screen: the light by which the eye then sees the screen is, of course, undulatory.

The phenomena in these exhausted tubes reveal to physical science a new world—a world where matter exists in a fourth state, where the corpuscular theory of light holds good, and where light does not always move in a straight line; but where we can never enter, and in which we must be content to observe and experiment from the outside.

II. "On a Machine for the Solution of Simultaneous Linear Equations." By Sir WILLIAM THOMSON, LL.D., F.R.S., President of the Royal Society, Edinburgh. Received August 30, 1878.

Let $B_1, B_2, \ldots B_n$ be n bodies each supported on a fixed axis (in practice each is to be supported on knife-edges like the beam of a balance).

Let $P_{11}, P_{21}, P_{31}, \ldots P_{n1}$, n pulleys each pivoted on B_1;
$P_{12}, P_{22}, P_{32}, \ldots P_{n2}$,, ,, B_2;
$P_{13}, P_{23}, P_{33}, \ldots P_{n3}$,, ,, B_3;
. . . .
$C_1, C_2, C_3, \ldots C_n$, n cords passing over the pulleys;
$D_1, P_{11}, P_{12}, P_{13}, \ldots P_{1n}, E_1$, the course of C_1;
$D_2, P_{21}, P_{22}, P_{23}, \ldots P_{2n}, E_2$, ,, C_2;
. . . .
$D_1, E_1, D_2, E_2, \ldots D_n, E_n$, fixed points;

$l_1, l_2, l_3, \ldots l_n$ the lengths of the cords between D_1, E_1, and D_2, E_2, . . . and D_n, E_n, along the courses stated above, when $B_1, B_2, \ldots B_n$, are in particular positions which will be called their zero positions;

l_1+e_1, l_2+e_2, . . . l_n+e_n, their lengths between the same fixed points, when $B_1, B_2, \ldots B_n$ are turned through angles $x_1, x_2, \ldots x_n$ from their zero positions;

$$\begin{array}{l}(11), (12), (13), \ldots (1n),\\ (21), (22), (23), \ldots (2n),\\ (31), (32), (33), \ldots (3n),\\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\end{array}$$

quantities such that

$$\left.\begin{array}{l}(11)x_1+(12)x_2+\ \ldots\ +(1n)x_n=e_1\\ (21)x_1+(22)x_2+\ \ldots\ +(2n)x_n=e_2\\ (31)x_1+(32)x_2+\ \ldots\ +(3n)x_n=e_3\\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\ (n1)x_1+(n2)x_2+\ \ldots\ +(3n)x_n=e_n\end{array}\right\} \quad \ldots \quad (\mathrm{I})$$

We shall suppose $x_1, x_2, \ldots x_n$ to be each so small that (11), (12),

. . . (21), &c., do not vary sensibly from the values which they have where $x_1, x_2, \ldots x_n$, are each infinitely small. In practice it will be convenient to so place the axes of $B_1, B_2, \ldots B_n$, and the mountings of the pulleys on $B_1, B_2, \ldots B_n$, and the fixed points D_1, E_1, D_2, &c., that when $x_1, x_2, \ldots x_n$ are infinitely small, the straight parts of each cord and the lines of infinitesimal motion of the centres of the pulleys round which it passes are all parallel. Then $\frac{1}{2}(11), \frac{1}{2}(21), \ldots \frac{1}{2}(n)$ will be simply equal to the distances of the centres of the pulleys $P_{11}, P_{21}, \ldots P_n$, from the axis of B_1;

$\frac{1}{2}(12), \frac{1}{2}(22) \ldots \frac{1}{2}(n2)$ the distances of $P_{12}, P_{22}, \ldots P_{n2}$ from the axis of B_2, and so on.

In practice the mounting of the pulleys are to be adjustable by proper geometrical slides, to allow any prescribed positive or negative value to be given to each of the quantities (11), (12), . . . (21), &c.

