has at hand, and which are obtained from the cinders of a copper-furnace. The cavities when thus filled up are well rubbed so as to escape notice, but they may usually be detected by looking at the mirror obliquely.

It was perhaps the presence of these bits of copper in the mirror which Ou-tseu-hing saw broken up in the 13th century, that misled him into concluding that the phenomenon of the magic mirror was produced by the inlaying of denser copper in a portion of the face exactly corresponding with the design on the back.

When the face of the mirror has been made quite smooth, an amalgam consisting, according to the Tokio makers, of half tin and half mercury, with perhaps a trace of lead, or of

Tin	69·36	per cent.,
Mercury	30	,,
Lead	0·64	,,

according to the analysis of MM. Champion and Pellet ("Industries de l'Empire Chinois") is rubbed over the surface with a stiff straw brush or with the hand. The mirror is finally wiped clean with a soft kind of paper, *mino-gami*, "paper from the province Mino," which is considered to scratch the surface less than silk. Leather was formerly never employed in polishing, as it would have been considered impious to pollute so holy a thing as a mirror by touching it with the skin of an animal; for under the old feudal system in Japan, workers in skins, saddlers, and others, belonged to the Eta or pariah class.

When mirrors possessed by private people require brightening up, in consequence of the surface tarnishing, the paste produced when razors are sharpened on a hone is usually rubbed over the face of the mirror.

III. "On the Torsional Strain which remains in a Glass Fibre after release from Twisting Stress." By J. Hopkinson, D.Sc., F.R.S. Received October 4, 1878.

It has long been known that if a wire of metal or fibre of glass be for a time twisted, and be then released, it will not at once return to its initial position, but will exhibit a gradually decreasing torsion in the direction of the impressed twist. The subject has undergone a good deal of investigation, especially in Germany. The best method of approximating to an expression of the facts has been given by Boltzmann ("Akad. der Wissensch. Wien," 1874). He rests his theory upon the assumption that a stress acting for a short time will

leave after it has ceased a strain which decreases in amount as time elapses, and that the principle of superposition is applicable to these strains, that is to say, that we may add the after-effects of stresses, whether simultaneous or successive. Boltzmann also finds that, if $\phi(t)\tau$ be the strain at time t resulting from a twist lasting a very short time τ, at time $t=0$, $\phi(t)=\frac{A}{t}$, where A is constant for moderate values of t, but decreases when t is very large or very small. A year ago I made a few experiments on a glass fibre which showed a deviation from Boltzmann's law. A paper on this subject by Kohlrausch ("Pogg. Ann.," 1876) suggested using the results of these experiments to examine how Boltzmann's law must be modified to express them. Professor Kohlrausch's results indicate that in the cases of silver wire and of fibre of caoutchouc Boltzmann's principle of superposition is only approximate, and that in the case of a short duration of twisting $\phi(t)=\frac{A}{t^a}$, where a is less than unity; in case of a long duration of twisting he uses other formulæ, which pretty successfully express his results, owing in part no doubt to the fact that in most cases each determination of the constants applies only to the results of one duration of twisting. In a case like the present it appears best to adopt a simple form involving constants for the *material* only, and then see in what way it fails to express the varying conditions of experiment. In 1865 Sir W. Thomson published ("Proceedings of the Royal Society") the results of some experiments on the viscosity of metals, the method being to determine the rate at which the amplitude of torsional vibrations subsided. One of the results was that if the wire were kept vibrating for some time it exhibited much greater viscosity than when it had long been quiescent. This should guard us from expecting to attain great uniformity in experiments so roughly conducted as those of the present paper.

