phone, and attributed the effect to its true cause, viz., the expansion of bodies under the influence of heat, which, in fact, is the explanation of all microphone receivers.

Ader reproduced speech by the vibrations of a wire conveying currents of electricity, but he found that only magnetic metals were effective, and therefore, like De la Rive, he attributed the result to magnetic agencies. (*Vide* Count du Moncel, "Telegraphic Journal," March 1, 1879.)

These and many other sonorous effects of currents on wires may be really due to such heat effects as I have described.

II. "A Preliminary Account of the Reduction of Observations on Strained Material, Leyden Jars, and Voltameters." By JOHN PERRY and W. E. AYRTON. Communicated by Professor G. G. STOKES, Sec. R.S. Received April 17, 1880.

It has been shown by Dr. Hopkinson that, if two Leyden jars be made of the same glass, but of different thicknesses:—

1st. If they be charged with the same difference of potential for equal times, discharged for equal times, and then insulated, that the residual charge will, after equal times, have in both cases the same difference of potential.

2nd. That residual charge is proportional to exciting charge. These propositions may be included in one law—the superposition of simultaneous forces is applicable to the phenomena of residual charge. All the investigations in Dr. Hopkinson's paper in the "Transactions of the Royal Society," vol. 167, part 2, serve to prove this law, and, so far, they support the theory of residual charge, which we owe to the late Professor Clerk Maxwell. We should, therefore, be inclined to think that one of the best methods of investigating the relation between the relative powers of different glasses to possess residual charge would be simply to charge jars made of these glasses for the same great length of time, discharge for the same short intervals, and insulate, measuring in each case the time increase of soaking out of the residual charge. Whatever the thickness of the glass or the amount of the original charge, we know that the same glass will always give the same proportion of residual charge at the same times from insulation. Any change of the state of the glass caused by heat would be shown as a change on the curve of increasing residual charge. There seems to be no doubt that this method would give what may be called a measure of the specific power of producing residual charge phenomena of the glass experimented on.

In this way, since 1875, we have obtained a considerable number of

curves for residual charge in condensers of different dielectrics at different temperatures, and so we may say that we have measured the power of producing residual charge phenomena in the substances experimented upon in different states, and this power, which we call our "residual-charge-function," ought, according to Dr. Hopkinson, to be constant for the same substance at the same temperature. Thus, if v_t is the residual charge at the time t after insulation, and if V was the original long continued charge, $v_t \div \mathrm{V}$ is our residual-charge-function for the time t, which we may designate by F_t. Of course, it is evident that if F'_t is our function for a different dielectric, then $\mathrm{F}_t \div \mathrm{F}'_t$ is not necessarily the same as $\mathrm{F}_{t'} \div \mathrm{F}'_{t'}$; in fact, the residual-charge-function is only completely specified when a table is given of its values at different times after insulation. We have used the same name and symbol in the experiments we have been simultaneously making during the last five years on strained wires and beams, and on voltameters and magnets, and of course there is always the difficulty experienced by Dr. Hopkinson with his more complicated function "ψt," which renders it necessary to give a table of values instead of only one value of the function. Thus, for two particular kinds of glass, Dr. Hopkinson finds—

$$\frac{\psi(5)-\mathrm{B}}{\psi'(5)-\mathrm{B}}=3{\cdot}5,$$

whereas

$$\frac{\psi(60)-\mathrm{B}}{\psi'(60)-\mathrm{B}}=6,$$

showing how with the same two glasses the ratio is affected by varying the time from 5 to 60. Again, also in his investigation of the influence of temperature on residual charge in the same glass, he finds that at 175° F. the values of $\psi(1)-13$ and $\psi(5)-13$ are respectively 0·38 and 0·034, with a ratio one to the other of 11; whereas the values of these functions at 108° F. are 0·155 and 0·05 respectively with a ratio one to the other of 3·1. This method of measurement is subject, therefore, to exactly the same objections as the easier method of which we have spoken; in fact, we are not aware that hitherto any method has shown itself to be better than actually giving the curves of rise of potential after insulation, or the curves of change of current during charge and discharge of a condenser. This is the method which we have found the easiest for the discussion of such phenomena, and we may direct attention to such curves for voltameters in a paper communicated by us to the Society of Telegraph Engineers, and which appeared in their Journal, vol. v, 1877, Nos. 15, 16, p. 391. Curve EFGG′ there shown, in fig. 4, p. 396, is especially interesting, as we see from it that if a voltameter with dilute sulphuric acid be charged with one Minotto's cell for a certain time, then discharged for a

certain time, by removing the battery and connecting together the platinum plates of the voltameter through a resistance, and finally the platinum plates be insulated, that the difference of potentials between the plates gradually rises, presenting phenomena exactly like the soaking out of the residual charge in a Leyden jar that has been insulated after being discharged for a short time.

The unsatisfactory nature of the ordinary experiments with condensers has led us to try if we could find the actual constants in Professor Clerk Maxwell's differential equation—

$$\frac{d^n v}{dt^n}+a\frac{d^{n-1}v}{dt^{n-1}}+\ \ldots\ +mv=a'u+b'\frac{du}{dt}+\ \ldots\ +m'\frac{d^{n-1}u}{dt^{n-1}}\quad . \quad . \quad (1),$$

where v is the difference of potentials of the two surfaces of the condenser, and u the current flowing into or from the condenser at the time t, for it is quite evident that if his theory be correct these constants completely determine the residual charge phenomena of the substance. The difficulty in obtaining these constants has involved us in a very large amount of labour, and we have only yet reduced a small number of the curves we have experimentally obtained during the last five years; in addition we have found that many of our experiments will have to be repeated with somewhat different conditions for the investigation to be regarded as complete. But some of the results already obtained from this reduction are of interest. Thus, in one set of observations, we have tried to determine the constants from curves of the soaking out of the residual charge of a Leyden jar discharged for a very short time, having been previously kept charged for so long a time in a room of nearly constant temperature, that no loss was observed during two or three days, that is, the jar may be regarded as possessing almost infinite insulation.

