

PROCEEDINGS

OF

THE ROYAL SOCIETY.

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November 21, 1878.

Sir JOSEPH HOOKER, K.C.S.I., President, in the Chair.

In pursuance of the Statutes, notice was given from the Chair of the ensuing Anniversary Meeting, and the list of Officers and Council proposed for election was read as follows:—

*President.*—William Spottiswoode, M.A., D.C.L., LL.D.

*Treasurer.*—John Evans, F.G.S., F.S.A.

*Secretaries.*— { Professor George Gabriel Stokes, M.A., D.C.L., LL.D.  
 { Professor Thomas Henry Huxley, LL.D.

*Foreign Secretary.*—Alexander William Williamson, Ph.D.

*Other Members of the Council.*—Frederick A. Abel, C.B., V.P.C.S.; William Bowman, F.R.C.S.; William Carruthers, V.P.L.S.; Major-General Henry Clerk, R.A.; William Crookes, V.P.C.S.; Sir William Robert Grove, M.A.; Augustus G. Vernon Harcourt, F.C.S.; Sir Joseph Dalton Hooker, C.B., K.C.S.I., D.C.L.; Admiral Sir Astley Cooper Key, K.C.B.; Lieut.-General Sir Henry Lefroy, C.B.; Lord Lindsay, P.R.A.S.; Sir John Lubbock, Bart., V.P.L.S.; Lord Rayleigh, M.A.; Charles William Siemens, D.C.L.; John Simon, C.B., D.C.L.; Professor Allen Thomson, M.D., F.R.S.E.

The Presents received were laid on the table, and thanks ordered for them.

The Rev. Thomas George Bonney, Dr. John Hughlings Jackson, and Mr. Edward Alfred Schäfer were admitted into the Society.

General Boileau, General Clerk, Mr. J. Evans, Dr. Gladstone, and Mr. Simon having been nominated by the President, were elected by ballot Auditors of the Treasurer's Accounts on the part of the Society.

VOL. XXVIII.

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The following Papers were read:—

- I. "On a method of using the Balance with great delicacy, and on its employment to determine the Mean Density of the Earth." By J. H. POYNTING, B.A., Fellow of Trinity College, Cambridge, Demonstrator in the Physical Laboratory, Owens College, Manchester. Communicated by Professor B. STEWART, F.R.S. Received June 21, 1878.

[PLATE 1.]

In the ease and certainty with which we can determine by the balance a relatively small difference between two large quantities, it probably excels all other scientific instruments.

By the use of agate knife edges and planes, even ordinary chemical balances have been brought to such perfection that they will indicate one-millionth part of the weight in either pan, while the best bullion balances are still more accurate. The greatest degree of accuracy which has yet been attained was probably in Professor Miller's weighings for the construction of the standard pound, and its comparison with the kilogramme, in which he found that the probable error of a single comparison of two kilogrammes, by Gauss's method, was  $\frac{1}{14000000}$ th part of a kilogramme.\* ("Phil. Trans.," 1856.)

But, though the balance is peculiarly well fitted to detect the relatively small differences between large quantities, it has not hitherto been considered so well able to measure absolutely small quantities as the torsion balance. The latter, for instance, was used in the Cavendish experiment, when the force measured by Cavendish was the attraction of a large lead sphere upon a smaller sphere, weighing about  $1\frac{1}{2}$  lbs., the force only amounting to  $\frac{1}{50000000}$ th part of this weight, or about  $\frac{1}{5000}$ th part of a grain.

The two great sources of error, which render the balance inferior to the torsion balance in the measurement of small forces, are:—

1. Greater disturbing effects produced by change of temperature, such as convection currents and an unequal expansion of the two arms of the balance.

2. The errors arising from the raising of the beam on the supporting frame between each weighing, consisting of varying flexure of the beam and inconstancy of the points of contact of the knife edges and planes.

The disturbances due to convection currents interfere with the torsion balance as well as with the ordinary balance, though they are

\* Even so far back as 1787, Count Rumford used a balance which would indicate one in a million and measure one in seven hundred thousand. ("Phil. Trans.," 1739.)

more easily guarded against with the former, by reason of the nature of the experiments usually performed with it. They might, perhaps, as has been suggested by Mr. Crookes, be removed from both by using the instruments in a partial vacuum, in which the pressure is lowered to the "neutral point," where the convection currents cease, but the radiometer effects have not yet begun. But a vacuum balance requires such complicated apparatus to work it, that it is perhaps better to follow the course which Baily adopted in the Cavendish experiment. He sought to remove the disturbing forces as much as possible, and to render those remaining as nearly uniform as possible in their action during a series of experiments, so that they might be detected and eliminated. For this purpose the instrument was placed in a darkened draughtless room, and was protected by a thick wooden casing gilded on its outer surface. Most of the heat radiated from the surrounding bodies was reflected from the surface of the case by the gilding. The heat absorbed only slowly penetrated to the interior, and was so gradual in its action, that, for a considerable time, the effect might be supposed nearly uniform. Under this supposition it was then eliminated by the following method of taking the observations. The resting point (that is the central position of equilibrium, about which the oscillations were taking place) of the torsion rod, at the ends of which were the small attracted weights, was first observed when the two large masses pulled it in one direction. The masses were then moved round to the opposite side, when they pulled the rod in the opposite direction and the resting point was again observed. The masses were then replaced in their original position and the resting point was observed a third time. These three observations were made at equal intervals of time; if, then, the disturbing effect was uniform during the time, the mean of the first and third observations gave what the resting point would have been, had the rod been pulled in that one direction at the same time that it was actually observed when pulled in the opposite direction. The difference between the second resting point and the means of the first and third might, therefore, be considered as due to the attractions of the masses alone.

In the experiments of which this paper contains an account, I have endeavoured to apply this method of introducing time as an element to the ordinary balance. But, before it could be properly applied, it was necessary to remove the errors due to the raising of the beam between successive weighings, as they could not be considered to vary in any uniform way with the time. I think I have effected this satisfactorily, by doing away altogether with the raising of the beam by the supporting frame, between the weighings. For this purpose I have introduced a clamp underneath one of the pans, which the observer can bring into action at any time, to fix that pan in whatever position it may be. The weight can then be removed from the pan,

and another, which is to be compared with it, can be inserted in its place without altering the relative positions of the planes and knife edges. The counterpoise in the other pan, meanwhile, keeps the beam in the same state of flexure. The pan is then unclamped and the new position about which it oscillates is observed. The only changes are due to the change in the weight and the effect of the external disturbing forces; the latter we may consider as proportional to the time, if sufficient precautions have been taken, and by again changing the weights and again observing the position of the balance, we may eliminate their effects.

Though the method when applied to the balance does not yet give such good results as Baily obtained from the torsion balance—partly, I believe, because I have not yet been able to apply all his precautions to remove external disturbing forces—it still gives better results than would have been obtained without it. This may be seen by the numbers recorded in the tables, where a progressive motion of the resting point may be noticed in most cases, in the same direction, during a series of experiments. Even when this is not the case, the method at once shows when the disturbing forces are irregular, and when we are justified in rejecting an observation on that account.

I give in this paper two applications of the method, one to the comparison of two weights, the other to the determination of the mean density of the earth. The latter is given only as an example of the method, but I hope shortly to continue the experiments with a large bullion balance, for the construction of which I have had the honour to obtain a grant from the Society. The balance is now in course of construction, by Mr. Oertling, of London.

#### *Description of the Apparatus.*

The balance which I have employed is one of Oertling's chemical balances, with a beam of nearly 16 inches, and fitted with agate planes and knife edges. It will weigh up to a little more than 1 lb. To protect it from sudden changes of temperature, the glass panes of the case are covered with flannel, on both sides of which is pasted gilt paper, with the metallic surface outwards. This case is enclosed in another outer case, a large box of inch deal, lined inside and out with gilt paper. The experiments have been conducted in a darkened cellar under the chemical laboratory at Owens College, which was kindly placed at my disposal by Professor Roscoe. As the ceilings and floors of the building are of concrete, any movement near the room causes a considerable vibration of the floor and walls. It was necessary, therefore, to support the balance independently of the floor. For this purpose, six wooden posts (A, B, C, D, E, F, fig. 1) were erected resting on the ground underneath and passing freely through the floor to a height of 6 feet 6 inches above it. They are connected at

the top by a frame like that of the table, and stayed against each other to give firmness. The wider part of the frame, near the posts E and F, is boarded over to form a table for the telescope (*t*, fig. 1) and scale (*s*), by which the oscillations of the balance are observed. The box containing the balance rests on two cross pieces, on the narrower part, ABCD, of the frame, with the beam parallel to AD, and its right end towards the telescope.

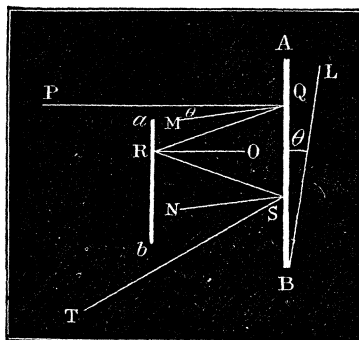
In order to observe the position of the beam, a mirror,  $1\frac{1}{4}$  inches by  $\frac{3}{4}$  inch, is fixed in the centre of the beam, and the reflection of a vertical scale (*s*, fig. 1) in this is viewed with a telescope (*t*) placed close to the scale. The light from the scale passes through two small windows cut in each of the cases of the balance and glazed with plate glass. The position of the beam is given by the division of the scale upon the cross line on the eyepiece of the telescope. The scale, which was photographed on glass, and reduced from a large scale, drawn very carefully, has 50 divisions to the inch. These are ruled diagonally with ten vertical cross lines. It is possible to read, with almost certainty, to a tenth of a division, or  $\frac{1}{500}$ th of an inch. Since the mirror is about 6 feet from the scale, a tenth of a division means an angular deflection of the beam of about 3".\*

The scale is illuminated from behind by a mirror (*m*), several inches in diameter, which reflects through it a parallel beam from a paraffin lamp (*l*). A plate of ground glass between the scale and mirror diffuses the light evenly over the scale and, by altering the position of the mirror, any desired degree of brilliancy may be given to the illumination of the scale. A screen (not shown in fig. 1) prevents stray light from striking the balance-case.

