Potassium Iodide.

·159 grm. gave....	20·75	at 0°	and 760 mm.	Mol. wt.....	171
·165 ,, ...	21·2	,,	,,	,, ...	174
·170 ,, ...	21·4	,,	,,	,, ...	177
·180 ,, ...	21·6	,,	,,	,, ...	186
·103 ,, ...	15·1	,,	,,	,, ...	152
·174 ,, ...	24·4	,,	,,	,, ...	159
				Mean molecular weight....	169·8

These numbers are fairly accordant, and seem to indicate that iodide of potassium is normal in its behaviour. The vapour, after each experiment, was blown out with a current of hydrogen. A crystalline deposit was obtained in each case, which was pure iodide of potassium, free from any trace of iron. Taken in connexion with the former experiments, this seems to show that, if free potassium is abnormal, its compounds are not altered. Before any final conclusions can be reached, further experiments must be made in platinum or iridium vessels, and it will be very important to control the results by examining the densities of the iodides of cæsium and rubidium.

IV. "On the Practical Solution of the Most General Problems in Continuous Beams." By John Perry and W. E. Ayrton. Communicated by Fleeming Jenkin, F.R.S.S. Lond. and Edin., Professor of Civil Engineering in the University of Edinburgh. Received November 27, 1879.

1. It is not necessary to enter into the question of the advisability of employing continuous girders in bridges with spans of less than 200 feet, but it is generally conceded that the increased economy due to the employment of continuous girders in longer spans more than counterbalances the well-known practical objections to continuity. Hence the practical solution of the general problem—given the conditions at the ends of a continuous girder, the spans, the moment of inertia of all cross sections, and the loading, to find the bending moment and shearing stress in every cross-section, is not unworthy of our attention.

2. Mr. Heppel has published the exact solution for cases in which each span may be supposed divided into two, three, four or five equal parts, in each of which the load and cross-section are supposed to be constant. As is well known, the difficulty of the general problem is due to the necessity of making certain summations in each span. Now, Mr.

Heppel found it convenient for his special case to make the summations twice for the same span, working forwards and backwards from every point of support, but in the following the summations are all effected working in the same direction, only *one* summation being necessary for each span.

3. Suppose OQ to be one span of a continuous beam which has loads and supporting forces beyond O and Q. To find the bending moment at a point P, it is sufficient to know what is the bending moment at O, and the shearing force at a very short distance inside O. If we suppose the bending moment to be called positive when it tends to make the beam convex upwards, and the shearing force at a section positive when it tends to elevate the part of the beam to the right of the section, then M, the bending moment at a point P, is equal to

$$\mathrm{M}_0 - \mathrm{S}_0 \times \mathrm{OP} + m,$$

where M_0 and S_0 are the bending moment and shear at O, and where m is the sum of all such expressions as—an element of load at any place P', between O and P, multiplied by $P'P$.

Again, if M is the bending moment at a section, E the modulus of elasticity, I the moment of inertia of the section about its neutral line, y the vertical displacement of a point in the neutral axis of the beam from its horizontal position, x the horizontal distance of this point from some fixed point in the neutral axis as O, then for all beams and girders used by engineers—

$$\frac{d^2y}{dx^2} = \frac{\mathrm{M}}{\mathrm{EI}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

The only restriction that we shall use is to suppose all points of support in the same straight line. But as is well known, if this case is solved, the unrestricted case may also be solved, and the case in which there is a yielding in the supports.

Let M_0 M_1 be the bending moments at the points of support O and Q. Let S_0 be the shearing force just inside O. When we know the nature of the loading—

$$\begin{array}{ccc} \mathrm{M}_0 & \mathrm{P} & \mathrm{M}_1 \\ \hline \wedge & & \wedge \\ \mathrm{O} & & \mathrm{Q} \\ < \; . \; . \; . \; . & l_1 & . \; . \; . \; . \; . \; > \end{array}$$

we can find m the moment at P due to the load between O and P. Thus if w be the load per unit length at any place, and if we know how w alters, we can find the sum—

$$\int_0^{\mathrm{OP}} w(\mathrm{OP}-x)dx = \int_0^{\mathrm{OP}} wzdz = m \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