The Leyden jar employed for these experiments was that of a Thomson's quadrant electrometer, and the measure of its charge was determined from the deflection produced in the needle of the electrometer itself when the electrodes of a Latimer Clark's constant cell attached to the electrometer quadrants were reversed. In reality three such cells in succession were used in order to detect any irregularity in the cells themselves. The Leyden jar was first rather highly charged, and then left with the replenisher untouched for some days until the loss of charge from day to day became imperceptible. The readings obtained with each of the three Clark's cells were then accurately taken, when one of them being left on the electrometer electrodes the Leyden jar was discharged for a very short time and then insulated; the soaking out of the residual charge was now measured by taking time-readings of the increase of deflection produced by the constant cell. To allow for alteration of zero of the electrometer arising from alterations of its charge frequent reversals of the cell

were taken. After some minutes the right and left readings due to the reversal of the cell could be observed on the ordinary scale placed two metres away from the electrometer; but at the commencement of the insulation after discharge, the soaking out was far too rapid to allow of numbers on the scale being read, consequently the following device was employed to allow of our obtaining the early part of the curve with accuracy. In the slit of the lamp were two cross-wires, an image of which by means of the mirror in the electrometer and of an auxiliary lens was formed on a cylinder 13 centims. in diameter and 100 centims. in length, revolved fairly rapidly by clockwork around a horizontal axis. The comparatively rapid motion of the image of the cross-wires was then accurately followed by an observer, who with a pencil dotted its position from to time on a sheet of paper wrapped round the revolving cylinder. On unwrapping the paper the dots made by the pencil enabled the curve of rise of potential to be drawn accurately from the beginning in each case.

In these experiments as the jar is insulated, u, $\frac{du}{dt}$, &c., equal 0, and the solution of the differential equation (1) is

$$v = \mathrm{A} + \mathrm{B}\epsilon^{-\beta t} + \mathrm{C}\epsilon^{-\gamma t} + \quad . \quad . \quad . \quad . \qquad . \quad (2),$$

where β, γ, &c., are such that $x+\beta$, $x+\gamma$, &c., are factors of the expression

$$x^n + ax^{n-1} + ax^{n-2} + \quad . \quad . \quad . \quad . \quad + mx.$$

Hence if we determine β, γ, &c., we determine the constants of the differential equation (1). Our method of reduction was as follows :—

If is less than γ, γ than δ, and so on in equation (2), then, after certain intervals of time, certain of the exponents became unimportant; and if the time is sufficiently great, equation (2) reduces itself to

$$v = \mathrm{A} + \mathrm{B}\epsilon^{-\beta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

If a curve is obtained experimentally connecting v and t, it is easy to determine A, B, and β. Thus from the observations given in the first two columns of the following table, corresponding with one of our several experiments with the Leyden jar (v being very slightly corrected by a curve), $\frac{dv}{dt}$ was determined for the various times. Then the values of v and $\frac{dv}{dt}$ for the same times being used as co-ordinates of points on a sheet of squared paper, we found that for the last 140 hours of our experiment the points lay, with very considerable accuracy, in a straight line; and hence during this time we may assume equation (3) to correctly represent the law of the soaking out of the residual charge. These plotted points enabled A to be deter-

mined. It is evident, also, that the values of log $(v-A)$ and t when plotted as co-ordinates of points give a straight line and enable B and β to be determined. We found that equation (3) for the particular experiment in question became

$$v'=474-177{\cdot}34\epsilon^{-0{\cdot}0000637t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

and giving values to t we have calculated column 3 in the following table. Proceeding in precisely the same way with the remainders $(v-v')$ given in column 4, we obtain the other coefficients and exponents in the equation (2).

Soaking out of the Residual Charge in the Leyden Jar.
June 16, 1876.

Time in minutes.	Observed value of v.	v' calculated from (4).	Difference $(v-v')$.
0	0	296·7	296·7
0·5	82	296·7	214·7
1	102	296·8	194·8
2	120	296·9	176·9
4	138	297·1	159·1
7	155	297·4	142·4
10	168	297·8	129·8
20	190	298·8	108·8
40	213	301·1	88·1
80	236	305·4	69·4
150	264	312·8	48·8
250	296	322·8	26·8
350	324	332·1	8·1
500	345	345·0	0
700	362	360·1	−1·9
1,000	398·4	380·2	1·8
1,500	404·5	405·8	1·3
2,000	423·9	424·4	0·5
2,500	437·9	437·9	0
3,000	447·6	447·7	0·1
4,000	459·0	460·1	1·1
5,000	465·4	466·6	1·2
6,000	469·5	470·1	0·6
7,000	471·8	471·9	0·1
8,000	472·9	472·6	−0·3
9,000	473·6	473·7	0·1

We have stated above that we slightly corrected, by means of a curve, the values of v before putting them in column 2. It would

make the present paper too long to describe in full the slightly wavy nature of the actual curves connecting v and t obtained in our experiments. That the variations are mainly due to slight fluctuations in the temperature of the room is seen from our finding a diurnal period in the waves. Indeed, we have found that small changes of temperature very materially affect all the phenomena with which we deal in this paper. But there is something more in these small waves than can be explained by changes of temperature, and possibly the solution (2) of the differential equation may have some coefficients which are sine and cosine functions of the time. A study of the constants on Professor Maxwell's theory ought to tell whether

$$x^n+ax^{n-1}+ \dots +mx=0$$

has any unreal roots.

In the same way as that above described, we have obtained the exponentials of

$$u=A+B\epsilon^{-bt}+C\epsilon^{-ct}+ \quad \dots \dots \quad (5),$$

the solution of equation (1) when v is kept constant, from actual observations of the current flowing into a condenser from a battery with a constant electromotive force. The curve for u, just like the curve described above, possesses a slight waviness, and when this is neglected its simple logarithmic nature subsequently to a certain period of time is very striking.

In former papers read before this and other Societies we have given our reasons for believing that the phenomena of polarisation in voltameters, under the action of an electromotive force insufficient to produce disruptive discharge in the liquid, that is rapid and visible decomposition, are of exactly the same nature as the phenomena exhibited by condensers having glass or other solid dielectrics. And we think that if our analogies fail for greater electromotive forces it is because rupture of a solid dielectric introduces instantaneously entirely different conditions. An examination of the following reductions, which we give as a sample of the observations of current flowing into and out of a voltameter containing ordinary water and charged with one Minotto's cell (which has an electromotive force less than will produce visible decomposition), will show that there is a striking adherence to the logarithmic law, and that we here add another analogy to those we have already given between condensers with liquid and with solid dielectrics.