Suppose this to be done, and each of the bodies $B_1, B_2, \ldots B_n$ to be placed in its zero position and held there. Attach now the cords firmly to the fixed points $D_1, D_2, \ldots D_n$ respectively; and passing them round their proper pulleys, bring them to the other fixed points $E_1, E_2, \ldots E_n$, and pass them through infinitely small smooth rings fixed at these points. Now hold the bodies $B_1, B_2, \ldots$ each fixed, and (in practice by weights hung on their ends, outside $E_1, E_2, \ldots E_n$) pull the cords through $E_1, E_2, \ldots E_n$ with any given tensions* $T_1, T_2, \ldots T_n$. Let $G_1, G_2, \ldots G_n$ be moments round the fixed axes of $B_1, B_2, \ldots B_n$ of the forces required to hold the bodies fixed when acted on by the cords thus stretched. The principle of "virtual velocities," just as it came from Lagrange (or the principle of "work"), gives immediately, in virtue of (I),

$$\left.\begin{array}{l} G_1=(11)T_1+(21)T_2+ \ \ldots \ +(n1)T_n \\ G_2=(12)T_1+(22)T_2+ \ \ldots \ +(n2)T_n \\ \ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ G_n=(1n)T_1+(2n)T_2+ \ \ldots \ +(n_n)T_n \end{array}\right\} \quad \ldots \quad \text{(II)}.$$

Apply and keep applied to each of the bodies, $B_1, B_2 \ldots B_n$ (in practice by the weights of the pulleys, and by counter-pulling springs), such forces as shall have for their moments the values $G_1, G_2 \ldots G_n$, calculated from equations (II) with whatever values seem desirable for the tensions $T_1, T_2 \ldots T_n$. (In practice, the straight parts of the cords are to be approximately vertical, and the bodies B_1, B_2, are to be each balanced on its axis when the pulleys belonging to it are

* The idea of force here first introduced is not essential, indeed is not technically admissible to the purely kinematic and algebraic part of the subject proposed. But it is not merely an ideal kinematic construction of the algebraic problem that is intended; and the design of a kinematic machine, for success in practice, essentially involves dynamical considerations. In the present case some of the most important of the purely algebraic questions concerned are very interestingly illustrated by these dynamical considerations.

removed, and it is advisable to make the tensions each equal to half the weight of one of the pulleys with its adjustable frame.) The machine is now ready for use. To use it, pull the cords simultaneously or successively till lengths equal to $e_1, e_2, \ldots e_n$ are passed through the rings $E_1, E_2, \ldots E_n$, respectively.

The *pulls* required to do this may be positive or negative; in practice, they will be infinitesimal, downward or upward pressures applied by hand to the stretching weights which (§) remain permanently hanging on the cords.

Observe the angles through which the bodies $B_1, B_2, \ldots B_n$ are turned by this given movement of the cords. These angles are the required values of the unknown $x_1, x_2, \ldots x_n$, satisfying the simultaneous equations (I).

The actual construction of a practically useful machine for calculating as many as eight or ten or more of unknowns from the same number of linear equations does not promise to be either difficult or over-elaborate. A fair approximation being found by a first application of the machine, a very moderate amount of straightforward arithmetical work (aided very advantageously by Crelle's multiplication tables) suffices to calculate the residual errors, and allow the machines (with the setting of the pulleys unchanged) to be re-applied to calculate the corrections (which may be treated decimally, for convenience): thus, 100 times the amount of the correction on each of the original unknowns, to be made the new unknowns, if the magnitudes thus falling to be dealt with are convenient for the machine. There is, of course, no limit to the accuracy thus obtainable by successive approximations. The exceeding easiness of each application of the machine promises well for its real usefulness, whether for cases in which a single application suffices, or for others in which the requisite accuracy is reached after two, three, or more of successive approximations.

December 12, 1878.

W. SPOTTISWOODE, M.A., D.C.L., President, in the Chair.

Dr. Philipp Hermann Sprengel was admitted into the Society.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—