2. The glass fibre examined was about 20 inches in length. Its diameter, which might vary somewhat from point to point, was not measured. The glass from which it was drawn was composed of silica, soda, and lime; in fact, was glass No. 1 of my paper on "Residual Charge of the Leyden Jar" ("Phil. Trans.," 1877. In all cases the twist given was one complete revolution. The deflection at any time was determined by the position on a scale of the image of a wire before a lamp, formed by reflection from a light concave mirror, as in Sir W. Thomson's galvanometers and quadrant electrometer. The extremities of the fibre were held in clamps of cork; in the first attempts the upper clamp was not disturbed during the experiment, and the upper extremity of the fibre was assumed to be fixed; the mirror also was attached to the lower clamp. This arrangement was unsatisfactory, as one could not be certain that a part of the

observed after-effect was not due to the fibre twisting within the clamps and then sticking. The difficulty was easily avoided by employing two mirrors, each cemented at a single point to the glass fibre itself, one just below the upper clamp, the other just above the lower clamp. The upper mirror merely served by means of a subsidiary lamp and scale to bring back the part of the fibre to which it was attached to its initial position. The motion of the lower clamp was damped by attaching to it a vane dipping into a vessel of oil. The temperature of the room when the experiments were tried ranged from 13° C. to 13·8° C., and for the present purpose may be regarded as constant. The lower or reading scale had forty divisions to the inch, and was distant from the glass fibre and mirror $38\frac{3}{4}$ inches, excepting in Experiment V, when it was at $37\frac{1}{2}$ inches. Sufficient time elapsed between the experiments to allow all sign of change due to after-effect of torsion to disappear. In all cases the first line of the table gives the time in minutes from release from torsion, the second the deflection of the image from its initial position in scale divisions.

Experiment I.—The twisting lasted 1 minute.

t	1	2	3	4	5	7	10	17	25
Scale divisions ..	22	13	9	7	$5\frac{1}{2}$	4	3	2	1

Experiment II.—The twisting lasted 2 minutes.

t	1	2	3	4	5	7	10	20	40
Scale divisions..	38	25	18	15	13	10	8	$4\frac{1}{2}$	$3\frac{1}{2}$

Experiment III.—Twisted for 5 minutes.

t	1	2	3	4	5	7
Scale divisions	64	51	$41\frac{1}{2}$	$35\frac{1}{2}$	32	$26\frac{1}{2}$
....	10	15	22	58	15	
Scale divisions	$21\frac{1}{2}$	17	14	7	2	

Experiment IV.—Twisted for 10 minutes.

t	$\frac{1}{2}$	1	2	3	4	7	10
Scale divisions	106	85	66	57	$49\frac{1}{2}$	$37\frac{1}{2}$	31
t	15	25	45	120	170		
Scale divisions	$24\frac{1}{2}$	18	13	7	6		

Experiment V.—Twisted for 20 minutes.

t..................	1	2	3	4	5	7	10
Scale divisions	110	89	75	68	$61\frac{1}{2}$	52	44
t..................	15	25	40	60	80	100	
Scale divisions......	$35\frac{1}{2}$	$26\frac{1}{2}$	21	18	$13\frac{1}{2}$	$12\frac{1}{2}$	

Experiment VI.—Twisted for 121 minutes.

t	$\frac{1}{2}$	1	2	3	4	5	7
Scale divisions .	191	170	148	136	$126\frac{1}{2}$	$119\frac{1}{2}$	$108\frac{1}{2}$
t	10	15	30	65	90	120	589
Scale divisions .	97	$84\frac{1}{2}$	$63\frac{1}{2}$	$41\frac{1}{2}$	34	28	$3\frac{1}{2}$

It should be mentioned that the operation of putting on the twist and of releasing each occupied about two seconds, and was performed half in the second before the epoch $t = 0$, and half in the second after or as nearly so as could be managed. The time was taken by ear from a clock beating seconds very distinctly.

3. The first point to be ascertained from these results is whether or not the principle of superposition, assumed by Boltzmann, holds for torsions of the magnitude here used.