This method of reading—which, of course, doubles the deflection—has been so far sufficiently accurate for my purpose; that is to say, the errors arising from other sources are far greater than those arising from imperfections of reading. But in a long series of preliminary experiments I used the following plan to multiply the deflection still further. A rather smaller fixed mirror, *ab*, is placed opposite to and facing the beam-mirror, AB, fixed on the beam, and a few inches from it. Suppose the beam-mirror to be deflected from the position BL, parallel to *ab*, through an angle,  $\theta$ , to the position AB. If a ray, PQ, perpendicular to *ab* strikes AB at Q, it will make an angle,  $\theta$ , with QM, the normal at Q, and will be reflected along QR, making an angle,  $2\theta$ , with its original direction, and therefore with the normal RO, at R, when it strikes it. If it be reflected again to AB at S, it will make an angle,  $3\theta$ , with the normal SN, and the reflected ray, ST, will make an angle,  $4\theta$ , with the original direction, PQ, of the ray. It may be still further reflected between the two mirrors, if

\* The numbers on the scale run from below upwards, so that an increase in the weight in the right hand pan is indicated by a lower number on the scale.

desirable, each reflection at the mirror, AB, adding  $2\theta$  to the deflection of the ray. I have, for instance, employed three reflections from the



beam-mirror, so multiplying the deflection six times. In this case, one division of my scale, at the distance at which it was placed from the beam, corresponded to a deflection of 7" in the beam, and this could be subdivided to tenths by the eye. The only limit to the multiplication arises from the imperfection of the mirrors and the decrease in the illumination of the successive reflections.\*

The chair of the observer is placed on a raised platform, and a small table rising from the platform and free from the frame on which the instruments rest, is between the observer and the telescope. On this he can rest his note-book during an experiment. As the differences of weight observed are sometimes exceedingly minute, the balance is made very sensitive—usually vibrating in periods between 30" and 50". The value of a division of the scale cannot be determined by adding known small weights to one pan, as the deflection would usually be too great. Any approach of the observer to the case causes great disturbances, so that the ordinary method of moving a rider an observed distance along the beam is inapplicable. In some experiments made last year I calculated the force equivalent to the small differences in weight, in absolute measure, by observing the actual angular deflection and the time of vibration. With a knowledge of the moment of inertia of the beam and treating it as a case of small oscillations, it was possible to calculate the value of the scale. But the observations and subsequent calculations were so complicated that the following method of employing riders was ultimately adopted.

A small bridge about an inch long (fig. II, 1) is fitted on to the beam. The sides of the bridge are prolonged about half an inch above the

\* This method was used in the seventh and eighth series here recorded. Two reflections from the beam mirror were employed, giving four times the actual deflection.

arch which fits on to the beam, as shown in the end view (fig. II, 2). In each of these sides are cut two V-shaped notches directly opposite to each other, one of the opposite pairs being 6.654 millims. (about  $\frac{1}{4}$  inch) distant from the other pair. Two equal riders of the shape shown in fig. II, 3, are placed across the bridge, and are of such a size that they will just fit into the bottom of the notches. When one of these rests across the bridge the other is raised up from it. The lowering of one rider and the raising of the other corresponds heretofore to a transference of a single rider from one pair of notches to the other. The length of the half beam being 202.716 millims. and the distance between the notches 6.654 millims., this transference will be equivalent to the addition to one pair of 3.03284 of the weight of the rider used. As I have generally used a centigramme rider this means 0.3284 mgms.

Two levers *ll'* (fig. II, 4), with hooks *hh'* are used to raise one rider while the other is lowered. These levers are worked by two cams *cc'* on a rod *R*, which is prolonged out of the balance case to the observer. By turning this rod round, the one lever is raised while the other is depressed. The hook at the end of the raised lever picks up its rider while the other hook deposits its rider on the bridge, and then sinks down between the raised sides (as shown in fig. II, 4), leaving the rider resting freely on the bridge.

The levers are so adjusted that the beam even in its greatest oscillations never comes in contact with the hooks.

This arrangement might probably be still further perfected by introducing two small frames for the riders to rest upon, the frames resting on the beam by knife edges. It would then be certain that the movement of the riders was equivalent to a transference from one knife edge to the other, whereas the rider at present may not rest exactly over the centre of the notch. But I find that I get fairly consistent results by lowering the rider somewhat suddenly so as to give it sufficient impetus to go to the bottom of the notch, and have not therefore thought it necessary as yet to introduce more complicated apparatus.

In place of the right hand pan of the usual shape, another of the shape shown in fig. III, 1, is employed. To the centre of the pan underneath is attached a vertical brass rod which passes downwards through the bottom of the inner case of the balance. To the under side of this case is attached the clamping arrangement before referred to. This consists of two sliding pieces (fig. IV, 1, *ss*) working horizontally in a slot cut in a thick brass plate which is fastened to the case. Through a circular aperture in this plate (the slot is not cut through the whole thickness of the plate, but only as shown in fig. IV, 2) and about the middle of the slot hangs the rod *r* attached to the scale pan.

By means of right and left handed screw on a rod *R*, which is pro-

longed out of the case to the observer, these two sliding pieces can be made to approach, and clamp the rod, or to recede and free it. By having the opposite surfaces of the sliding pieces and the rod polished and clean, it is possible to clamp and unclamp without producing any disturbance. The clamp is of great use also to lessen the vibrations when they are too large, as it may be brought into action at any moment, and on releasing carefully the beam will start again from rest without any impetus. It may be used too to increase the vibrations by releasing suddenly when the beam will have a slight impetus in one direction or the other.

The weights which I have compared are two brass pounds avoirdupois, made for me by Mr. Oertling, and marked A and B respectively. They are of the usual cylindrical shape with a knob at the top (fig. III, 2). Two small brass pans (fig. III, 3) with a wire arch by which they can be suspended, are used to carry them; these are called respectively X and Y. I found on beginning to use them that there was too great a difference between A and B. I therefore adjusted them by putting a very small piece of wax upon A, the lighter. But the difference between them increased by 0.0782 mgm. in two days, which I thought was probably due to the wax. After the fourth series I therefore removed it and scraped B till it was more nearly equal to A. The weighings I—IV have, however, been retained, for though the differences on different days vary they are fairly constant on the same day.

The weights are changed by the following apparatus which has been designed to effect the change as simply and quickly as possible.

A horizontal "side rod" or link (*ss*, fig. V) is worked by two cranks (*cc*, fig. V, 2), which are attached to the axles of two equal toothed wheels (*tt*) with a pinion (*p*) connecting them. A second pinion (*q*), on a rod prolonged out of the case to the observer, gears with one of the toothed wheels. By turning this rod the toothed wheels are set in motion, both in the same direction, moving the horizontal "side rod" from the right say upwards and over to the left. A pin (*pn*) stops its motion downwards further than is shown in fig. V, 1. Near each end of the rod is cut a notch, and across these are hung the pans carrying the weights. The apparatus is fastened to the floor of the case between the central upright supporting the beam and the scale pan, the side rod being perpendicular to the direction of the beam, and exactly over the centre of the pan. In fig. V, 1, one of the weights B is supposed to be resting on the scale pan (the wires suspending the pan from the beam not being shown), the side rod having moved down so far below the wire of the smaller pan carrying the weight that it leaves it quite free. If, now, it is desired to change the weights the rod R is turned, setting the wheels in motion, the side rod moves up, picks up B—the notch catching the wire—then travels



over round to the extreme right, when A will be just over and nearly touching the scale pan. By continuing the motion slightly A will be gently deposited on the pan, and the side rod will move slightly down leaving the weight quite free. On the scale pan are four pins, turned slightly outwards, acting as guides for the small pan, and ensuring that it shall always come into the same position. The wheels and pinions are of such a size that two revolutions of the rod just suffice to change one weight for the other.

It will be seen that all the manipulation required from the observer during a series of weighings is the simple turning of three rods, which are prolonged out of the balance case to where he is stationed at the telescope. By turning one of these he can change the position of the rider on the beam by a known amount, and so find the value of his scale. By turning a second he clamps the scale pan, and so steadies the balance while the weights are changed by turning a third rod. I have made this arrangement not only because it seems as simple as possible to secure the end required, but also because it seemed more applicable to a vacuum balance (with which I hope ultimately to test it).

I take this opportunity of expressing my thanks to Mr. Thomas Foster, mechanician of Owens College, for his aid in the construction of the apparatus, and in the planning of many of its details.

*Method of conducting a Series of Weighings.*

After the counterpoise has been adjusted so that the beam swings nearly about its horizontal position, the frame is lowered so that the balance is ready for use. The pan is then clamped and the balance is left to come to a nearly permanent state of flexure if possible, sometimes for the night or even longer. The lamp is lighted usually half-an-hour or more before beginning to observe, so that its effect on the balance may attain a more or less steady state. It is necessary also to wait some time after coming into the room, for the opening of the door will always cause a considerable and immediate deflection of the beam. When a sufficient time has elapsed, the observations are commenced with a determination of the value of one scale division by means of the riders. The three extremities of two successive oscillations are observed with one of the riders resting on the beam. These are then combined as follows:—The mean of the first and third is taken, and the mean again of this and the second, this constituting the “resting point,” that is, the position of equilibrium of the beam at the middle of the time. For instance, in weighing No. I (see tables at the end) the three extremities of successive oscillations were 280·5, 312·0, and 286·0 (column 2). The resting point was taken as—

$$\frac{280\cdot5 + 286\cdot0 + 2 \times 312\cdot0}{4} = 297\cdot62,$$

the rider on the beam being the right hand one denoted by R, column 1. The balance is the clamped, and the other rider is brought on to the beam while the first is taken up. The resting point is again observed. In No. I it was 270·05. The balance is again clamped, and the first rider again brought on the beam, and in unclamping the resting point again observed. In the same weighing it was 296·75. These three are sufficient to give one determination of the deflection due to the transference of a rider. This will be the difference between the second resting point and the mean of the first and third. For instance,  $\frac{297\cdot62+296\cdot75}{2}-297\cdot18=27\cdot13$  divisions. This number is found in the fifth column.

This process is continued, the resting points being combined in threes till several values of the deflection due to the rider have been obtained, and the mean of these is taken as the true value. This plan of combining the resting points requires that the observations should be taken at nearly equal intervals. After a little practice it will always take the observer about the same time to go through the same operations of clamping, changing the riders, unclamping, clamping again to lessen the vibrations about the new resting point, and then beginning to observe, and I have considered that this was a sufficiently correct method of timing the observations.

When a series has been taken it will at once be seen whether they were begun too soon after entering the room, or whether any irregular disturbing force has acted. For instance, in weighing No. II, determination of one scale division, the first resting point is so much lower than the succeeding with the same rider that evidently the balance was still affected by my entrance into the room. It was, therefore, rejected. Again, in weighing No. III determination of the difference between the weights, the fourth resting point was much lower than the others with the same weight in the pan. The resting points, when the other weight was in the pan, showed no similar sudden drop of such magnitude. This observation was, therefore, rejected as being affected by some irregular disturbance.

When the value of the deflection is determined, the value of one scale division is at once found by dividing ·3282 mgm. by the number of divisions of the deflection, since the charge of the sides is equivalent to the addition of ·3282 mgm. to one pan.

The determination of the difference between the weights is then begun. This is carried on in a precisely similar manner, the only difference being, that the rod changing the weights is now turned round in place of the rod changing the riders. I have usually taken a greater number of observations of the difference between the weights than of the deflection due to the riders, as the former is somewhat more irregular than the latter. This irregularity I believe to arise

from slight differences of temperature of the two weights, and perhaps from air currents caused by their motion inside the case. They do not seem to be due to any fault in the clamping arrangement, since that is employed equally in both, and the changing of the weights, if effected gently, does not move the beam at all.