In the following table t is the time, in minutes, after applying one Minotto's cell, u is the current flowing through the voltameter, the unit of u being 0·371 microfarad per second; u' is the current calculated from the formula

$$u'=180{\cdot}1+426{\cdot}6\epsilon^{-0{\cdot}01773t} \quad \dots \dots \quad (6),$$

which we have obtained from the general solution (5) of the differential equation by a method explained towards the commencement of this paper.

Charging of a Voltameter, consisting of Platinum Plates dipping in ordinary Water, with one Minotto's Cell.

February 14, 1878.

Time in minutes.	Observed value of u.	u' calculated from (6).	Difference $(u-u')$.
4	690	587	103
10	618	537	81
20	526	479	47
30	455	431	14
40	398	390	8
50	355	356	−1
60	322	327	−5
70	297	303	−6
80	279	283	−4
90	265	267	−2
100	252	253	−1
110	240	241	−1
120	230	231	−1
130	223	223	0
140	216	216	0
150	211	210	1
160	205	205	0
170	201	201	0
180	198	198	0
190	195	195	0

After the deflection had diminished from 690 to 195 in rather more than three hours, the Minotto's cell was suddenly removed, and the voltameter discharged through the galvanometer, the terminals of which at the same moment were reversed, so that the deflections for discharge should be on the same side as those for charging, in order to somewhat diminish the swinging of the needle.

The general differential equation (1) when applied to the discharge curve thus obtained by experiment, leads to the solution—

$$u'=11{\cdot}4+158{\cdot}5\epsilon^{-0{\cdot}0579t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

from which u' given in the third column of the following table has been calculated.

Discharging of the Voltameter.

February 14, 1878.

Time in minutes.	Observed value of u.	u' calculated from (7).	Difference $(u-u')$.
6	244·0	123·4	120·6
10	156·8	100·2	56·6
15	91·4	77·9	13·5
20	62·0	61·2	0·8
30	37·8	39·3	−1·5
40	26·6	27·0	−0·4
50	20·6	20·2	0·4
60	16·4	16·3	0·1
70	14·0	14·1	−0·1
80	13·0	12·9	0·1

The reason why the exponentials in the equation connecting currents and time when charging differ from those connecting currents and time when discharging the voltameter, is as follows:—If r is the resistance of the battery, galvanometer, connexions, &c., but excluding the resistance of the voltameter itself, then—

$$u=\frac{V-v}{r},$$

where V is the electromotive force of the battery, and v that of the voltameter, at any time t during the charging—

$$\therefore \frac{dv}{dt}=-r\frac{du}{dt},$$

$$\frac{d^2v}{dt^2}=-r\frac{d^2u}{dt^2},$$

which when substituted in the general differential equation, enable the integration to be effected. But during the discharge after the removal of the battery, if r' is the resistance of the galvanometer, connexions, &c., but excluding that of the voltameter itself, then—

$$-u=\frac{v}{r'},$$

$$\therefore \frac{dv}{dt}=-r'\frac{du}{dt},$$

&c.

Consequently the constants in the differential equation, and therefore the exponentials in the equations of our curves, must be different.

Guided by this, we have arranged in our later experiments that r

and r' should be both very small, so that the exponentials in the equations both of the charge and discharge curves should be the same.

When a beam or wire is suddenly bent or twisted, the forces producing the strain being maintained constant, it is known that the strain increases with the time, and if the forces suddenly cease to act, that the strain does not altogether disappear at once, but diminishes gradually. This strain w, we have found can, like differences of potential and current, be most easily expressed as the sum of certain exponentials of the time. There are other analogies with condensers. Rapid tremors given to a strained body, our experiments show, cause it to attain its ultimate state more rapidly; one example of this is the more rapid soaking out of the residual charge in a Leyden jar, as noticed by Dr. Hopkinson; our curves connecting w and t for strained beams, like those connecting v and t in condensers, or u and t in voltameters, we find to be of a slightly wavy nature, and to be all similarly affected by change of temperature. Like several other experimenters, we have found that the strain in a body at any time depends not only on the forces acting upon the body at that time, but also on all the forces which have acted on the body during all previous time; so that, for instance, if a beam fixed at the ends has been loaded negatively for some time, if this load be removed and a positive load applied and soon taken off, the beam may be found to have a negative deflection; or again, if a current be sent for some time through a voltameter in one direction, and then reversed for some time, and finally the battery removed, the discharge from the voltameter, through a galvanometer, will, in certain cases, first be found opposite to the latter charging current, but subsequently it will become nought, and then become the opposite of the first charging current, this experiment being best seen with electrolytes having a certain amount of rigidity. Similar phenomena are also evidenced with iron under magnetisation, and Dr. Hopkinson's experiments on Leyden jars also lead to similar results.

We have consequently, while investigating the results we have experimentally obtained during the last few years with strained substances, been led to construct the following theory, which is analogous with that of the late Professor Clerk Maxwell for Leyden jars.

A perfect fluid is such that the only stress which can exist at any small interface which may be drawn in the fluid is a stress normal to the interface, and hence if the fluid is contained between two parallel planes which have a motion in their planes relatively to one another there is no force required to cause this motion, or to maintain it, in the case of a perfect fluid. If the fluid is a gas or a vapour, we can calculate the force which would be required to maintain a given relative

velocity of the plates, and our students have obtained experimentally some materials (not yet published) for the calculation of such forces when the fluid is water. If the fluid is tar, or a mixture of tar and pitch, in any proportion, or even if it is the more solid substance pitch or sealing-wax, we find that although the viscosity is almost infinitely greater than in the case of water, still the phenomena exhibited are exactly the same. And just as experiments on water enable us to examine best some phenomena exhibited by viscous substances, so experiments on sealing-wax enable us to make the best examination of other phenomena.

When external forces suddenly act on a viscous fluid (and we know that all substances, whether solid, liquid, or gaseous, come under the head of "viscous fluids,"), tending to cause strains in the material, we can calculate in the well-known way the strains and stresses. If the forces do not alter, and if the shape of the body varies very little with time, the stresses do not alter, but the strains increase according to a law—

$$X = r\frac{ds}{dt},$$

where X is the stress at any interface, s the corresponding strain, t the time, and r a constant. Of this strain, if the part which was suddenly produced, and which is suddenly removable, is f, then—

$$X = kf,$$

k being the coefficient usually given in books to connect the corresponding stress and strain.

During the increase of strain, mechanical energy is being converted into heat, through the agency of internal friction or viscosity at the rate—

$$X\frac{ds}{dt}$$

per unit time.