If the fibre be twisted for time T through angle X, then the torsion at time t after release will be X $\{\psi(\mathrm{T}+t)-\psi(t)\}$ where

$$\psi(t) = \int\phi(t)\, dt.$$

If now $\mathrm{T} = t_1 + t_2 + t_3 + \ldots$ we may express the effect of one long twist in terms of several shorter twists by simply noticing that

$$\mathrm{X}\{\psi(t)-\psi(t+\mathrm{T})\} = \mathrm{X}[\{\psi(t)-\psi(t+t_1)\}+\{\psi(t+t_1)-\psi(t+t_1+t_2)\} + \{\psi(t+t_1+t_2)-\psi(t+t_1+t_2+t_3)\}+, \&c.]$$

Apply this to the preceding results, calculating each experment from its predecessor. Let x_t be the value of $\psi(\mathrm{T}+t)-\psi(t)$, that is, the torsion at time t, when free, divided by the impressed twist measured in same unit; we obtain the following five tables of comparison.

Results for T=2 compared with those from T=1.

t	1	2	3	4	5	7
x_t observed....	0·00195	128	092	077	066	051
x_t calculated...	0·00199	112	082	064	051	040
t	10	20	40			
x_t observed....	041	023	018			
x_t calculated ..	029	016				

Results for T=5 compared with those from T=2 and T=1.

t	1	2	3	4	5	7	10
x_t observed....	0·00328	262	212	182	164	136	110
x_t calculated...	0·00323	233	181	156	136	108	193
t	15	22	58	151			
x_t observed....	087	072	036	010			
x_t calculated...	066	047					

Results for T=10 compared with those from T=5.

t	$\frac{1}{2}$	1	2	3	4	7	10
x_t observed....	0·00544	435	338	292	253	192	159
x_t calculated...	..	469	398	339	300	236	197

t	15	25	45	120	170
x_t observed....	125	092	067	036	031
x_t calculated...	161	130	088		

Results for T=20 compared with those from T=10.

t	1	2	3	4	5	7	10
x_t observed....	0·00580	470	398	358	327	276	234
x_t calculated...	0·00587	483	430	384	356	312	266

t	15	25	40	60	80	100
x_t observed....	188	140	111	085	072	066
x_t calculated...	217	167	135	100	084	

Results for T=121 compared with those from T=20.

t	$\frac{1}{2}$	1	2	3	4	5	7
x_t observed...	0·00979	871	758	697	648	612	556
x_t calculated..	..	1070	950	880	830	780	730

t	10	15	30	65	90	120	589
x_t observed...	497	433	325	212	174	144	18
x_t calculated..	670	600	500	380	350		

In examining these results it must be remembered that those for small values of T are much less accurate than when T is greater, for the quantity observed is smaller but is subject to the same absolute error; any irregularity in putting on or releasing from the stress will cause an error which is a material proportion of the observed deflection. For this reason it would be unsafe to base a conclusion on the experiments with T=1 and T=2. The three last tables agree in indicating a large deviation from the principle of superposition, the actual effect being *less* than the sum of the separate effects of the periods o stress into which the actual period may be broken up. Kohlrausch finds the same to be the case for india-rubber, either greater torsions or longer durations give less after-effects than would be expected from smaller torsions and shorter periods.

4. Assuming with Boltzmann that $\phi(t)=\frac{A}{t}$, we have at time t after termination of a twist lasting time T,

$$x_t=A\{\log(T+t)-\log t\},$$

the logarithms being taken to any base we please. The results were

plotted on paper, x_t being the ordinate and $\log \frac{T+t}{t}$ the abscissa; if the law be true we should find the points all lying on a straight line through the origin. For each value for T they do lie on straight lines very nearly for moderate values of t; but if T is not small these lines pass above the origin. When t becomes large the points drop below the straight line in a curve making towards the origin. This deviation appears to indicate the form $\phi(t)=\frac{A}{t^a}$, a being less than, but near to, unity. If $a=0{\cdot}95$ we have a fairly satisfactory formula.