When the deflection has been determined, it is multiplied by the number of milligrammes corresponding to one scale division, and this, of course, gives the difference between the weights. I have interchanged the weights in the two pans X and Y, between the series of weighings, in order to make the experiments like those conducted in the weighings for the standard pound. But my object has not been to show at all that the method gives consistent results day after day, and, in fact, the difference between the weights has varied. For instance, according to weighings I and II,  $A - B = \cdot 0446$ , while, according to weighings III and IV,  $A - B = \cdot 0232$ . There is a greater difference between these than can be accounted for by errors of experiment, and it probably arose from the small piece of wax with which I made A nearly equal to B. The difference between the weights when measured to such a degree of accuracy as that which I have attempted, will, no doubt, vary from time to time, partly with deposits of dust, partly with changes in the moisture in the atmosphere, and so on.

But I think the numbers which are given in the tables are sufficient to show that the difference between two weights in any one series of weighings can be measured with a greater degree of accuracy than has hitherto been supposed possible. I give in the tables a full account of the weighings, each series containing a determination of the value of one scale division and a determination of the difference between the weights. The greatest deviation of any one of a series from the mean of that series of differences is always given. This I consider a better test of accuracy of weighing than the probable error. What is wanted in weighing is rather a method which will give at once a good determination of the difference between two weights. But I may state, that if the error of any one of a series be taken as its difference from the mean of that series, the probable error of a single determination of the difference between the weights in the first four series is  $\cdot 4344$  of a division, or  $\cdot 0054$  mgm., that is,  $\frac{1}{240000000}$ th of the total weight, while the greatest error is  $1\cdot 8$  divisions, or  $\cdot 0224$  mgm., that is  $\frac{1}{200000000}$ th of the total weight. It may be remarked that these weighings were all made during peculiarly unfavourable weather when there were frequent heavy showers, causing sudden changes of temperature, and thus seriously affecting the working of the balance. In the series V—VIII the greatest error is only  $\frac{1}{200000000}$  of the total weight, the weather having improved considerably.

*On the Employment of the Balance to determine the Mean Density of the Earth.*

In the Cavendish experiment, the attraction of a large sphere of lead of known mass and dimensions upon another smaller sphere also of known mass and dimensions, is measured when the two are an observed distance apart. Comparing this attraction with the weight of the small sphere—that is the attraction of the earth upon it—and knowing the dimensions of the earth, we can deduce the mass of the earth in terms of the mass of the large lead sphere, and so obtain its mean density. The torsion balance, which was invented for the purpose by Mitchell, the original contriver of the experiment, has hitherto been used to determine the force exerted by the mass upon the small sphere. In the arrangement here described, I have replaced the torsion balance by the ordinary balance, and have so been able to compare the attraction of a lead sphere with that of the earth upon the same mass somewhat more directly. The results which I have obtained have no value in themselves, but they serve as an example of the employment of the balance for more delicate work than any which it has as yet been supposed able to perform.

The method is shortly this:—A lead weight (called “the weight”) weighing 452.92 grms. (nearly 1 lb.) hangs down by a fine wire from one arm of a balance, from which the pan has been removed at a distance of about six feet below it, and is accurately counterpoised in the other pan, suspended from the other arm. A large lead mass (called “the mass”) weighing 154,220.6 grms. (340 lbs.) is then introduced directly under the hanging weight. The attraction of this mass increases the weight slightly and the beam is deflected through an angle which is observed. The value of this deflection in milligrammes is measured by the employment of riders in the manner described above, and so the attraction of the mass is known. The increase of the weight caused by the mass has been in my experiments about .01 of a milligramme, or  $\frac{1}{40000000}$ th of the whole weight.

The balance which I have used is that which I have described above. It was placed in the same room and in the same position as in the weighing experiments. The same method was used to observe the oscillations with a single mirror on the beam. The scale was a simple one etched on glass and not diagonally ruled. It had about 50 divisions to the inch, and the numbers increased from above downwards, so that an increase in the weight hanging from the left arm was indicated by a lower number on the scale.

The weight which is suspended by a very fine brass wire from the left arm, passing through a hole in the bottom of the balance case, hangs in a double tin tube, 4 inches in diameter, to protect it from air currents. At the bottom of the tube is a window, through which can

be seen the bottom of the weight as it hangs. The weight is 4.248 centims. in diameter and is gilded. The mass is a sphere of an alloy of lead and antimony. It was cast with a "head" on and then accurately turned. Its vertical diameter is 30.477 centims. (about 1 foot). The specific gravity of a specimen of the metal was found to be 10.422. Its weight given by a weighing machine is 340 lbs. about, and this agrees very nearly with the weight calculated from the specific gravity. I am obliged to accept this as the true weight provisionally, until it is found more correctly by the large balance referred to above and now being constructed.

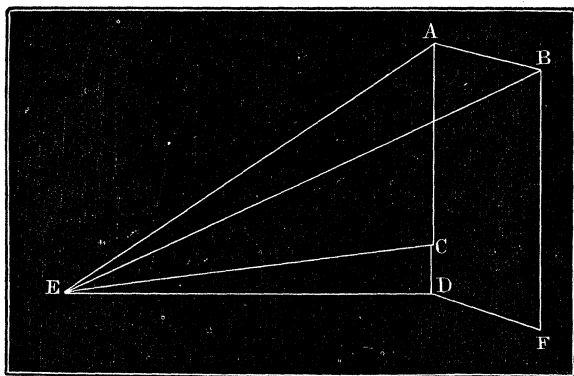
This mass (fig. I, M) is placed in a shallow wood cup at one end of a 2-inch plank, 8 inches wide and 6 feet 11 inches long, mounted on four flanged brass wheels, and serving as a carriage for it (fig. I). A plank about 12 feet long nailed to the floor in a direction perpendicular to the beam of the balance, as shown in fig. I, *pp*, acts as a railway for the carriage, and a firm stop at each end prevents the carriage from running off the rail. The distance between the stops is rather less than twice the length of the carriage, and the weights hangs down from the balance exactly midway between the stops. The mass is placed on the carriage so that it is exactly under the weight when the carriage is at one end of its excursion against one of the stops. An empty cup (*c*, fig. 1) of the same dimensions as that in which the mass rests is placed at the other end of the carriage, and is just under the weight when the carriage is against the other stop. By this arrangement no correction is needed for the attraction of the carriage upon the weight or counterpoise, and the effect caused by the removal of the carriage from one end of its excursion to the other is entirely due to the difference of attractions of the mass upon the weight and counterpoise in its two positions. The position of the mass when directly under the weight is called its "in position," and that when it is at the other end of its excursion is called the "out position." The length of the excursion is 5 feet 7.3 inches.

To draw the carriage along the rail a vertical iron shaft with a wood cylinder at the lower end pivots on the floor, and is prolonged up to the level of the observer as he sits at the telescope with a handle by which he can turn it. The two ends of a rope which winds round the cylinder pass through pulleys on the stops, and are attached to the ends of the carriage. The observer can then move the mass with great ease by turning the handle, even while looking through the telescope.

When a series of observations is made, the general method is this. The deflection (*r*) due to the transference of a rider from one notch to the other on the beam is first observed exactly in the manner before described, the mean of four or five values being taken as the true value. Then the deflection (*n*) due to the difference of attraction of

the mass in its two positions is found in exactly the manner in which the difference between two weights is found, except that now when three successive extremities of oscillations have been observed for a resting point the mass is moved from one position to the other where the weights were changed in the former experiments, the clamp not being brought into action. The second extremity of the oscillation which is proceeding while the mass is moved, is observed as the first of the next three. When nine or more resting points have been observed they are combined in threes, and the mean of the resulting values of the deflection  $n$  is used in the subsequent calculation.

This deflection is, of course, less than that which would be observed were there no attraction on the counterpoise, and were the out position of the mass at an infinite distance. To find the factor  $f$  by which the deflection  $n$  due to the change of position of the mass must be multiplied in order to reduce it to the deflection which would be observed under these conditions, let AB be equal and parallel to the beam



of the balance at the level of the counterpoise of which B is the centre. Let C be the centre of the weight, D that of the mass in its in position, E that of the mass in its out position. Draw BF, DF, parallel to AD, AB. Let  $\mu$  = the mass.

The vertical attraction of the mass in its in position will be—

$$\frac{\mu}{CD^2} - \frac{\mu BF}{BD^3}.$$

The vertical attraction in its out position will be—

$$\frac{\mu CD}{CE^3} - \frac{\mu BF}{BE^3}.$$

The difference between these is actually observed, viz. :

$$\frac{\mu}{CD^2} \left\{ 1 - \frac{CD^2 BF}{BD^3} - \frac{CD^3}{CE^3} + \frac{CD^2 BF}{BE^3} \right\}.$$

The factor by which we must multiply the observed difference to reduce it to the attraction of the mass on the weight in its position is therefore—

$$f = 1 + \frac{CD^2 BF}{BD^3} + \frac{CD^3}{CE^3} - \frac{CD^2 BF}{BE^3}$$

$$= 1.0185$$

since CD = 22.13 centimetres.

|             |   |
|-------------|---|
| BD = 192.03 | „ |
| BF = 187.70 | „ |
| CE = 172.36 | „ |
| BE = 257.09 | „ |

The values of  $r$  and  $n$  being observed, the distance between the centres of the mass and weight  $d$  is then measured by adding 17.362 centims. to the sum of their radii, the distance from the top of the mass to the bottom of the weight as measured by a cathetometer.

It now remains to explain the calculation of the mean density  $\Delta$  from the observed values of  $r$ ,  $n$ , and  $d$ . We have—

$$\frac{f \times \text{increase in weight observed}}{\text{Weight of weight}} = \frac{\text{Attraction of mass on weight when "in"}}{\text{Attraction of earth on weight}}.$$

But the increase in weight is  $\frac{n \times 6654}{r \times 202716}$  mgms., since the distance between the notches is 6.654 millims., and the half beam 202.716 millims. The weight of the weight is 453.92 grammes.

The attraction of the mass in centimetres

$$= \frac{\text{Volume} \times \text{density}}{(\text{distance between centres of mass and weight } r)^2},$$

$$= \frac{\text{Weight in grammes}}{d^2},$$

$$= \frac{154220.6}{d^2}.$$

The attraction of the earth is

$$\Delta \times \frac{4}{3} \pi R \left\{ 1 + M - \frac{5}{2} (M - \epsilon) \right\} \cos^2 \lambda,$$

where  $\Delta$  = mean density of the earth,

$R$  = earth's polar radius in centimetres,

$M$  =  $\frac{\text{centrifugal force at the equator}}{\text{Equatorial gravity}},$

$\epsilon$  = ellipticity.

$\lambda$  = latitude.

The logarithm of the coefficient of  $\Delta$  when  $R$  is in inches is 9.0209985 (“Astronomical Soc. Mem.,” xiv, p. 118), or if  $R$  is in centimetres it is 9.4258322.