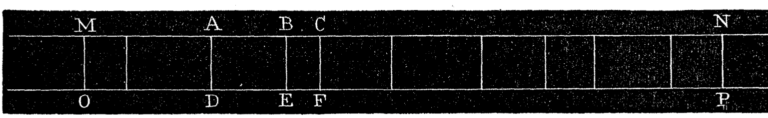
If the external forces are not kept constant, the stresses X alter, and also the strains; the second relation given above remains the same, but the first becomes—

$$X = r\frac{d}{dt}(s-f).$$

Now the first and last relations constitute our hypothesis; we have arrived at it by analogy, and not yet by experiment. We know that for the motions of pendulums, &c., in air, this law holds, and as the velocity or $\frac{ds}{dt}$ is less and less, the law is found to be more and more true. For the motion of steamships it is usual to assume the law—

$$X = r\left(\frac{ds}{dt}\right)^n,$$

where n varies from 2 to 2·28, but as motions in water become slower and slower, n becomes more and more nearly equal to unity. In the following investigation, we assume that the rates of deformation of the materials, after the first instant, are so slow, that n always equals unity. Our hypothesis is exactly analogous with Ohm's law in electricity; and just as Ohm's law has only been proved for metallic conductors of electricity, so our hypothesis is only known to be true for gases and liquids. All the analogies which we have observed cannot be discussed in this short paper, but they are such that we are led to believe that if our hypothesis proves to be untrue for materials subjected to small strains, then Ohm's law will prove untrue for currents of electricity in bad conductors. It will be seen that it leads to the conclusion that just as there are no perfect insulators of electricity, so no material, however rigid it may appear to be to us, can for an infinite time resist the effect of even small forces tending to change its form.



Let MNOP be a large prism of unit square section, formed of blocks of different materials of lengths, a_1, a_2, &c. Let them be subjected to shear stress by the action of tangential force v distributed over the surface MN, and an equal and opposite tangential force distributed over the parallel plane OP. Let the compound prism be so long that we need not speak of the terminal couple which is required to produce equilibrium. Let f_1, f_2, &c., be the strains existing in the blocks which would be instantaneously destroyed if the stress disappeared, so that if the shear stresses in the blocks are respectively X_1, X_2, &c., then—

$$X_1 = k_1 f_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8),$$

$$X_2 = k_2 f_2, \text{ \&c.,}$$

where k_1, k_2, &c., are the moduli of elasticity of the separate blocks. The distributed external force acting on either of the upper or lower sides of the block being of course $a_1 X_1$.

If the velocity of OP with regard to MN is u, then u is the rate at which each block is gaining strain, because the motion of OP with regard to MN is the measure of the common strain which exists in all the blocks. Consequently, by our hypothesis for any block, we have—

$$X_1 = r_1\left(u - \frac{df_1}{dt}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

where r_1 is a constant for the material of which the particular block is composed. Hence, combining (8) and (9) we have—

$$u=\frac{1}{r_1}X_1+\frac{1}{k_1}\frac{dX_1}{dt},$$

from which by denoting in the usual way an inverse operation

$$X_1=\left(\frac{1}{r_1}+\frac{1}{k_1}\frac{d}{dt}\right)^{-1}u.$$

The total distributed force v is of course equal to the sum of all the forces a_1X_1, a_2X_2, &c., so that

$$v=\left\{a_1\left(\frac{1}{r_1}+\frac{1}{k_1}\frac{d}{dt}\right)^{-1}+a_2\left(\frac{1}{r_2}+\frac{1}{k_2}\frac{d}{dt}\right)^{-1}+\ \ldots\ldots\ \&c.\right\}u,$$

an equation between the force distributed over either of the parallel planes, and the rate of motion of the planes relatively to one another.

If any number of blocks are of the same material, or if, although not of the same material, the ratio of r to k is the same in them, then it is evident that for all such blocks there will only be one inverse term of the above equation. Hence if there are n different materials having different ratios of r to k, we have the general equation

$$\frac{d^nv}{dt^n}+b_1\frac{d^{n-1}v}{dt^{n-1}}+\ \ldots\ +b_{n-1}\frac{dv}{dt}+b_nv$$
$$=c_nu+c_{n-1}\frac{du}{dt}+\ \ldots\ +c_2\frac{d^{n-2}u}{dt^{n-2}}+c_1\frac{d^{n-1}u}{dt^{n-1}}\quad\ldots\quad(10).*$$

In the preceding we have followed Professor Clerk Maxwell's method of investigating the charge of a condenser, and the currents flowing into it, the method of reasoning from our hypothesis regarding the action of a viscous block being the same as his reasoning from Ohm's law. From the form of the equation it is evident that the order

* If we put $m_1=\frac{r_1}{k_1}$, &c., and let

s^r represent the sum of products of m_1, m_2, &c., r at a time,

s^r_p " " " r at a time, but only such as involve m_p,

then the differential equation is evidently as follows :—

$$v+s'\frac{dv}{dt}+s^2\frac{d^2v}{dt^2}+\ \ldots\ldots\ +s^n\frac{d^nv}{dt^n}=\sigma u+\left(s'\sigma-\sum_{p=1}^{p=n}a_pr_ps_p'\right)\frac{du}{dt}$$
$$+\left(s^2\sigma-\sum_{p=1}^{p=n}a_pr_ps_p{}^2\right)\frac{d^2u}{dt^2}+\ \ldots\ldots\ +\left(s^{n-1}\sigma-\sum_{p=1}^{p=n}a_pr_ps^{n-1}_p\right)\frac{d^{n-1}u}{dt^{n-1}},$$
where $\sigma=a_1r_1+a_2r_2+\ \ldots\ldots\ +a_nr_n$,

Now if the body consists of only two substances, then we have

$$v+\left(\frac{r_1}{k_1}+\frac{r_2}{k_2}\right)\frac{dv}{dt}+\frac{r_1r_2}{k_1k_2}\frac{d^2v}{dt^2}=(a_1r_1+a_2r_2)u+\left\{r_1r_2\left(\frac{a_2}{k_1}+\frac{a_1}{k_2}\right)\right\}\frac{du}{dt}$$

of the materials is indifferent, so that if there are several blocks of the same substance, they may be subdivided, or united without altering the phenomena. An investigation of the cases in which the materials are arranged otherwise than in rectangular blocks of unit depth, would lead to similar results, and if we may assume our hypothesis to be true for other kinds of strain than shear, then the above equation may be proved to be true for any strain of any heterogeneous body strained in any way.