And also, if we know how EI alters, we can find the following sums, each of which is denoted by a symbol:—

$$\left.\begin{array}{ll} \int_0^x \frac{dx}{\mathrm{EI}}=d & \int_0^{l_1} \frac{dx}{\mathrm{EI}}=d_1 \\ \int_0^x \frac{xdx}{\mathrm{EI}}=f & \int_0^{l_1} \frac{xdx}{\mathrm{EI}}=f_1 \\ \int_0^x \frac{mdx}{\mathrm{EI}}=g & \int_0^{l_1} \frac{mdx}{\mathrm{EI}}=g_1 \end{array}\right\} \quad \ldots \ldots \quad (3),$$

and also

$$\left.\begin{array}{ll} \int_0^x d\,.\,dx=n & \int_0^{l_1} d\,.\,dx=n_1 \\ \int_0^x f\,.\,dx=q & \int_0^{l_1} f\,.\,dx=q_1 \\ \int_0^x g\,.\,dx=\mathrm{F} & \int_0^{l_1} g\,.\,dx=\mathrm{F}_1 \end{array}\right\} \quad \ldots \ldots \quad (4).$$

It is very easy to write out the values of these summations in certain simple cases, and the calculation in the most general cases can always be made with a very fair approximation to accuracy, and with not much risk of mistakes being made, by the graphic methods described below.

The bending moment at any point P is—

$$\mathrm{M}=\mathrm{M}_0-\mathrm{S}_0x+m \quad \ldots \ldots \quad (5),$$

but at Q we have $x=l_1$,

and $\mathrm{M}=\mathrm{M}_1$,

and hence $\mathrm{M}_1=\mathrm{M}_0-\mathrm{S}_0l_1+m_1$,

therefore

$$\mathrm{S}_0=\frac{\mathrm{M}_0-\mathrm{M}_1+m_1}{l_1} \quad \ldots \ldots \quad (6).$$

So that expression (5) becomes

$$\mathrm{M}=\mathrm{M}_0+m-x\frac{\mathrm{M}_0-\mathrm{M}_1+m_1}{l_1} \quad \ldots \ldots \quad (7).$$

Substituting this value for M in the differential equation (1), and integrating once, remembering that when x equals nought, d, f, and g vanish, we see that if α_0 is the inclination (very small) of the beam downwards from the horizontal at O, and if α is the inclination of the beam at any point P, then

$$\frac{dy}{dx} \text{ or } \tan\alpha \text{ or } \alpha=\alpha_0+\mathrm{M}_0d-\mathrm{S}_0f+g \quad \ldots \quad (8).$$

If α_1 is the inclination at the support Q, then

$$\alpha_1=\alpha_0+\mathrm{M}_0 d-\mathrm{S}_0 f+g_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

Integrating (8) again we see that the deflection at any point P is

$$y=x\alpha_0+\mathrm{M}_0 n+\mathrm{F}-\mathrm{S}_0 q \quad . \quad . \quad . \quad . \quad . \quad . \quad (10),$$

and, as the supports are on the same level,

$$y=0$$

when

$$x=l_1,$$

so that

$$0=l_1\alpha_0+\mathrm{M}_0 n_1+\mathrm{F}_1-\mathrm{S}_0 q_1,$$

and hence

$$\alpha_0=\frac{\mathrm{S}_0 q_1-\mathrm{F}_1-\mathrm{M}_0 n_1}{l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

Substituting the value of S_0 from (6) we have therefore

$$\alpha_0=\frac{(\mathrm{M}_0+m_1-\mathrm{M}_1)q_1-l_1\mathrm{F}_1-\mathrm{M}_0 n_1 l_1}{l_1^2} \quad . \quad . \quad . \quad . \quad (12),$$

and using this in equation (9) we find

$$\alpha_1=\frac{(\mathrm{M}_0+m_1-\mathrm{M}_1)q_1-l_1\mathrm{F}_1-\mathrm{M}_0 n_1 l_1}{l_1^2}+\mathrm{M}_0 d_1-\mathrm{S}_0 f_1+g_1 \quad . \quad (13),$$