Inserting these values in equation A we obtain

$$\Delta = \frac{154220 \cdot 6}{d^2} \times \frac{453290}{f \times \frac{n}{r} \frac{6654}{202716}} \cdot \frac{4}{3} \pi R \left\{ 1 + M - \left( \frac{5}{2} M - \epsilon \right) \cos^2 \lambda \right\}$$

$$\therefore \Delta = C \times \frac{r}{nd^2}$$

$$\text{where } C = \frac{15220 \cdot 6 \times 453290 \times 202716}{\frac{4}{3} \pi R \left( 1 + M - \frac{5}{2} M - \epsilon \right) \cos^2 \lambda \times f \times 6654}$$

$$\text{and } \log C = 1 \cdot 8951337,$$

$$\therefore \log \Delta = 1 \cdot 8951337,$$

$$+ \log r,$$

$$- \log n,$$

$$- 2 \log d.$$

The following table is an account of an experiment made on May 30th, 1878, and will serve as a specimen of the method of making the observations. It is the best which I have yet made in the closeness with which all the values of  $n$  agree with each other.

VII.—May 30, 1878.—Determination of  $r$ .

| Rider on beam. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points. | Of $r$ differences<br>= $r$ . |
|----------------|------------------------------|----------------|--------------------------------------------------|-------------------------------|
| R              | 260·9<br>250·9<br>260·6      | 255·82         |                                                  |                               |
| L              | 214·6<br>205·0<br>212·3      | 2·9·22         | 256·74                                           | 47·52                         |
| R              | 271·9<br>246·7<br>269·4      | 257·07         | 210·31                                           | 47·36                         |
| L              | 214·3<br>209·2<br>212·9      | 211·40         | 257·62                                           | 46·22                         |
| R              | 249·7<br>263·8<br>253·0      | 257·57         | 212·23                                           | 45·34                         |
| L              | 204·9<br>220·7<br>206·0      | 213·07         |                                                  |                               |

Mean  $r = 46 \cdot 61$ .



Determination of  $n$ .

| Position of mass. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points. | Differences = $n$ . |
|-------------------|------------------------------|----------------|--------------------------------------------------|---------------------|
| In                | 216·8<br>208·1<br>215·9      | 212·22         |                                                  |                     |
| Out               | 210·9<br>216·1<br>211·0      | 213·52         | 212·27                                           | 1·25                |
| In                | 213·9<br>210·8<br>213·8      | 212·32         | 213·61                                           | 1·29                |
| Out               | 212·7<br>214·6<br>212·9      | 213·70         | 212·41                                           | 1·29                |
| In                | 211·5<br>213·6<br>211·3      | 212·50         | 213·78                                           | 1·28                |
| Out               | 215·9<br>211·8<br>216·0      | 213·87         | 212·60                                           | 1·27                |
| In                | 210·2<br>215·0<br>210·6      | 212·70         | 213·93                                           | 1·23                |
| Out               | 216·7<br>211·6<br>216·1      | 214·00         | 212·73                                           | 1·27                |
| In                | 209·9<br>218·4<br>210·4      | 212·77         | 214·08                                           | 1·31                |
| Out               | 217·0<br>211·4<br>216·9      | 214·17         | 212·98                                           | 1·19                |
| In                | 209·8<br>216·2<br>210·6      | 213·20         |                                                  |                     |

$\therefore$  mean  $n=1\cdot26$ .

At the close of the experiment  $d$  was found to be 22·216 centimetres.  
We have therefore—

$$\begin{aligned}
 \log \Delta &= \log C. \\
 &+ \log 46\cdot61. \\
 &- \log 1\cdot26. \\
 &- 2 \log 22\cdot16. \\
 &= 1\cdot8951337. \\
 &+ 1\cdot6684791. \\
 &- 0\cdot1003705. \\
 &- 2\cdot6937226. \\
 &= \cdot7695197. \\
 \therefore \Delta &= 5\cdot882.
 \end{aligned}$$

I have made in all 11 experiments with this method. The resulting values of  $\Delta$  are—

|    |       |        |       |        |
|----|-------|--------|-------|--------|
| 1  | ..... | May 20 | ..... | 5·393. |
| 2  | ..... | „ 23   | ..... | 5·570. |
| 3  | ..... | „ 24   | ..... | 4·415. |
| 4  | ..... | „ 28   | ..... | 7·172. |
| 5  | ..... | „ 29   | ..... | 5·109. |
| 6  | ..... | „ 29   | ..... | 6·075. |
| 7  | ..... | „ 30   | ..... | 5·882. |
| 8  | ..... | „ 30   | ..... | 6·336. |
| 9  | ..... | June 5 | ..... | 5·977. |
| 10 | ..... | „ 5    | ..... | 5·580. |
| 11 | ..... | „ 6    | ..... | 5·100. |

The resulting mean value of the mean density of the earth is 5·69.

If the eleven determinations be supposed to have equal weight, the probable error of their value is 0·15.

The various determinations differ very much among themselves, but they seem to me sufficiently close to justify the hope that with a large balance and a large weight, which will not be so easily affected by air currents, and with greater precautions to prevent those air currents, a good determination of the mean density of the earth may ultimately be obtained by this method.

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## I.—June 12.—Determination of Value of 1 Scale Division.

| Rider on the beam. | Extremities of three successive oscillations. | Resting point. | Means of preceding and successive resting points. | Differences, i.e., deflection due to .3282 mgm. |
|--------------------|-----------------------------------------------|----------------|---------------------------------------------------|-------------------------------------------------|
| L                  | 280·5<br>312·0<br>286·0                       | 297·62         |                                                   |                                                 |
| R                  | 253·1<br>284·9<br>257·3                       | 270·05         | 297·18                                            | 27·13                                           |
| L                  | 306·2<br>288·5<br>303·8                       | 296·75         | 268·68                                            | 28·07                                           |
| R                  | 258·8<br>275·0<br>260·5                       | 267·32         | 294·81                                            | 27·49                                           |
| L                  | 285·8<br>298·7<br>288·3                       | 292·87         | 266·79                                            | 26·08                                           |
| R                  | 272·9<br>260·6<br>271·0                       | 266·27         | 292·88                                            | 26·61                                           |
| L                  | 296·1<br>290·0<br>295·5                       | 292·90         |                                                   |                                                 |

Mean R—L=27·08 divisions.

$$\therefore 1 \text{ division} = \frac{0.3282}{27.08} = 0.01212 \text{ milligramme.}$$

Determination of  $(B + X) - (A + Y)$ .

| Weight on pan. | Extremities of oscillations. | Resting point. | Means of preceding and succeeding resting points. | Differences.            |
|----------------|------------------------------|----------------|---------------------------------------------------|-------------------------|
| A + Y          | 254·5<br>272·9<br>257·3      | 264·40         |                                                   |                         |
| B + X          | 267·5<br>249·8<br>264·8      | 257·97         | 263·27                                            | 5·30                    |
| A + Y          | 258·5<br>265·4<br>259·3      | 262·15         | 257·04                                            | 5·11                    |
| B + X          | 259·3<br>253·6<br>258·0      | 256·12         | 261·87                                            | 5·75                    |
| A + Y          | 253·5<br>268·5<br>255·9      | 261·60         | 255·63                                            | 5·97                    |
| B + X          | 244·6<br>264·5<br>247·0      | 255·15         | 261·87                                            | 6·72                    |
| A + Y          | 273·6<br>252·0<br>271·0      | 262·15         | 255·85                                            | 6·30                    |
| B + X          | 252·7<br>260<br>253·5        | 256·55         | 262·90                                            | 6·37                    |
| A + Y          | 275<br>253·7<br>272·4        | 263·70         | ..                                                | Mean difference<br>5·93 |

$$\therefore (B + X) - (A + Y) = .01212 \times 5.93.$$

$$= .0718 \text{ milligramme.}$$

Greatest deviation from the mean—

$$= .82 \text{ division.}$$

$$= .0099 \text{ milligramme.}$$

The weather during this series of weighings was very unfavourable, with frequent heavy showers.

## II.—June 13.—Determination of 1 Scale Division.

| Rider on beam. | Extremities of oscillations. | Resting point. | Means of preceding and succeeding resting points.            | Differences due to R—L. |
|----------------|------------------------------|----------------|--------------------------------------------------------------|-------------------------|
| L              | 302·8<br>299·9<br>302·9      | 301·37         | This is rejected as it is so much lower than the succeeding. |                         |
| R              | 321·8<br>335·7<br>326·8      | 330·00         |                                                              |                         |
| L              | 299·6<br>308·7<br>301·0      | 304·50         | 330·68                                                       | 26·18                   |
| R              | 322·9<br>337·5<br>327·6      | 331·37         | 305·10                                                       | 26·27                   |
| L              | 299·5<br>310·9<br>301·5      | 305·70         | 331·77                                                       | 26·07                   |
| R              | 325·2<br>337·1<br>329·3      | 332·17         | 305·90                                                       | 26·27                   |
| L              | 290·4<br>319·4<br>295·2      | 306·10         | ..                                                           |                         |

$\therefore$  mean R—L=26·20.

$\therefore$  1 division=.01252 milligramme.

The weather was as unfavourable as on the previous day.

The weights were changed shortly before the commencement of this series and the balance then worked so irregularly that for some

time I was unable to begin the rider determination. Even then the first resting point had to be rejected.

Determination of  $(A + X) - (B + Y)$ .

| Weight in pan. | Extremities of oscillations. | Resting point. | Means of preceding and succeeding resting points.                                                                                                              | Differences. |
|----------------|------------------------------|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
| B + Y          | 307·8<br>326·0<br>311·6      | 317·85         | These are all rejected, as the motion was so irregular. The weights had been changed a short time before, and had probably not reached an uniform temperature. |              |
| A + X          | 293·7<br>307·8<br>295·8      | 301·27         |                                                                                                                                                                |              |
| B + Y          | 309·6<br>322·3<br>312·6      | 316·70         |                                                                                                                                                                |              |
| A + X          | 295·5<br>309·6<br>297·6      | 303·17         |                                                                                                                                                                |              |
| B + Y          | 304·1<br>322·7<br>308·1      | 314·4          |                                                                                                                                                                |              |
| A + X          | 289·3<br>304·4<br>291·5      | 297·4          |                                                                                                                                                                |              |
| B + Y          | 305·5<br>315·0<br>307·5      | 310·75         | 297·35                                                                                                                                                         | 13·40        |
| A + X          | 294·0<br>300·1<br>294·8      | 297·30         | 309·95                                                                                                                                                         | 12·65        |
| B + Y          | 304·6<br>312·7<br>306·6      | 309·15         | 296·51                                                                                                                                                         | 12·54        |
| A + X          | 290·0<br>300·7<br>291·5      | 295·72         |                                                                                                                                                                |              |

Mean  $(A + X) - (B + Y) = 12·86$  divisions.

$\therefore (A + X) - (B + Y) = 0·1610$  milligramme.

Greatest deviation from mean  $= 0·54$  division  $= 0·0067$  milligramme.