$$x_t=A'\left(\overline{T+t}^{\frac{1}{20}}-t^{\frac{1}{20}}\right),\ \text{where } A'=\frac{A}{t^a} \text{ when } T=121.$$

In the following Table the observed and calculated values of x_t when T=121 are compared, A′ being taken as 0·032.

t	$\frac{1}{2}$	1	2	3	4	5	7
x_t observed....	0·00979	871	758	697	648	612	556
x_t calculated...	0·00976	870	755	691	643	600	550

t	10	15	30	65	90	120	589
x_t observed....	497	433	325	212	174	144	18
x_t calculated...	493	429	320	218	176	147	42

To show the fact that A′ decreases as T increases if a be assumed constant, I add a comparison when T=20, it being then necessary to take A′=0·037.

t	1	2	3	4	5	7	10
x_t observed....	0·00580	470	398	358	327	276	234
x_t calculated...	0·00607	485	422	370	337	285	233

t	15	25	40	60	80	100
x_t observed....	188	140	111	085	072	066
x_t calculated...	185	125	089	067	052	041

A better result would in this case be obtained by assuming $a=0{\cdot}92$, or $=0{\cdot}93$ in the former case with A′=0·021. Probably the best result would be given by taking A constant, and assuming that a increases with T.

Taking the formula $\phi(t)=\frac{A}{t}$ these experiments give values of A ranging from 0·0017 to 0·0022. Boltzmann for a fibre, probably of a quite different composition, gives numbers from which it follows that A=0·0036.

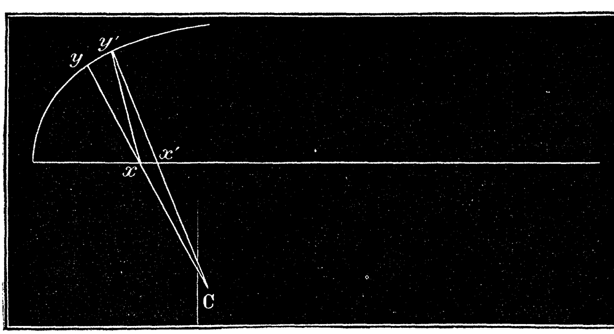
5. In my paper on "Residual Charge of the Leyden Jar" that

subject is discussed in the same manner as Boltzmann discusses the after-effect of torsion on a fibre, and it is worth remarking that the results of my experiments can be roughly expressed by a formula in which $\phi(t)=\frac{A}{t^{\alpha}}$. For glass No. 5 (soft crown) $\alpha=0{\cdot}65$, whilst for No. 7 (light flint) it is greater; but in the electrical experiment no sign of a definite deviation from the law of superposition was detected.

IV. "Note in correction of an Error in the Rev. Dr. Haughton's Paper 'Notes on Physical Geology. No. V" ("Proc. Roy. Soc.," vol. xxvii, p. 447). By the Rev. SAMUEL HAUGHTON, M.D., Professor of Geology in the University of Dublin, F.R.S. Received October 9, 1878.

In my paper read 20th June last, and published in the "Journal of the Royal Society," there is an error in p. 450 which I wish to correct.

Referring to the geometrical proof of Mr. Darwin's theorem, I state that from cusp to cusp of the cycloidal wabble occupies $152\frac{1}{2}$ days; this is an error, as it should be 305 days, as can be shown geometrically.



Let yx, $y'x'$, be two successive positions of the line joining the axes of rotation and figure; produce them to meet at C, which will be the centre of curvature, because yx and $y'x'$, are normals to the cycloidal arc yy'; it is well known that yC, (radius of curvature) is double yx (chord of generating circle) or double $y'x$; therefore the angle yxy' is double the angle yCy'; but yxy' measures the angular velocity of the wabble, when x is supposed at rest; therefore the angular velocity of yx is only half that of the wabble, if the axis of figure were at rest. Hence in 305 days, yx will turn through 180° only, and not 360°.