Now, let m_2, d_2, f_2, g_2, n_2, q_2, F_2, be the values of the summations made in the next span, QR, Q corresponding with O, and R with Q. If M_2 is the bending moment at the support R, then equation (12) becomes, putting α_1 instead of α_0, M_1 instead of M_0, M_2 instead of M_1,*

$$\alpha_1=\frac{(\mathrm{M}_1+m_2-\mathrm{M}_2)q_2-l_2\mathrm{F}_2-\mathrm{M}_1 n_2 l_2}{l_2^2} \quad . \quad . \quad . \quad . \quad (14)$$

M_0		M_1	P	M_2
$\wedge$		$\wedge$ x		$\wedge$
O		Q		R
<	l_1	×	l_2	>

Thus we have in equations (13) and (14) distinct values of the angle of inclination at Q. Putting these values equal to one another

* If θ is the small angle of discontinuity of the neutral line of the beam at Q, that is if θ is equal to

$$\left.\begin{matrix}\text{inclination downwards of the}\\ \text{neutral line of } QR \text{ at } Q\end{matrix}\right\}-\left\{\begin{matrix}\text{inclination downwards of the}\\ \text{neutral line of } OQ \text{ at } Q,\end{matrix}\right.$$

then we see that α_1 in (14) minus α_1 in (13) equals θ, and hence we must put $-\theta$ instead of 0 on the right hand side of equation (15). The angle θ is not supposed to vary with alteration of loading.

and simplifying, we find an equation connecting M_0, M_1, and M_2, and this is our form of the General Theorem of the Three Moments.*

$$M_0\left(\frac{q_1}{l_1^2}-\frac{n_1}{l_1}+d_1-\frac{f_1}{l_1}\right)-M_1\left(\frac{q_2}{l_2^2}-\frac{n_2}{l_2}+\frac{q_1}{l_1^2}-\frac{f_1}{l_1}\right)+M_2\frac{q_2}{l_2^2}$$

$$+\frac{m_1q_1}{l_1^2}-\frac{m_2q_2}{l_2^2}+g_1-\frac{f_1m_1}{l_1}+\frac{F_2}{l_2}-\frac{F_1}{l_1}=0\ . \quad . \quad . \quad . \quad (15).$$

4. We have in the theorem of three moments a relation between the bending moments M_0, M_1, M_2, at any three consecutive points of support; therefore in a continuous girder of N spans there are $N-1$ such equations, and $N-1$ unknown moments, because the moments at the end supports are each equal to nought. Hence the moments at all the supports are found by the easy solution of these simple simultaneous equations.

We may now suppose that the moments at the supports O and Q at the two ends of a span are found, then the shearing force at O can be calculated by equation (6), the bending moment at any point P by equation (7), and the deflection at any point P by equation (10). Points of maximum and minimum bending moment are obtained by equating to nought the differential coefficient of M with respect to x. At a point of inflexion we know that the bending moment must be equal to nought.

We can prove that, in any span OQ of a continuous beam when we know the angles of inclination of the girder at the two points of support, we have sufficient data for making all the necessary calculations. For

$$\alpha_0 \text{ at } O=\frac{(M_0+m_1-M_1)q_1-l_1F_1-M_0n_1l_1}{l_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12),$$

$$\alpha_1 \text{ at } Q=\frac{(M_0+m_1-M_1)q_1-l_1F_1-M_0n_1l_1}{l_1^2}+M_0d_1-S_0f_1+g_1 \quad (13),$$

so that, knowing α_0 and α_1, we can find M_0 and M_1; and then the bending moment at any point P, if x equals OP, is

$$M=M_0+m-x\frac{M_0-M_1+m_1}{l_1} \quad . \quad . \quad . \quad . \quad . \quad (7),$$

so that we can draw the diagram of bending moment. Also the shear at O is

$$S_0=\frac{M_0-M_1+m_1}{l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

* Although the theorem of the three moments as given in equation (15) involves more expressions than Mr. Heppel's form as given by Professor Rankine ("Proc. Roy. Soc.," vol. xviii, p. 178), still it will be observed that it is really more convenient. In any case the summations d, f, g have to be calculated in order to obtain n, q, and F, but with our form of the theorem there is this advantage, that when there are more than two spans the summations have only to be made *once* for any span.