## III.—June 13.—Determination of 1 Scale Division.

| Rider on beam. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points. | Differences due to R—L. |
|----------------|------------------------------|----------------|--------------------------------------------------|-------------------------|
| R              | 301·3<br>290·6<br>299·7      | 295·55         |                                                  |                         |
| L              | 313·7<br>328·7<br>318·1      | 322·30         | 296·38                                           | 25·92                   |
| R              | 293·6<br>300·4<br>294·5      | 297·22         | 322·47                                           | 25·25                   |
| L              | 311·0<br>331·6<br>316·4      | 322·65         | 297·88                                           | 24·77                   |
| R              | 281·6<br>313·2<br>286·2      | 298·55         | 323·57                                           | 25·02                   |
| L              | 310·1<br>335·8<br>316·3      | 324·50         | 298·62                                           | 25·88                   |
| R              | 287·8<br>308·1<br>290·8      | 298·7          |                                                  |                         |

Mean R—L=25·37 divisions.

$$\therefore 1 \text{ division} = \frac{3282}{25 \cdot 37} = \cdot 01293.$$

Determination of  $(A+X) - (B+Y)$ .

| Weight in pan. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points.                                     | Differences. |
|----------------|------------------------------|----------------|--------------------------------------------------------------------------------------|--------------|
| A + X          | 303·3<br>294·5<br>301·8      | 298·52         |                                                                                      |              |
| B + Y          | 303·6<br>315·8<br>306·2      | 310·35         | 297·07                                                                               | 13·28        |
| A + X          | 288·9<br>301·5<br>290·6      | 295·62         |                                                                                      |              |
| B + Y          | 300·2<br>311·6<br>302·5      | 306·47         | This is evidently due to some irregular and short disturbing cause, and is rejected. |              |
| A + X          | 288·8<br>304·1<br>291·1      | 297·07         |                                                                                      |              |
| B + Y          | 301·5<br>319·3<br>303·9      | 311·0          | 297·16                                                                               | 13·84        |
| A + X          | 291·7<br>302·0<br>293·2      | 297·25         | 310·33                                                                               | 13·08        |
| B + Y          | 301·0<br>316·8<br>304·1      | 309·67         |                                                                                      |              |

Mean  $(A+X) - (B+Y) = 13·40$  divisions.

$\therefore (A+X) - (B+Y) = 0·1732$  milligramme.

Greatest deviation from mean =  $·44$  division =  $·0057$  milligramme.



## IV.—June 14.—Determination of 1 Value of Scale Division.

| Rider on beam. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points.                                                          | Differences due to R—L. |
|----------------|------------------------------|----------------|-----------------------------------------------------------------------------------------------------------|-------------------------|
| R              | 252·5<br>253·4<br>251·7      | 252·5          | This was taken soon after entering the room. It is so much lower than the succeeding that it is rejected. |                         |
| L              | 290·8<br>273·7<br>288·8      | 281·75         |                                                                                                           |                         |
| R              | 242·4<br>266·7<br>245·1      | 255·22         | 281·96                                                                                                    | 26·74                   |
| L              | 272·6<br>289·7<br>276·7      | 282·17         | 255·89                                                                                                    | 26·28                   |
| R              | 252·8<br>253·9<br>258·7      | 256·57         | 282·67                                                                                                    | 26·10                   |
| L              | 286·2<br>280·0<br>286·5      | 283·17         |                                                                                                           |                         |

Mean R—L=26·37 division.

$$1 \text{ division} = \frac{3282}{26 \cdot 37} = \cdot 01244.$$

Being interrupted, I could not continue the series of rider determinations further.

Determination of  $(B+X)-(A+Y)$ .

| Weight in pan. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points. | Differences. |
|----------------|------------------------------|----------------|--------------------------------------------------|--------------|
| A + Y          | 291·7<br>275<br>289·5        | 282·80         |                                                  |              |
| B + X          | 261·0<br>280·3<br>263·4      | 271·25         | 282·85                                           | 11·60        |
| A + Y          | 285·7<br>280·4<br>285·1      | 282·90         | 271·61                                           | 11·29        |
| B + X          | 280·2<br>265·3<br>277·5      | 272·07         | 283·90                                           | 11·83        |
| A + Y          | 294·9<br>276·1<br>292·5      | 284·90         | 273·43                                           | 11·47        |
| B + X          | 276·0<br>273·9<br>275·4      | 274·8          | 286·40                                           | 11·60        |
| A + Y          | 267·3<br>305·4<br>273·5      | 287·90         | 275·11                                           | 12·79        |
| B + X          | 278·5<br>272·9<br>277·4      | 275·42         | 289·28                                           | 13·86        |
| A + Y          | 296·6<br>285·5<br>295·1      | 290·67         |                                                  |              |

Mean  $(B+X)-(A+Y)=12\cdot06$  divisions.

$\therefore (B+X)-(A+Y)=0\cdot1500$  milligramme.

Greatest deviation from the mean  $=1\cdot8$  division  $=0\cdot01224$  mgm.

The previous determination of  $(B+X)-(A+Y)$  was  $\cdot0718$  mgm. The difference is too great,  $\cdot0782$  mgm., to be accounted for by errors of experiment. There must have been some deposit on one of the weights, either of dust or moisture. I therefore took them out, cleaned and adjusted them by scraping B till nearly equal to A, and removing the wax from A.

## V.—June 14.—Determination of Value of 1 Scale Division.

| Rider on beam. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points. | Differences due to R—L. |
|----------------|------------------------------|----------------|--------------------------------------------------|-------------------------|
| R              | 212·6<br>230·3<br>214·7      | 221·97         |                                                  |                         |
| L              | 252·3<br>242·5<br>251·2      | 247·12         | 222·16                                           | 24·96                   |
| R              | 225·0<br>220·5<br>223·6      | 222·35         | 246·67                                           | 24·32                   |
| L              | 238·6<br>252·8<br>240·7      | 246·22         | 221·87                                           | 24·35                   |
| R              | 224·9<br>218·7<br>223·3      | 221·40         | 245·98                                           | 24·58                   |
| L              | 249·7<br>242·2<br>248·9      | 245·75         | 221·26                                           | 24·49                   |
| R              | 226·8<br>216·7<br>224·3      | 221·12         |                                                  |                         |

∴ mean R—L=24·54 divisions.

$$\therefore 1 \text{ division} = \frac{0.3282}{24.54} = 0.01337 \text{ milligramme.}$$

Determination of  $(B + Y) - (A + X)$ .

| Weight in pan. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points.         | Differences. |
|----------------|------------------------------|----------------|----------------------------------------------------------|--------------|
| A + X          | 206·3<br>212·7<br>206·8      | 209·62         | This was rejected as being so much higher than the rest. |              |
| B + Y          | 209·0<br>206·4<br>208·6      | 207·6          |                                                          |              |
| A + X          | 212·4<br>204·0<br>211·3      | 207·92         | 207·71                                                   | ·21          |
| B + Y          | 206·8<br>208·8<br>206·9      | 207·82         | 207·88                                                   | ·06          |
| A + X          | 211·8<br>204·4<br>210·8      | 207·85         | 207·64                                                   | ·21          |
| B + Y          | 209·1<br>206·1<br>208·6      | 207·47         | 207·86                                                   | ·39          |
| A + X          | 210·7<br>205·5<br>209·8      | 207·87         | 207·11                                                   | ·76          |
| B + Y          | 208·5<br>205·3<br>207·9      | 206·75         | 207·83                                                   | ·78          |
| A + X          | 209·7<br>205·1<br>208·9      | 207·20         | 206·45                                                   | ·75          |
| B + Y          | 203·3<br>208·7<br>203·9      | 206·15         |                                                          |              |

Mean  $(B + Y) - (A + X) = \cdot 49$  division  $= 0\cdot 00655$  milligramme.

Greatest deviation from mean  $= \cdot 43$  division  $= 0\cdot 00575$  mgm.

A and B had here been cleaned and B readjusted by scraping. A small vessel containing calcium chloride was put inside the balance to dry the air. This improved the action of the clamp, diminishing the cohesion.

## VI.—June 17.—Determination of Value of 1 Scale Division.

| Rider on beam. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points. | Differences due to R—L. |
|----------------|------------------------------|----------------|--------------------------------------------------|-------------------------|
| R              | 210·4<br>214·3<br>210·7      | 212·42         |                                                  |                         |
| L              | 238·4<br>233·3<br>238·1      | 235·77         | 212·93                                           | 22·84                   |
| R              | 209·5<br>217·0<br>210·3      | 213·45         | 236·17                                           | 22·72                   |
| L              | 246·2<br>228·1<br>243·9      | 236·57         | 213·38                                           | 23·19                   |
| R              | 224·2<br>204<br>221·1        | 213·32         | 236·59                                           | 23·27                   |
| L              | 233·7<br>239·1<br>234·6      | 236·62         | 213·52                                           | 23·09                   |
| R              | 207·6<br>219·4<br>208·6      | 213·75         |                                                  |                         |

Mean R—L=23·02 divisions.

$$\therefore 1 \text{ division} = \frac{.3282}{23.02} = .01425 \text{ milligramme.}$$

Determination of  $(B + X) - (A + Y)$ .

| Weight in pan. | Extremities of Oscillations. | Resting point. | Mean of pre-<br>ceding and<br>succeeding<br>resting points. | Differences. |
|----------------|------------------------------|----------------|-------------------------------------------------------------|--------------|
| B + X          | 216·9<br>214·6<br>216·6      | 215·67         |                                                             |              |
| A + Y          | 256·7<br>233·8<br>253·9      | 244·55         | 216·08                                                      | 28·47        |
| B + X          | 220·1<br>213·7<br>218·5      | 216·50         | 244·90                                                      | 28·40        |
| A + Y          | 254·8<br>236·7<br>252·8      | 245·25         | 217·07                                                      | 28·18        |
| B + X          | 221·5<br>214·5<br>220·1      | 217·65         | 245·48                                                      | 27·83        |
| A + Y          | 257·5<br>235·3<br>254·8      | 245·72         | 217·37                                                      | 28·35        |
| B + X          | 220·9<br>214·0<br>219·5      | 217·1          | 245·68                                                      | 28·58        |
| A + Y          | 246·8<br>244·5<br>246·8      | 245·65         | 216·83                                                      | 28·82        |
| B + X          | 223·7<br>210·5<br>221·6      | 216·57         |                                                             |              |

$\therefore$  mean  $(B + X) - (A + Y) = 28.38$  divisions  $= 0.4043$  milligramme.

Greatest deviation from mean  $= .55$  division  $= .00784$  milligramme.