The integration of the general differential equation (10), where v is constant, is, putting $\frac{dw}{dt}$ for u,

$$w = A + Bt + C\epsilon^{-\gamma t} + D\epsilon^{-\delta t} + \ldots.$$

where γ, δ, &c., are roots of the equation

$$x^n + \frac{c_2}{c_1}x^{n-1} + \ldots. + \frac{c_n}{c_1}x - \frac{b_n v}{c_1} = 0,$$

and when v is maintained at zero during recovery after removal of the load

$$w = A_1 + C_1\epsilon^{-\gamma t} + D_1\epsilon^{-\delta t} + \ldots.$$

Of course if complete tables of the values of u and v, and their differential coefficients, could be given for all times during any experiment, it would be best to use the general differential equation in determining the constants. The method we have adopted is essentially the same as that we have employed in the analogous cases for Leyden jars, voltameters, &c. The wavy nature of some of the curves we have experimentally obtained connecting strain and time in strained beams, indicates that perhaps the equation given above may have imaginary roots, and that possibly there may be terms in our solutions of the shape

$$C \cos bt\, \epsilon^{-\beta t}.$$

Much, however, of the waviness we have observed, we have traced to variations of temperature, as it generally has a periodic time equal to 24 hours.

Our theory for strained beams and twisted wires we have tested to about the same extent as Professor Clerk Maxwell's theory for Leyden

where v is the load on the beam or couple twisting the wire, and u is $\frac{dw}{dt}$ if w is the deflection of the beam or twist of the wire.

If v is constant then the equation becomes

$$\frac{d^2w}{dt^2} + g_1\frac{dw}{dt} + g_2 = 0,$$

g_1 and g_2 being certain functions of a, k, and r, and the solution of this is

$$w = A + \frac{g_2}{g_1}t - B\epsilon^{-g_1 t}.$$

jars has been tested, and we find the same kind of confirmation. For example, the following is one of the many experiments we tried :—

September, 1877.

A circular glass rod 128 centims. long, 0·832 centim. in diameter, rested on two knife edges 117 centims. apart, and was loaded with a weight of 400 grms. hung at its centre. The deflections of the rod were observed by a ray of light coming from a rigidly fixed lamp, falling on a mirror rotated by the bending of the rod, and reflected on to a rigidly fixed vertical scale. To prevent errors arising from slight alterations of the position of the flame, it was always focussed so that the image of two cross-wires fixed in the lamp slit and formed by a fixed lens always occupied the same position, when not reflected by the mirror attached to the deflected rod, and it was the position of this image, after being reflected by this mirror, which was observed on the vertical scale. After the beam had been loaded for 145·55 hours, observations of the increasing deflection being taken all the time, the load was removed. For several hours after the load had been removed, the actual observations themselves needed very little correction by drawing a curve, afterwards the slight waviness of the true curve due to changes of temperature had to be eliminated. In the several tables of observations of loaded and relieved beams (each experiment lasting two or three weeks), we have given the temperature of the room at the time of making the observation.

By the method of reduction adopted, we find that during the recovery of the rod from t equal 20, to t equal 240 (1=1·25 hours) the curve is almost purely logarithmic, and we find by calculation that

$$w_1 = 10{\cdot}6 + 224{\cdot}79\epsilon^{-0{\cdot}01296t},$$

and the column of values of $(w-w_1)$, in the following table shows that this expression satisfies the observations, very accurately for a very long period, namely, 220 on our scale of time, or 275 hours. Taking these numbers, we have calculated w_2, the next term in the solution of the differential equation, and find that

$$w_2 = 138{\cdot}7\epsilon^{-0{\cdot}3125t},$$

so that the last column of the table shows the inaccuracy of the assumption—

$$w = 10{\cdot}6 + 224{\cdot}79\epsilon^{-0{\cdot}01296t} + 138{\cdot}7\epsilon^{-0{\cdot}3125t},$$

an inaccuracy which is of very little importance from

$$t = 2{\cdot}4 \text{ to } t = 240,$$

and even this inaccuracy we have found is to some extent due to the fact that the temperature of the room was steadily falling at that time from 26° C. to 21° C.

Recovery of the Glass Beam after being loaded for 145·55 hours with 400 grms.

Time (1 = 1·25 hours) reckoned from removal of load.	Observed value of w.	$w - w_1$.	$w - w_1 - w_2$.
0 load on	6475·0		
load off	....		
0·005	600		
0·4	411·3	177·0	154·4
1·4	340·8	109·4	19·9
2·4	300·6	71·9	6·5
3·4	275·6	49·9	1·9
4·4	278·5	35·6	0·5
5·4	246·3	26·1	0·4
6·4	237·1	19·7	0·9
8·0	224·6	11·4	0·0
10·0	214·2	6·2	0·1
12·0	206·9	3·9	0·6
14·0	200·6	2·5	0·8
16	195·0	1·7	0·8
18	189·8	1·2	0·7
20	184·5	0·4	0·1
30	162·8	−0·2	−0·2
40	145·3	0·8	0·8
50	131·0	2·8	2·8
60	116·3	2·4	2·4
70	103·2	1·9	1·9
80	91·5	0·2	0·2
90	81·2	0·6	0·6
100	72·3	0·2	0·2
110	64·7	0·1	0·1
120	58·0	0·0	0·0
130	52·5	0·2	0·2
140	47·7	0·5	0·5
150	43·3	0·5	0·5
160	39·3	0·4	0·4
170	35·8	0·4	0·4
180	32·7	0·3	0·3
190	29·8	0·0	0·0
200	27·3	−0·1	−0·1
210	25·1	−0·3	−0·3
220	23·2	−0·4	−0·4
230	21·6	−0·4	−0·4
240	20·0	−0·4	−0·4

There cannot be much doubt of the fact, that electric induction through dielectrics, and magnetisation of iron, are really phenomena of stress and strain, and it is probable that the correctness of Ohm's law when applied to dielectrics is dependent on the theory of continued strain, which we have just given, and which we are attempting to prove. In addition to experiments such as we have described, we have been making others to find whether beams which are kept in a state of strain rise in temperature through internal friction, which is of course analogous with electric resistance, and in addition to see whether a soft steel magnet acted upon by a constant current also rises in temperature; but hitherto these heating effects, if they exist, have been too small for our instruments to detect. In speaking of beams we refer especially to glass beams; a beam of sealing-wax would probably under the same circumstances perceptibly rise in temperature.