and the shear at any point P is equal to S_0 minus the load between O and P, therefore we can draw the diagram of shearing force. Also the deflection at any point P equals

$$M_0 n + F - S_0 q.$$

As a familiar example, we may consider a beam OQ, of which the ends are fixed horizontally and at the same level, here

$$\alpha_0 = 0,$$

and
$$\alpha_1 = 0.$$

We can prove that, when we know the angle of the beam at one end of a span, and the bending moment at the other end, we have sufficient data for calculation, having the above summations m, &c. This applies to the familiar case of a beam fixed horizontally at one end and merely supported at the other end, the supports being level, as before.

The following well-known examples illustrate the use of the formulæ :—

In all cases when the cross-section of the beam remains constant, the summations d, f, n, and q may easily be shown to have the following values :—

$$d = \frac{x}{EI}, \qquad d_1 = \frac{l_1}{EI}, \qquad d_2 = \frac{l_2}{EI},$$

$$f = \frac{x^2}{2EI}, \qquad f_1 = \frac{l_1^2}{2EI}, \qquad f_2 = \frac{l_2^2}{2EI},$$

$$n = \frac{x^2}{2EI}, \qquad n_1 = \frac{l_1^2}{2EI}, \qquad n_2 = \frac{l_2^2}{2EI},$$

$$q = \frac{x^3}{6EI}, \qquad q_1 = \frac{l_1^3}{6EI}, \qquad q_2 = \frac{l_2^3}{6EI},$$

so that the theorem of three moments becomes—

$$M_0 l_1 + 2M_1(l_1 + l_2) + M_2 l_2 - 2m_1 l_1 - m_2 l_2 + 6EI\left(g_1 + \frac{F_2}{l_2} - \frac{F_1}{l_1}\right) = 0.$$

In all cases where the load is spread uniformly over the span, being w per unit length, it is easy to show that—

$$m = \frac{wx^2}{2}, \qquad m_1 = \frac{wl^2}{2},$$

$$g = \frac{wx^3}{6EI}, \qquad g_1 = \frac{wl^3}{6EI},$$ the section being constant on the span.

$$F = \frac{wx^4}{24EI}, \qquad F_1 = \frac{wl^4}{24EI}.$$

When the spans are equal in length, and all cross-sections are the

same, and the loading over both spans is uniform and equal to w per unit length, then the theorem becomes—

$$M_0 + 4M_1 + M_2 - \frac{wl^2}{2} = 0.$$

When in the two consecutive spans, OQ and QR, there is the same common cross-section, and when there is a uniform load, w, per unit length over the first span, and w_2 over the second, we get the theorem in M. Clapeyron's form—

$$M_0 l_1 + 2M_1(l_1 + l_2) + M_2 l_2 - \frac{w_1 l_1^3}{4} - \frac{w_2 l_2^3}{4} = 0.$$

Graphic Determination of the Summations of paragraph 3.

5. It has hitherto been usual for engineers to assume that the load and section are uniform in each span, because, as we have seen, the calculations in this case are easy, whereas those necessary when attempting the general solution are exceedingly complicated; in fact, an examination of Mr. Heppel's solution of a comparatively simple case is sufficient to deter engineers from such calculations. But by the following graphic method the solution of the most general case only requires a very elementary mathematical knowledge, and may be completed in a few hours, or in a much shorter time if a Thomson's integrating machine, or even a good planimeter be available.

We employ a link polygon method for the calculation of m, and it may be expedient to draw the deflected beam by Mohr's method. To employ, however, a link polygon method throughout the whole of the investigation would lead to much waste of time, however interesting it might be from a mathematical point of view; and as in many other engineering problems, for instance, the finding of centres of gravity and moments of inertia, so in this we see that the use of a little arithmetic, and the actual measurement of lines gives quicker and more accurate integrations.