## VII.—June 18.—Determination of 1 Scale Division.

| Rider on beam. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points. | Differences due to R—L. |
|----------------|------------------------------|----------------|--------------------------------------------------|-------------------------|
| R              | 171·6<br>190·7<br>172·7      | 181·42         |                                                  |                         |
| L              | 210·3<br>225·3<br>212·4      | 218·32         | 180·47                                           | 37·85                   |
| R              | 189·2<br>171·7<br>185·5      | 179·52         | 217·54                                           | 38·02                   |
| L              | 223·2<br>210·8<br>222·3      | 216·77         | 179·04                                           | 37·73                   |
| R              | 185·0<br>173·2<br>182·9      | 178·57         | 216·17                                           | 37·60                   |
| L              | 220·2<br>211·3<br>219·5      | 215·57         | 178·22                                           | 37·35                   |
| R              | 189·1<br>168·8<br>184·8      | 177·87         |                                                  |                         |

Mean R—L=37·71 divisions.

$$\therefore 1 \text{ scale division} = \frac{.3282}{3771} = .00870 \text{ milligramme.}$$

Determination of  $(B + Y) - (A + X)$ .

| Weight in pan. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points.                                                                                                              | Differences. |
|----------------|------------------------------|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
| B + Y          | 208·5<br>221·7<br>209·5      | 215·35         |                                                                                                                                                               |              |
| A + X          | 246·9<br>214·4<br>244·1      | 229·95         | 216·32                                                                                                                                                        | 13·73        |
| B + Y          | 219·2<br>215·6<br>218·0      | 217·1          | 230·91                                                                                                                                                        | 13·81        |
| A + X          | 240·8<br>221·0<br>238·7      | 231·87         | 217·12                                                                                                                                                        | 14·75        |
| B + Y          | 226·1<br>209·4<br>223·7      | 217·15         | 232·24                                                                                                                                                        | 15·09        |
| A + X          | 221·8<br>242·3<br>224·1      | 232·62         | 216·96                                                                                                                                                        | 15·66        |
| B + Y          | 211·5<br>221·8<br>212·0      | 216·77         | 232·21                                                                                                                                                        | 15·44        |
| A + X          | 219·0<br>243·0<br>222·2      | 231·80         |                                                                                                                                                               |              |
| B + Y          | 208·3<br>229·0<br>209·9      | 219·05         | This sudden change of resting point must be due to some irregular disturbance. It is therefore rejected. It was slowly returning to nearly its former values. |              |

Mean  $(B + X) - (A + Y) = 14·75$  divisions =  $·12831$  milligramme.

Greatest deviation from the mean =  $1·02$  division =  $·0089$  mgm.

The great difference between the result here and that in series V is probably due to deposit of dust. The new mirrors had to be fixed up just before the experiment began, and the doors were open for some time. At the conclusion of the weighing I found a good deal of dust on the weights.



## VIII.—June 19.—Determination of 1 Scale Division.

| Rider on beam. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points.                                                                          | Difference due to R—L. |
|----------------|------------------------------|----------------|---------------------------------------------------------------------------------------------------------------------------|------------------------|
| L              | 235·8<br>222·3<br>233·3      | 228·42         | This is so much higher than the rest, probably through being observed soon after I entered the room, that it is rejected. |                        |
| R              | 197<br>181·2<br>193·9        | 188·32         |                                                                                                                           |                        |
| L              | 228·2<br>222·7<br>228·0      | 225·40         | 188·24                                                                                                                    | 37·16                  |
| R              | 197·4<br>180·7<br>193·9      | 188·17         | 225·46                                                                                                                    | 37·29                  |
| L              | 233·6<br>218·0<br>232·5      | 225·52         | 187·63                                                                                                                    | 37·89                  |
| R              | 191·7<br>183·3<br>190·1      | 187·10         | 225·28                                                                                                                    | 38·18                  |
| L              | 230·2<br>220·3<br>229·4      | 225·05         | 187·35                                                                                                                    | 37·70                  |
| R              | 193·8<br>182·5<br>191·8      | 187·60         |                                                                                                                           |                        |

Mean R—L=37·64 divisions.

$$1 \text{ scale division} = \frac{.3282}{37.64} = .00872.$$

Determination of  $(B+X) - (A+Y)$ .

| Weight in pan. | Extremities of oscillations. | Resting point. | Mean of preceding and succeeding resting points.                                                                                                                                                                                                | Differences. |
|----------------|------------------------------|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
| $A + Y_1$      | 245·3<br>238·7<br>244·2      | 241·72         | In one observation not recorded just before this the clamp had been loose, and the scale pan had slipped, and the resting point was thereby changed. The disturbance had apparently not subsided when this was taken, it is therefore rejected. |              |
| $B + X$        | 193·0<br>185·7<br>191·4      | 188·95         |                                                                                                                                                                                                                                                 |              |
| $A + Y$        | 226·6 *<br>243·3<br>229·2    | 235·60         | 188·22                                                                                                                                                                                                                                          | 47·38        |
| $B + X$        | 182·2<br>192·8<br>182·4      | 187·55         | 235·22                                                                                                                                                                                                                                          | 47·67        |
| $A + Y$        | 229·4<br>239·2<br>231·6      | 234·85         | 187·47                                                                                                                                                                                                                                          | 47·38        |
| $B + X$        | 194·4<br>182·5<br>192·2      | 187·4          | 234·71                                                                                                                                                                                                                                          | 47·31        |
| $A + Y$        | 239·8<br>229·7<br>239·1      | 234·57         | 187·50                                                                                                                                                                                                                                          | 47·07        |
| $B + X$        | 194·7<br>181·6<br>192·5      | 187·60         | 234·51                                                                                                                                                                                                                                          | 46·91        |
| $A + Y$        | 230·6<br>237·4<br>232·4      | 234·45         | 187·50                                                                                                                                                                                                                                          | 46·95        |
| $B + X$        | 186·0<br>189·0<br>185·6      | 187·40         |                                                                                                                                                                                                                                                 |              |

Mean  $(B+X) - (A+Y) = 47·24$  divisions  $= 0·4119$  milligramme.Greatest deviation from the mean  $= ·43$  division  $= ·00375$  mgm.

*Poynting.*

Fig 1.  
Plan of the Room.

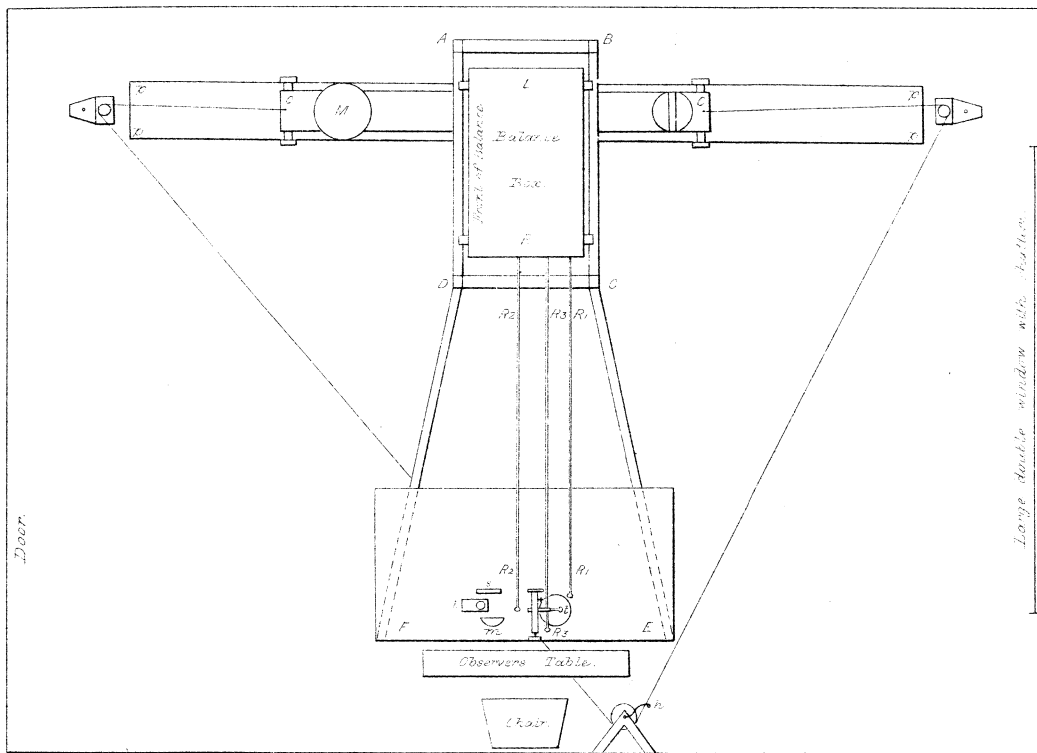
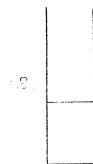
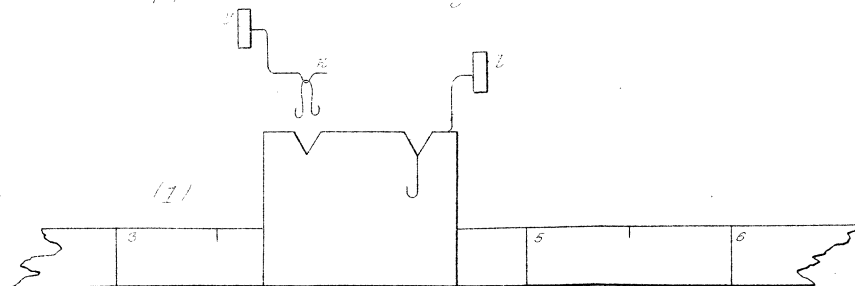
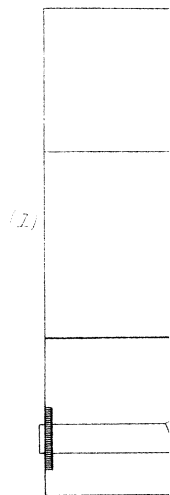


Fig. II  
Apparatus to change the riders.



*Ad. J.*



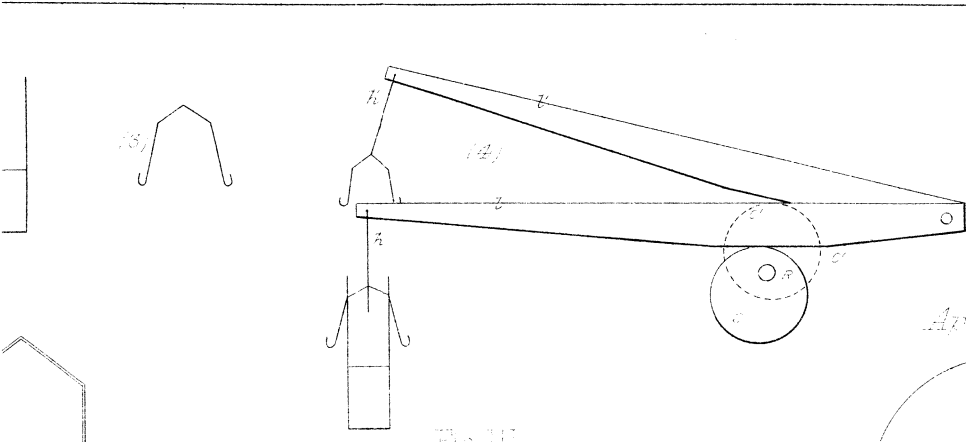


Fig. III

(2) Weight Carrying Pan and Scale Pan.

