-1}\mathrm{T}_i,$$

$$\varpi=\frac{i(i+2)a^2-(i-1)(i+3)r^2}{2(i-1)[2(i+1)^2+1]\mu}\times r^{i-1}\frac{d\mathrm{T}_i}{d\theta},$$

$$\nu=\qquad\text{the same}\qquad\times\frac{r^{i-1}}{\sin\theta}\,\frac{d\mathrm{T}_i}{d\phi},$$

where $\mathrm{T}_i=w\left(\mathrm{S}_i-2g\dfrac{i-1}{2i+1}\,\dfrac{\sigma_i}{a^i}\right)$, a surface harmonic of order i. A differential equation is then found, which gives the form of the free surface at any time under the action of any disturbing potential

which satisfies the condition of expansibility as a series of solid harmonics.

When the disturbing potential $wr^i S_i$ is zero, and when $r=a+s_i$ is the equation to the free surface initially, then the equation to the surface at the time t is given by $r=a+\sigma_i$, where

$$\sigma_i=s_i \text{ exp. } \left(-\frac{gwai}{2(i+1)^2+1}t\right).*$$

This gives the law of the subsidence of inequalities on the surface of a viscous globe under the influence of simple gravitation; and it is suggested that some light may possibly be thrown thereby on the laws of geological subsidence and upheaval. It appears from this formula that inequalities of wide extent will subside much more quickly than wrinkles.

The rate is found at which a rotating spheroid would adjust itself to a new form of equilibrium, when its axis of figure is not coincident with that of rotation; and the law is established which was assumed in a former paper.†

The case is next considered where S_i is a surface harmonic of the second order, multiplied by a simple time harmonic—that is to say, $S_i=S\cos(vt+\eta)$. This is the assumption appropriate for the tidal problem. The forces in this case do not form a rigorously equilibrating system; but there is a couple of the second order of small quantities called into existence, the consideration of which is deferred to a future paper.

It is then shown that if $wr^2 S\cos(vt+\eta)$ be a term in the tide-generating potential, and if $\tan\epsilon=\frac{19\mu v}{2gwa}$, the tide of the viscous spheroid is equal in height to the equilibrium tide of a perfectly fluid spheroid multiplied by $\cos\epsilon$, and the tide is retarded by $\epsilon\div v$. It is next proved that the equilibrium tide of a shallow ocean overlying the nucleus is equal to the like tide on a rigid nucleus multiplied by $\sin\epsilon$, and that there is an acceleration of the time of high water equal to $\frac{\pi}{2v}-\frac{\epsilon}{v}$.

This theory is then applied to the lunar semi-diurnal and fortnightly tides, and tables are given from which the following is extracted—the coefficient of viscosity being expressed in gramme-weights, centimeters, and seconds.

* I write "exp." for "*e* to the power of."

† "On the Influence of Geological Changes on the Earth's Axis of Rotation." Phil. Trans., vol. clxvii, Pt. I, p. 282. I take this opportunity of correcting a slight mistake in that paper; the formula in the fourth line from the bottom of p. 301, should run $\frac{D}{2d\rho hc^4}=\frac{E}{2e\rho hc^4}=-\frac{1}{qc}\cot qc+\&c.$ The mistake arose in copying out the formula, and does not affect the subsequent arithmetical results.

Lunar Semi-diurnal Tide.

Coefficient of viscosity $\times 10^{-10}$.	Retardation of bodily tide.	Height of bodily tide is tide of fluid spheroid multiplied by	Height of ocean tide is tide on rigid nucleus multiplied by	Acceleration of high water of ocean tide.
Fluidity 0	0	1·000	000	3 hrs. 6 min.
96	41 min.	·940	·342	2 hrs. 25 min.
721	2 hr. 25 min.	·342	·940	41 min.
Rigidity ∞	3 hr. 6 min.	·000	1·000	0

Fortnightly Tide.

Coefficient of viscosity $\times 10^{-10}$.	Retardation of bodily tide.	Height of bodily tide is tide of fluid spheroid multiplied by	Height of ocean tide is tide on rigid nucleus multiplied by	Acceleration of high water of ocean tide.
1,200	9 hrs.	·985	·174	3 days 1 hr.
12,000	2 days 6 hrs.	·500	·866	1 day 3 hrs.
Rigidity ∞	3 days 10 hrs.	·000	1·000	0

A comparison of the numbers in the first column with the viscosity of pitch at near the freezing temperature (when I found by rough experiments that its viscosity was about $1·3 \times 10^8$), shows how enormously stiff the earth must be to resist the tidally distorting influence of the moon. It may be remarked that pitch at this temperature is hard, apparently solid and brittle; and if the earth was not very far stiffer than pitch, it would comport itself sensibly like a perfect fluid, and there would be no ocean tides at all. It follows, therefore, that no very considerable portion of the interior of the earth can even distantly approach the fluid condition.

This does not, however, seem conclusive against the existence of bodily tides in the earth of the kind here considered; for, under the enormous pressures which must exist in the interior of the earth, even the solidest substances might be induced to flow to some extent like a fluid of great viscosity.