Our students in Japan made the necessary summations for spans of 300 and 200 feet respectively. Their drawings were exhibited at the meeting, and one of them is shown on a reduced scale in fig. 6. It is advisable to use some well-known scale for the measurement of the lines; for example, a scale of centimetres. The distance AB, fig. 6, represents the span of 200 feet to the scale—one centimetre stands for $3\frac{1}{3}$ feet. The ordinate of the diagram $AEEE$, &c., shows the value of $\frac{1}{\mathrm{EI}}$ at every point of the span, and in all cases, except where there is a sudden change of inclination of the beam over a pier (see the end of § 3), any unit of measurement whatever may be employed for $\frac{1}{\mathrm{EI}}$.

6. *To find the curve for* m *and the value of* m.—In fig. 1, AB is the

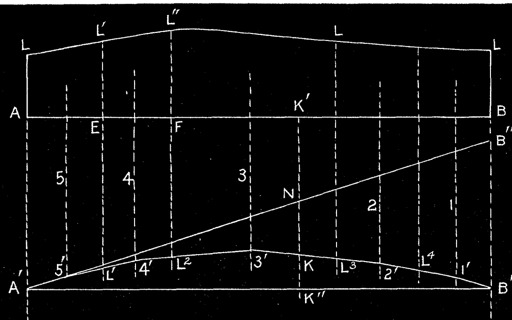


Fig. 1.

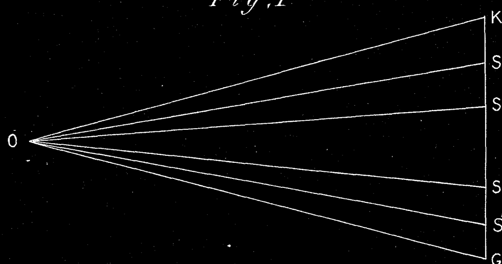


Fig. 2.

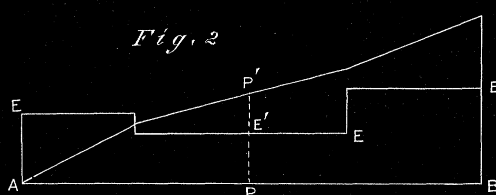


Fig. 4.

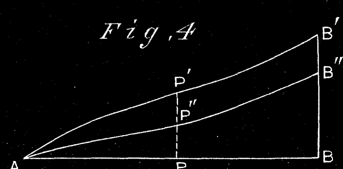
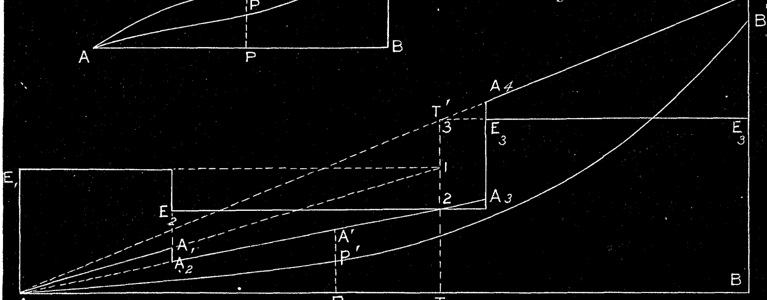
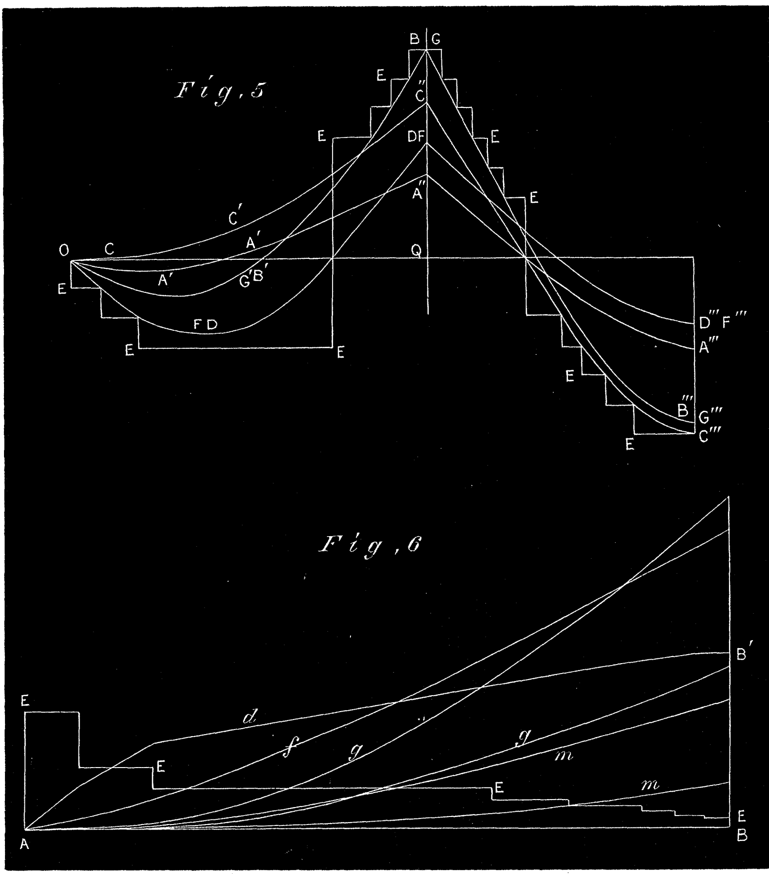