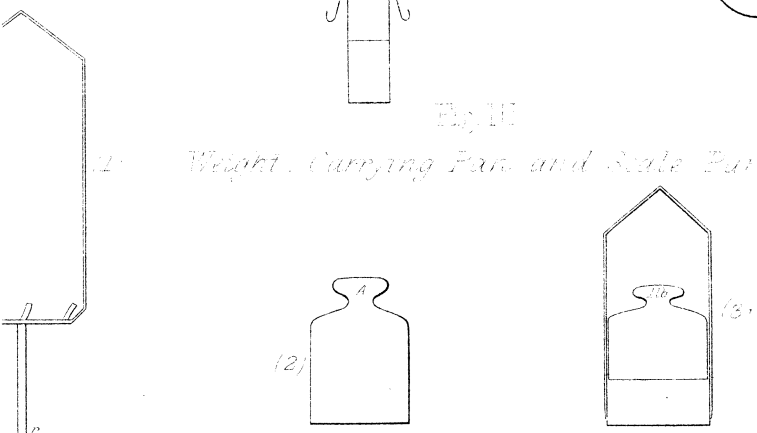


Fig. IV

Apparatus to clamp the Scale Pan.

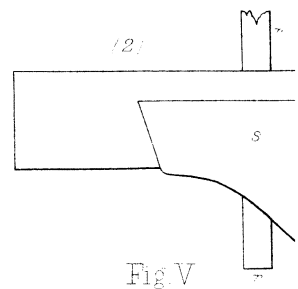
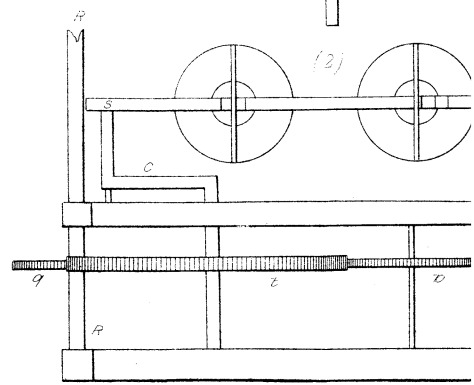
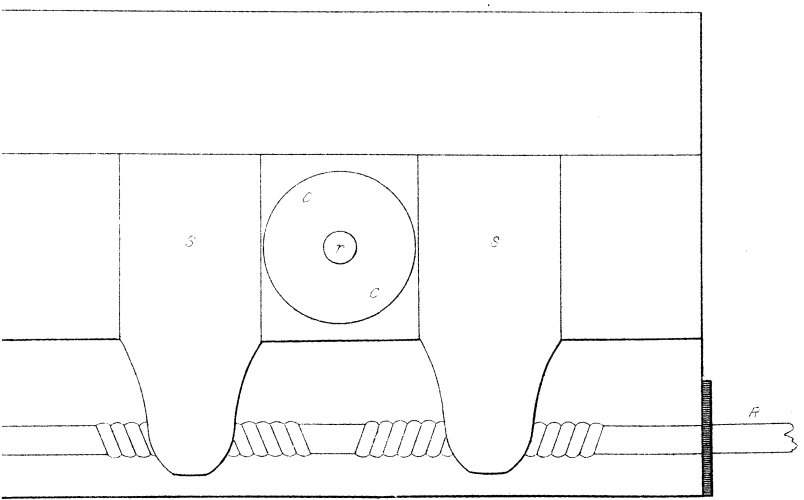
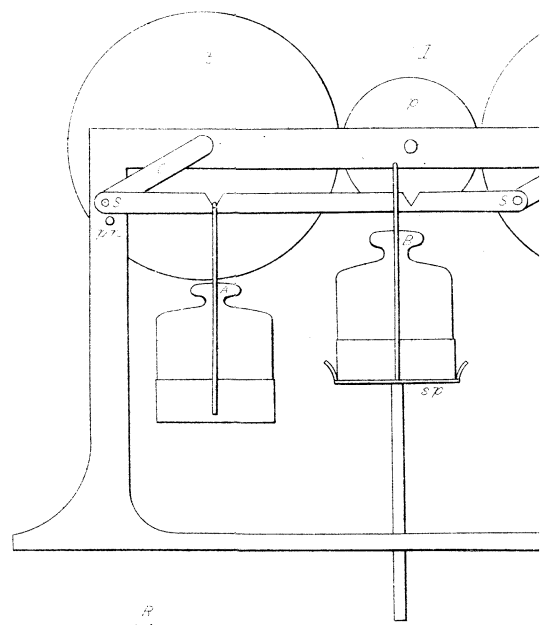
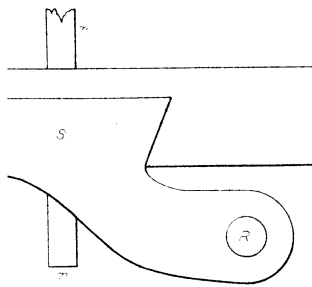


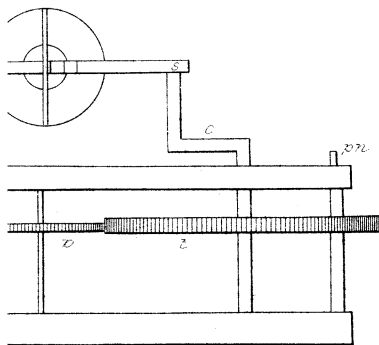
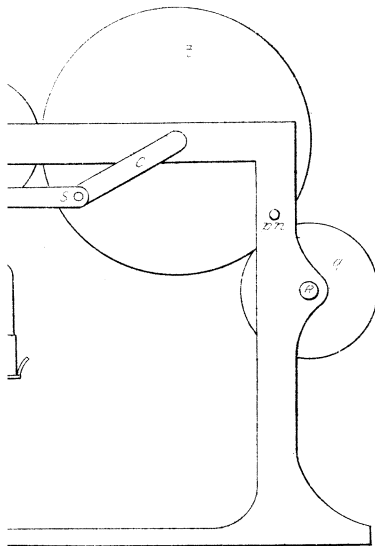
Fig. V

Apparatus to change the...





ge the weights.



*Summary.*

| Series.         | Mgms. | Greatest deviation            |                |
|-----------------|-------|-------------------------------|----------------|
|                 |       | from mean in<br>milligrammes. |                |
| 1 (B+X)−(A+Y) = | ·0718 | ·0099 }                       | A=B+·0446 mgm. |
| 2 (A+X)−(B+Y) = | ·1610 | ·0067 }                       |                |
| 3 (A+X)−(B+Y) = | ·1732 | ·0057 }                       | A=B+·0116 mgm. |
| 4 (B+X)−(A+Y) = | ·1500 | ·0122 }                       |                |
| 5 (B+Y)−(A+X) = | ·0065 | ·0057 }                       | B=A+·1989 mgm. |
| 6 (B+X)−(A+Y) = | ·4043 | ·0078 }                       |                |
| 7 (B+Y)−(A+X) = | ·1283 | ·0089 }                       | B=A+·1418 mgm. |
| 8 (B+X)−(A+Y) = | ·4119 | ·0037 }                       |                |

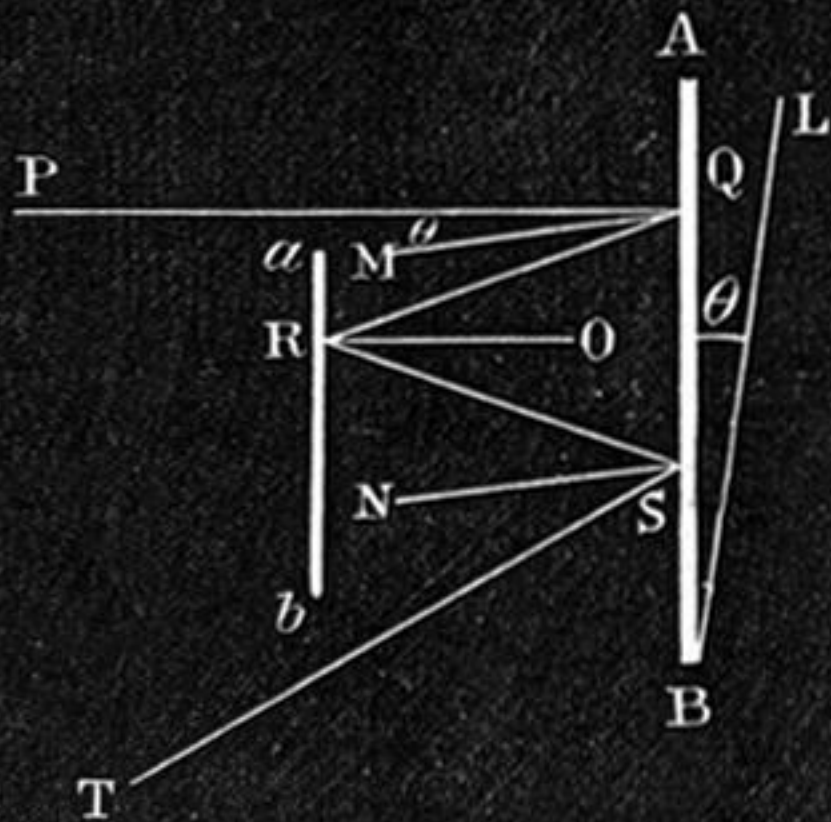
The greatest error—that is the greatest deviation of any one value from the mean of its series—in the first four series is  $\frac{1}{200000000}$ th of a pound. The greatest error in the four series Nos. 5—8 is  $\frac{1}{500000000}$ th of a pound.

## II. “On Repulsion resulting from Radiation.” Part VI. By WILLIAM CROOKES, F.R.S., V.P.C.S.

*(Abstract.)*

In this part, with which the research closes, the author first examines the action of thin mica screens fixed on the fly of an ordinary radiometer, in modifying the movements. It is found that when a disk of thin clear mica is attached 1 millim. in front of the blacked side of the vanes of an ordinary radiometer, the fly moves negatively, the black side approaching instead of retreating from the light. When a thin mica disk is fixed on each side of the vanes of a radiometer, the result is an almost total loss of sensitiveness.

In order to examine the action of screens still further an instrument is described having the screens movable, and working on a pivot independent of the one carrying the fly, so that the screens can move freely and come close either to the black or to the white surfaces of the disks. By gentle tapping the screens can be brought within 2 millims. of the black surfaces. A candle is now brought near, shaded so that the light has to pass through one of the clear disks and fall on the black surface. The black side immediately retreats, the clear disk remaining stationary for a moment and then approaching the light. If the candle is allowed to shine on the plain side of the black disk, no immediate movement takes place. Very soon, however, both disks move in the same direction away from the candle, the speed of the clear disk gradually increasing over that of the blacked disk.