As a further example of the reductions which we are describing in this paper, we give a table of observed and calculated values of the diminishing deflection of a glass fibre, which had been twisted for some days, and then released. The fibre was 85 centims. long, of a mean diameter of 0·034 centim. suspended vertically, so that its upper end could not turn in the support, and carrying a weight of 211·6 grms. Attached to this weight was a vane dipping into water to diminish the rapidity of the swings.

The untwisting of the vertical glass wire was observed by the reflection of a ray of light, coming from a fixed lamp, by a mirror rigidly attached to the wire at its lower extremity. The spot of light, or rather the image of the cross-wires in the lamp slit, moved over a horizontal semicircular scale graduated in millimetres, and one metre in radius, with the mirror turning at its centre. An arrangement similar to that employed in our deflected beam experiments was used to obviate errors arising from difference of position of the flame of the lamp on different days.

The mirror and the weight having been kept twisted through a right angle for about a week, the deflecting couple was suddenly removed, and time readings of the diminishing deflections taken. As at first the deflection diminished with considerable rapidity, and as in addition the spot of light described a series of oscillations about the varying mean position, the system of registration we employed was as follows:—The position of the spot of light on the scale at the end of every swing was observed as accurately as possible, and the time recorded by simultaneously pressing down the key of a "break circuit chronograph." From these successive time observations of the swings

about a varying mean position, the true curve was drawn in accordance with a method mathematically developed by us.*

The reduction of this curve by the method previously explained leads to the equation

$$w_1 = 72{\cdot}8 + 35{\cdot}55\epsilon^{-0{\cdot}0569t},$$

where w_1 is the deflection, and t the time from the moment of removing the deflecting couple, the unit of t being 20 seconds. Several of the earlier observations have been omitted in the following table, as it is not until after 40 seconds that the above equation represents accurately the torsional strain. From this time, however, until the end of the experiment, the equation is very accurate.

Recovery of Glass Wire previously subjected to Torsional Strain.

November 16, 1876.

Time from moment of removing deflecting couple.	Observed deflection w.	w_1 calculated from formula.	Difference $(w-w_1)$.
0·7	143·5	107·0	36·5
1·0	131·5	106·4	25·1
1·5	118·4	105·4	13·0
2	110·4	104·5	5·9
3	102·9	102·8	0·1
4	100·4	101·1	0·3
5	98·7	99·5	−0·8
6	97·2	98·1	−0·9
7	96·0	96·7	−0·7
8	95·0	95·4	−0·4
9	94·0	94·1	−0·1
10	92·9	92·9	0·0
11	91·9	91·8	0·1
12	90·8	90·8	0·0
13	89·9	89·8	0·1
14	88·8	88·8	0·0
15	88·0	88·0	0·0
16	87·1	87·1	0·0
17	86·3	86·3	0·0
18	85·6	85·6	−0·1
19	84·9	84·9	0·0
20	84·3	84·2	0·1
21	83·6	83·6	0·0
22	82·9	83·0	−0·1
23	82·2	82·4	−0·2
24	81·9	81·9	0·0

* "Journal Soc. of Telegraph Engineers," vol. v, 1877, p. 391.

As a further example of the reduction by our method of the results of observation, we give five tables of experiments made with a special form of voltameter devised so as to enable experiments to be made with a gas-freed liquid, or a liquid more or less saturated with any particular gas, and with the platinum plates previously free from gas, or made to occlude any special gas. The voltameter itself was something like the upper part of a Geissler's mercury pump; that is, by lowering a column of mercury the electrolyte could be lowered from the platinum plates, leaving them in a vacuous space. The electrolyte of course commenced to boil, and to rapidly free itself from gas. By a valve this gas was allowed to escape, on again raising the mercury column, and the liquid above it. This operation being repeated a number of times, the liquid being left for a considerable period under the vacuous space towards the end of the operation, a large portion of the gas in the liquid could be removed. In order to remove the gas occluded in the platinums they were not, as is usual in voltameters, made in the form of plates, but each was a platinum spiral, the two ends of which protruded to the outside of the voltameter. By means of a small Grove's battery, each of these spirals could be made red hot in the vacuous space produced when the electrolyte was lowered, and thus a voltameter was obtained with gas-freed platinums in a gas-freed liquid, or by subsequently bubbling through a stream of a particular gas, with platinums in a liquid, with more or less at will of that gas occluded.

As these platinum spirals presented but a small surface, and as they were necessarily some little distance apart, the currents charging and discharging such a voltameter, when only one Minotto's cell was employed, were rather small, so that it was necessary to employ a fairly delicate reflecting galvanometer. From the first elongation of the spot of light on charging or discharging the voltameter, we could calculate the time integral of the current in the first half swing of the needle, and the subsequent time readings of the diminishing deflection were ascertained by an experienced observer, dotting with a pencil on the horizontal cylinder revolving uniformly (referred to at the commencement of the paper) the limits of the excursions of the spot of light in the successive swings about the varying mean position, the locus of which was subsequently drawn in by our method already referred to.

In the following table $\dot{u}$ is the observed current flowing into the voltameter to charge it at the time t, from the moment of applying one Minotto's cell; the voltameter contained distilled water well freed from gas, and the platinum spirals were kept red hot for some time in the vacuous space, before immersion in the liquid. The solution of the general differential equation leads to

$$u' = 86{\cdot}2 + 85{\cdot}7\epsilon^{-0{\cdot}06245t},$$

which from the following table, it will be seen accurately represents the charging current from t equal 7 to t equal 25. After the current had flowed into the voltameter for 1 hour and $7\frac{1}{3}$ minutes, the cell was removed, and the voltameter discharged through the galvanometer, the connexions of which were simultaneously reversed, so that the discharge deflection might be on the same side as the charge deflection, and therefore less swinging about of the needle produced at the moment of discharge. In the second of the following tables u is the discharge current at a time t from the commencement of the discharge, and u' is calculated from the equation

$$u' = 30{\cdot}90\epsilon^{-0{\cdot}0553t},$$

which accurately represents the discharge current from t equal 5 to t equal 25. The two exponents in the solutions of the differential equation for the current flowing in during charge, and the current flowing out during discharge, are

$$-0{\cdot}06245t \text{ and } -0{\cdot}0553t,$$

which, considering that the resistance of the galvanometer, battery, and connexions could not be neglected in comparison with that of the voltameter itself, are remarkably nearly equal to one another, being both—$0{\cdot}06t$ to two places of decimals.