Fig. 3.



span, and the ordinate of the figure *ALLLB*, shows at any point to a given scale the intensity of the load; that is, the amount of load per foot of the beam at that place. Divide the space *ALLLB* into any convenient number of spaces by vertical lines, and assume that the load on *EF*, for instance, is not a distributed load, but a single one, acting through the centre of gravity of the area $EL'L''F$, and numerically equal to the total load on *EF*, and similarly with the other parts, so



that now the span has concentrated loads 1, 2, 3, 4, 5. Draw *GK* representing the loads to a given scale, and join the points *S* with any point *O*, as shown in the figure. We shall call *GK* our force polygon, as usual, and we now proceed to draw our link polygon, drawing $A'5B''$ parallel to *OK*, 5′4′ parallel to *OS*, &c. Then A′5′4′3′2′1′B′ is our link polygon for concentrated loads, and for the real loading we must draw a curve through $A'L_1L_2L_3L_4B'$. Producing $A'5'$ we shall find

it quite easy to prove that any ordinate, as NK for instance, represents the value of m for the point K', and $B''B'$ is m_1.*

In fig. 6 the diagram for m has been altered in shape by the student, but this extra labour was not necessary.

7. Any ordinate to the curve $AEEEB$, fig. 2, represents the value at that place of $\frac{1}{\mathrm{EI}}$, where E is the modulus of elasticity of the substance of which the beam is composed, and I is the moment of inertia of the section about its neutral line, any unit of measurement being employed.

The value of d at any point P is shown by the ordinate PP' to the curve $AP'B'$, and BB' equals d_1, the curve $AP'B'$ being obtained by raising at a number of points, such as P, the ordinate PP', measured to any convenient scale, and representing the area $AEE'P$. It is evident that n_1 is the total area of the figure $AP'B'B$, the scale of measurement being computed in the manner described below.

8. *To find* f.—$A\ E_1\ E_2\ E_3\ B$, fig. 3, shows the value of $\frac{1}{\mathrm{EI}}$ at every point. Take $A\ T$ any convenient distance, and raise the perpendicular TT'; make $T3$ equal to BE_3, $T2$ equal to the ordinate at E_2 and $T1$ equal to the ordinate at E_1. Join A with the points, 1, 2, 3, &c., and produce if necessary. Now it is quite evident that any ordinate of the figure $AA_1A_2A_3A_4B''B$ represents $\frac{x}{\mathrm{EI}}$ to a known scale; thus, for example, PA' represents PA divided by EI at the point P. (This diagram might, of course, have been easily obtained numerically.) Now at any point make the ordinate PP' represent to any suitable scale the area $AA_1A_2A'P$, and it is evident that any ordinate of the curve through all such points as P', say $AP'B'$, is f to a known scale. It is also evident that g_1 is the total area of the figure $AP'B'B$.†