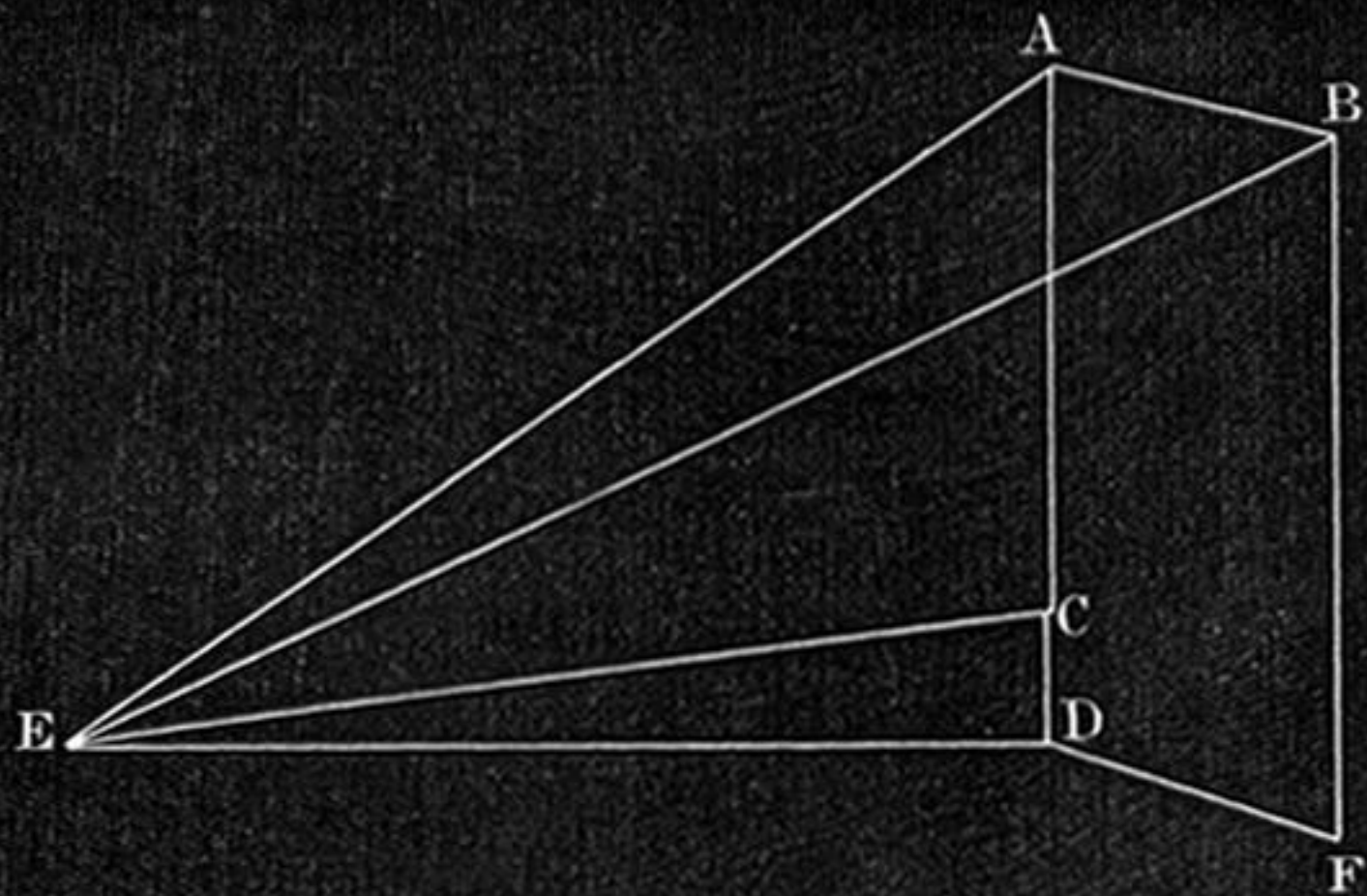




Fig 1  
Plan of the Room

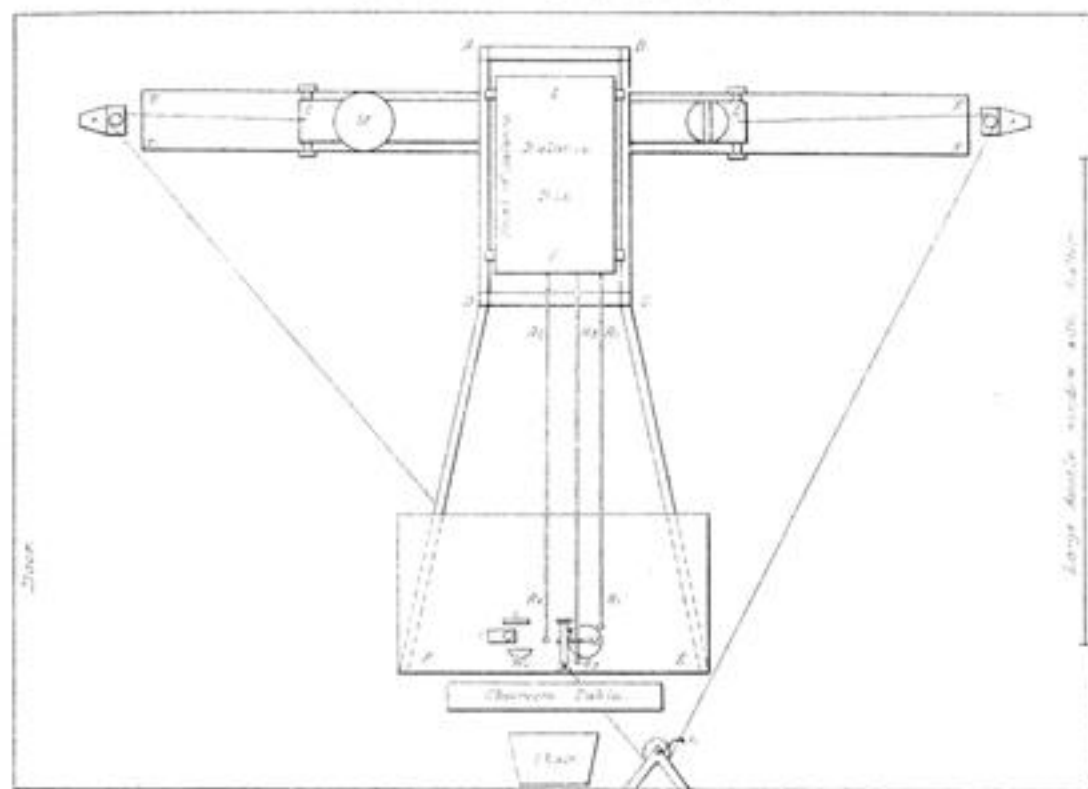


Fig 2  
Apparatus to change the riders

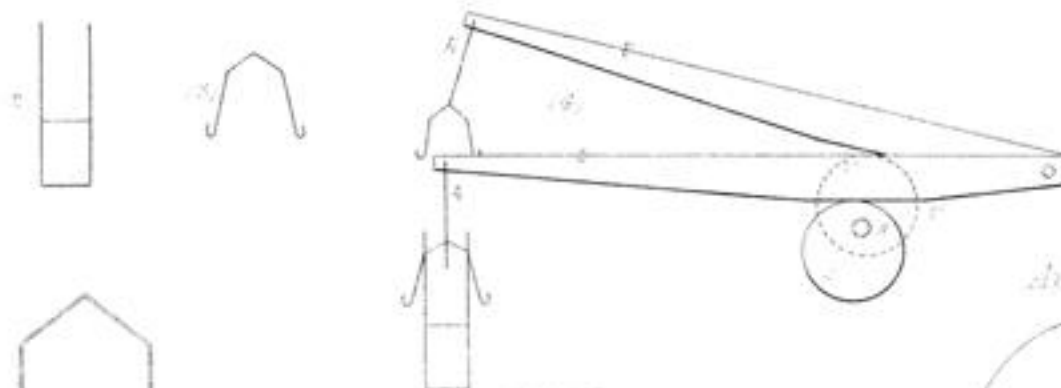
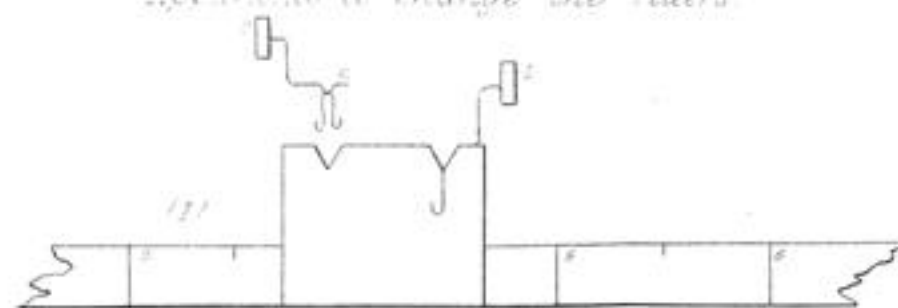


Fig 3  
Weight Carrying Pan and Scale Bar



Fig 4  
Apparatus to clamp the Scale Pan

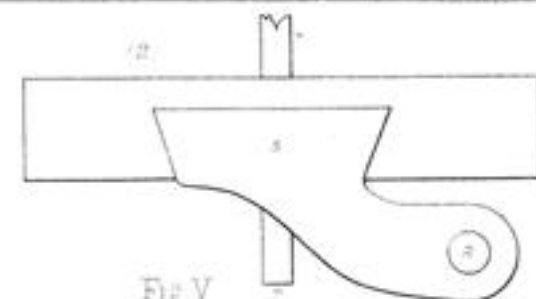
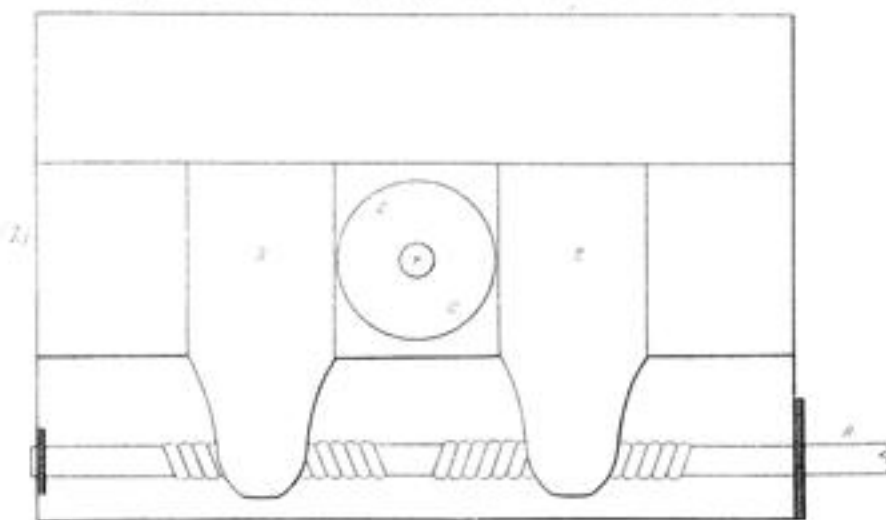


Fig 6  
Apparatus to change the weights