When it was desired to measure the time integral of the discharge current during the first half swing of the galvanometer needle, rather than the deflections, a short time after the commencement of the discharge the galvanometer was short circuited for a sufficient length of time before the Minotto's cell was removed for the needle to come to rest at zero.

Charging with one Minotto's Cell of a Voltameter containing Distilled Water well freed from Gas, and the Platinum Spirals heated red hot *in vacuo*.

August, 1876.

Time in minutes.	Observed current u.	u' calculated from the formula.	Difference $(u-u')$.
1·5	200	164·2	35·8
2	187·4	161·9	25·5
3	173·7	157·3	16·4
4	163·9	153·1	10·8
5	156·0	148·9	7·1
6	149·3	145·1	4·2
7	143·5	141·6	1·9
8	138·8	138·2	0·6
9	134·9	136·0	−1·1

Time in minutes.	Observed current u.	u' calculated from the formula.	Difference $(u-u')$.
10	131·7	132·1	−0·4
11	129·0	129·3	−0·3
12	126·5	126·7	−0·2
13	124·0	124·3	−0·3
14	121·7	121·9	−0·2
15	119·7	119·8	−0·1
16	117·8	117·8	0·0
17	115·9	115·9	0·0
18	114·0	114·0	0·0
19	112·4	112·4	−0·1
20	110·7	110·8	−0·1
21	109·2	109·3	−0·1
22	107·8	107·9	−0·1
23	106·5	106·6	−0·1
24	105·3	105·3	0·0
25	104·2	104·2	0·0

Discharge of the same Voltameter after being charged with one Minotto's Cell for 1 hour and $7\frac{1}{3}$ minutes.

Time in minutes from the moment of discharging.	Observed current u.	u' calculated from the formula.	Difference $(u-u')$.
0 before discharge	286·0		
0·5 after discharge	79·5	30·1	49·4
1	60·8	29·2	31·6
2	41·0	27·7	13·3
3	32·8	26·2	6·6
4	28·0	24·8	3·2
5	24·7	23·4	1·3
6	22·5	22·2	0·3
7	20·5	21·0	−0·5
8	19·4	19·9	−0·5
9	18·2	18·8	−0·6
10	17·0	17·8	−0·8
11	16·5	16·8	−0·3
12	15·9	15·9	0·0
13	15·1	15·1	0·0
14	14·3	14·3	0·0
15	13·5	13·5	0·0
16	12·8	12·8	0·0
17	12·1	12·1	0·0
18	11·4	11·4	0·0

Time in minutes from the moment of discharging.	Observed current u.	u' calculated from the formula.	Difference $(u-u')$.
19	10·8	10·8	0·0
20	10·2	10·2	0·0
21	9·7	9·7	0·0
22	9·2	9·2	0·0
23	8·7	8·7	0·0
24	8·2	8·2	0·0
25	7·8	7·8	0·0

If the platinums after absorbing gas were merely kept for a long time in the vacuous space above the distilled water, but not *heated* to assist the driving out of the gas, then after raising the water to surround the wires, the charge current as observed on applying the Minotto's cell is given under u in the following table; while u' is calculated from

$$u' = 96{\cdot}3 + 79{\cdot}43\epsilon^{-0{\cdot}64t},$$

which is the solution of the differential equation, and which represents the charging phenomena almost from the moment of applying the Minotto's cell.

Charging; Gas-freed Distilled Water, Platinum Spirals not heated.

August, 1876.

Time in minutes.	Observed current u.	u' calculated from the formula.	Difference $(u-u')$.
0·5	165·0	153·9	11·1
1	138·5	138·1	0·4
2	118·0	118·3	−0·3
3	107·5	107·9	−0·4
4	102·4	102·4	0·0
5	99·5	99·5	0·0
6	98·0	98·0	0·0
7	97·2	97·2	0·0
8	96·7	96·7	0·0
9	96·5	96·5	0·0

When the distilled water and the platinum spirals were throughly aërated, the charging current measured experimentally is shown under u in the following table; while u_1 is calculated from

$$u' = 40{\cdot}25 + 101{\cdot}25\epsilon^{-0{\cdot}087t}.$$

Charging with one Minotto's Cell, Distilled Water and Platinum Spirals thoroughly aërated.

August, 1876.

Time in minutes.	Observed current u.	u' calculated from the formula.	Difference $(u-u')$.
1·5	144·0	129·3	14·7
2	134·4	125·5	8·9
3	120·9	118·3	2·6
4	111·7	111·8	−0·1
5	105·0	105·8	−0·8
6	99·7	100·3	−0·6
7	95·3	95·3	0
8	90·7	90·7	0
9	86·5	86·5	0
10	82·6	82·6	0
11	79·1	79·1	0
12	75·8	75·8	0
13	72·8	72·8	0
14	70·1	70·1	0
15	67·6	67·6	0
16	65·3	65·3	0
17	63·2	63·2	0
18	61·3	61·3	0
19	59·1	59·1	0
20	57·9	57·9	0
21	55·4	55·4	0
22	55·1	55·1	0
23	53·8	53·8	0
24	52·7	52·7	0
25	51·7	51·7	0

When the distilled water and the platinum spirals were thoroughly freed from air, and subsequently saturated with hydrogen, the charging current measured experimentally is given under u in the following table; and u' is calculated from

$$u' = 120 + 38{\cdot}02\epsilon^{-0{\cdot}125t}.$$

Charging with one Minotto's Cell, Distilled Water, and Platinum Spirals, thoroughly freed from Air and then saturated with Hydrogen.

August, 1876.