The theory of the bodily tides of an "elastico-viscous" spheroid is next developed. The kind of imperfection of elasticity considered is where the forces requisite to maintain the body in any strained configuration diminish in geometrical progression, as the time increases in arithmetical progression. There are two constants which define the mechanical nature of this sort of solid: first, the coefficient of rigidity n, at the instant immediately after the body has been strained; and second, "the modulus of the time of relaxation of rigidity" $\mathfrak{t}$, which is the time in which the force requisite to maintain the body in its strained position has diminished to e^{-1} or ·368 of its initial value. I am not aware that there is any experimental justification for the assumption of such a law; but after considering the various physical

objections which may be raised to it, I came to the conclusion that the investigation was still of some value.

The equations of flow of such an ideal solid have been given (with some assistance from Professor Maxwell) by Mr. Butcher* and they are such that, if the body be incompressible, and if inertia be neglected, they may be written in exactly the same form as the equations of flow of a purely viscous fluid, the coefficient $n\left(\frac{1}{\mathfrak{t}}+\frac{d}{dt}\right)^{-1}$ merely replacing the coefficient of viscosity. Hence it follows that the solution previously found may be at once adapted to the new hypothesis.

In the application to the tidal problem, if the tide-generating potential be as before, $wr^2\mathrm{S}\cos(vt+\eta)$, and if $\tan\psi=v\mathfrak{t}$, $\tan\chi=v\mathfrak{t}\left(1+\frac{19n}{2gwa}\right)$, and $\tan\epsilon=\tan\chi-\tan\psi$, it appears that the bodily tide raised by this potential is equal to the corresponding tide of a perfectly fluid spheroid multiplied by $\frac{\cos\chi}{\cos\psi}$, and the tide is retarded by a time $\frac{\chi-\psi}{v}$. Also the equilibrium tide of a shallow ocean overlying the elastico-viscous nucleus is equal to the corresponding tide on a rigid nucleus multiplied by $\cos\chi\tan\epsilon$, and there is an acceleration of the time of high water equal to $\frac{\pi}{2v}-\frac{\chi}{v}$.

If $\mathfrak{t}$ be taken as zero, whilst n is infinite, but $n\mathfrak{t}$ (the coefficient of viscosity) finite, the solution becomes that already found for a purely viscous spheroid. If, on the other hand, $\mathfrak{t}$ be infinite, the solution is that of Sir W. Thomson's problem of the purely elastic sphere. This hypothesis is therefore intermediate between those of pure viscosity and pure elasticity.

Sir William Thomson worked out numerically the bodily tides of elastic spheres with the rigidities of glass and of iron; and tables of results are here given for those rigidities, with various times of relaxation of rigidity, for the semi-diurnal and fortnightly tides.

It appears that if the time of relaxation of rigidity is about one quarter of the tidal period, then the reduction of ocean tide does not differ much from what it would be if the spheroid were perfectly elastic. The acceleration of high tide, however, still remains considerable; and a like observation may be made in the case of pure viscosity approaching rigidity. This leads me to think that one of the most promising ways of detecting such tides in the earth, would be by the determination of the periods of maximum and minimum in a tide of long period in a high latitude. But I am unfortunately unacquainted with practical tidal observation, and therefore cannot tell how far it would be possible to carry out this suggestion.

* Proc. Lond. Math. Soc., Dec. 14, 1876, pp. 107–9.

It is then shown that the effects of inertia, which had been neglected in finding the laws of the tidal movements, cannot be such as to materially affect the accuracy of the results.

In the first part of this paper I followed Sir W. Thomson in using the equilibrium theory for the determination of the amount of reduction of ocean tides. But that theory is acknowledged on all hands to be very faulty in its explanation of tides of short period; hence a dynamical investigation of the effects of a bodily yielding of the earth on a tide of short period in a shallow equatorial canal appeared likely to be interesting. This investigation is carried out in the second part of the paper. The problem is simplified by supposing the circular canal developed into a straight canal, whose bottom is constrained to execute a simple harmonic wave motion.

The result shows that the height of the ocean tide relatively to the nucleus bears the same relation to the height of tide on a rigid nucleus as in the equilibrium theory, and that the alteration of phase is the same. This seems to increase the force of Sir W. Thomson's argument as to the rigidity of the earth.

The chief practical result of this paper may be summed up by saying, that it is strongly confirmatory of the view that the earth has a very great effective rigidity; but its chief value is, that it forms a necessary first chapter to the investigation of the precession of viscous and imperfectly elastic spheroids—an investigation which I hope to complete very shortly.

VI. "On the Formation of Chlor-iodide and Brom-iodide of Ethylidene." By Dr. MAXWELL SIMPSON, F.R.S., Professor of Chemistry, Queen's College, Cork. Received May 7, 1878.

(Preliminary Notice.)

Chlor-iodide of ethylidene $\begin{matrix} CH_3 \\ | \\ CHClI \end{matrix}$. This body I have succeeded in preparing by two processes.

First Process.—A quantity of iodide of ethylidene, which had been prepared by Gustavson's method, and heated to 160° C. but not distilled, was vigorously agitated for some time with a weak solution of chloride of iodine without the application of heat. The excess of chloride was then poured off, and the product well washed with dilute potash and distilled. Almost the entire quantity passed over between 110 and 150° C. This yielded, on fractioning, a large quantity of fluid boiling between 116 and 120° C., most between 117 and 119.° This was the body in question. The chloride of iodine used in this process was