Time in minutes.	Observed current u.	u' calculated from the formula.	Difference $(u-u')$.
1·35	196·0	152·1	43·9
2	170·5	149·6	20·9
3	155·8	146·1	9·7
4	148·0	143·0	5·0
5	142·5	140·3	2·2
6	138·5	137·9	0·6
7	135·5	135·8	−0·3
8	133·5	134·0	−0·5
9	132·2	132·3	−0·1
10	130·9	130·9	0
11	129·6	129·6	0
12	128·5	128·5	0
13	127·5	127·5	0
14	126·6	126·6	0
15	125·8	125·8	0
16	125·1	125·1	0
17	124·5	124·5	0
18	120·0	124·0	0
19	123·5	123·5	0
20	123·1	123·1	0

The equations, therefore, which we have obtained, as solutions of the general differential equation, to represent the phenomena of *charging* our voltameter with one Minotto's cell, when the distilled water and the platinum spirals were in different states as regards the gas absorbed are:—

I. Distilled water freed from gas, platinum spirals left for some time in the vacuum above the water, but not heated—

$$u'=96{\cdot}3+79{\cdot}43\epsilon^{-0{\cdot}64t}.$$

II. Distilled water freed from gas, spirals made red hot *in vacuo*—

$$u'=86{\cdot}2+85{\cdot}7\epsilon^{-0{\cdot}06245t}.$$

III. Distilled water, and platinum spirals thoroughly aërated—

$$u'=40{\cdot}25+101{\cdot}25\epsilon^{-0{\cdot}087t}.$$

IV. Distilled water, and platinum spirals thoroughly freed from air, and then saturated with hydrogen—

$$u'=120-38{\cdot}02\epsilon^{-0{\cdot}125t}.$$

These equations, as has been seen, give with considerable accuracy the actual value of the current flowing into the voltameter at any moment during the greater period of the charging in the different cases. The difference between the curves of charge in the different cases is partly due to the different state of the distilled water and the platinums, and partly to the level of the mercury on which the water rested being slightly different in the different cases. To remove the possibility of this latter cause of variation in the curves, the construction of the voltameter was altered, so that the level of the mercury under the water was always so far away from the bottom of the platinum spirals, that any slight alteration in the level did not at all affect the curve of charging, or discharging. We refrain, however, in this preliminary note from giving any of the reductions of the curves we have experimentally obtained with this improved form of voltameter.

We may remark that the methods usually taken for measuring the resistance of liquids are quite misleading if there is any analogy between voltameters and condensers with solid dielectrics. For instance, on our assumption, the constant terms in the above expressions for u, measuring, as they do, the current after a great time has elapsed, really represent the ratio of the electromotive force of the battery to the total resistance of the voltameter and connexions, and therefore their reciprocals are measures of the several resistances. We need hardly mention that in ordinary experiments the current is measured for as short a time as possible, "to avoid polarisation, as it is said." For small values of the time the current flowing into a voltameter, or liquid condenser, is very great, and in our opinion may be regarded as nearly infinitely great; for the first rapid rush of electricity is due to charging the voltameter as a condenser is charged; and if this rush is not quite instantaneous, it is because the electromotive force charging the voltameter does not immediately reach its constant value. But just as the potential of the end of a cable suddenly attached to a battery will acquire its maximum value more and more rapidly as the resistance of the battery and connexions is made less and less, so by making the resistance of our single element used to charge the voltameter less and less, and by measuring the time integral of the current flowing in for shorter and shorter periods of time, that is, by using galvanometer needles of more and more quickness of vibration, a better and better approximation to the *static capacity* of a voltameter is arrived at. And we found that when we had reduced the resistance of our single cell to 0·04 of an ohm, by using in this single cell a zinc plate having 15 square feet in area, the more and more carefully we made the experiments, the more and more nearly did the first discharge of a voltameter, which is always less than the first charge, become equal to it. If the charge and discharge in voltameters do not prove to be as nearly equal as in glass Leyden jars, it

is because the true conduction current is so much greater proportionally in water than in glass. In certain experiments, which we hope to have the honour of describing fully in a future paper, we have been able to make an approximation to the time integral of this *conduction current* during the small times of first charge and discharge, and by subtracting it from the measured charge, or adding it to the measured discharge, we have obtained what we may reasonably call the true induction charge in a voltameter. We mention this now since, although years ago Mr. Cromwell Varley referred to the electrostatic capacity of voltameters, the investigation of this instantaneous charge has been neglected, because this charge is small compared with the residual one, just as the important investigation of the residual charge in Leyden jars was for a long time not carried out, because the residual charge was small compared with the instantaneous charge.

For assistance rendered in the carrying out of the series of experiments, the results of a few only of which are given in this preliminary account, we have to thank the following of our students:—Messrs. Fujioka, Igarashi, Iida, Inoguchi, Kasai, Kawaguchi, Mita, Nobechi, Oshima.

III. "On the Structure and Development of the Skull in the Batrachia. Part III." By W. K. PARKER, F.R.S. Received April 29, 1880.

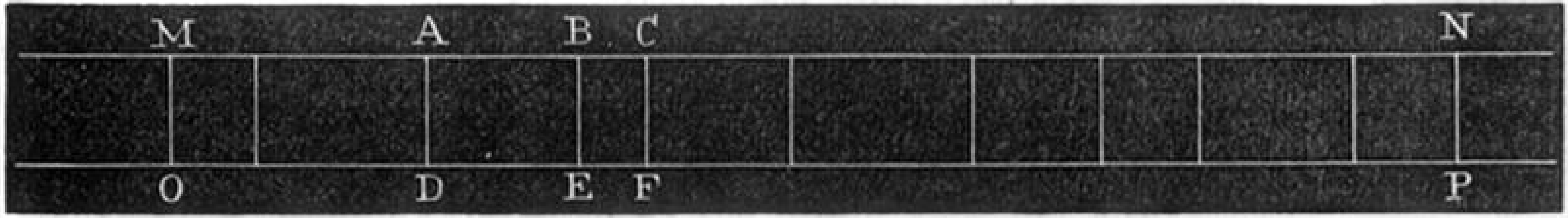
(Abstract.)

Some of the work brought forward in this paper was in hand before the first part was in print. That initial piece of work dealt only with the formation of the skull in the common frog, but it was followed by another which appeared in the "Philosophical Transactions" in 1876, which treated of the skulls of the *common* and of the "aglossal" toads.

Of the latter types only two kinds are known, viz., the nailed toad of the Cape (*Dactylethra*), and the monstrous toad of Surinam (*Pipa*). All the bulk of the Batrachia are included in the sub-group "Opisthoglossa," these have a tongue, and in most cases it is free *behind* and not in front; the "Proteroglossal" Batrachia are very few in number, and the character itself (as Dr. Günther informs me) is not well pronounced.

I have now worked out the skull, in one or more stages, in about a *tithe* of the known species, and in my second paper in both of the aberrant ("aglossal") types; in them this was done in various stages.

I am not aware that there is any "order" of any "class" in the Vertebrata where so large a percentage of species has been, or indeed,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