9. *To find* g.—At any point P we now know (see fig. 1) the value of m; we also know the value of $\frac{1}{\mathrm{EI}}$; we can, therefore, compute the

* For we know that the ordinate KK'' represents the bending moment at the point K' if the beam is merely supported at the ends. Also NK'' represents the bending moment about K' due to the proper supporting force at A, and therefore NK, the difference, is m the bending moment at K' due to the loading merely between A and K'.

† Generally as to scale :—When at the point P, fig. 4, we raise a perpendicular PP'' which represents, according to any scale, by its length the area of PAP', then if the ordinate PP' was to such a scale that one centimetre represents a units, and if AB is the span to such a scale that one centimetre represents b feet, if the area AP′P is c square centimetres, and if it is represented by a line PP'' whose length is e centimetres, it is evident that the ordinates of our new curve $AP''B''B$ have a scale such that one centimetre represents $\frac{abc}{e}$ units.

value of $\frac{m}{\mathrm{EI}}$ by actual measurement of lines and numerical work; and erect the perpendicular PP', fig. 4, representing $\frac{m}{\mathrm{EI}}$ to any suitable scale. Now make $P\ P''$ represent to a suitable scale the area PAP', then the curve passing through all such points as P'' has for its ordinates the values of g. It is also evident that F_1 is the total area of the figure $AP''B''B$.

Example.

10. As an example we may take the following, worked out by Messrs. Terauchi and Saiki, two of our students :—

Design a lattice-girder-bridge with two main girders, the girders being of a constant depth; to carry a double line of railway, and to consist of three spans, respectively 200, 300, and 200 feet long.

Consulting Baker, they assumed the total weight of cross-beams, permanent way, &c., to equal 0·55 ton per foot of the middle span, and 0·5 ton per foot in the end spans: the weight of main girders to be 1·35 and 0·7 ton per foot in the middle and end spans respectively, and the rolling load 2 tons per foot.

First Calculations.—Assume that the girders have everywhere the same cross-section, then we have merely to employ the following equation, § 4, for any two consecutive spans :—

$$\mathrm{M}_0 l_1 + 2\mathrm{M}_1(l_1+l_2) + \mathrm{M}_2 l_2 - \frac{w_1 l_1^3}{4} - \frac{w_2 l_2^3}{4} = 0.$$

1. *As regards the Weight of the Bridge itself.*

$$w_1 = 1{\cdot}2, \qquad w_2 = 1{\cdot}9, \qquad w_3 = 1{\cdot}2.$$

We find
$$\mathrm{M}_1 = \mathrm{M}_2 = 11711{\cdot}5 \text{ ton feet},$$
$$\mathrm{M}_0 = \mathrm{M}_3 = 0.$$

At any point in the first span

$$\mathrm{M} = 0{\cdot}6x^2 - 61{\cdot}4425x.$$

At any point in the second span

$$\mathrm{M} = 11711{\cdot}5 - 0{\cdot}95x^2 - 285x.$$

The curve $OAA'A''A'''$ (fig. 5) has been plotted to show this diagram of bending moments for half the bridge, the other half being exactly similar. O and Q are two supports, and H is the middle of the bridge.

2. *The Rolling Load covering the whole Bridge.*

We find that
$$\mathrm{M}_1 = \mathrm{M}_2 = 25173 \text{ ton feet},$$
$$\mathrm{M}_0 = \mathrm{M}_3 = 0.$$

At any point in the first span

$$M=1{\cdot}6x^2-194{\cdot}135x.$$

At any point in the second span

$$M=25173+1{\cdot}95x^2-585x,$$

and the diagram for half the bridge is $OBB'B''B'''$,

3. *The Rolling Load covering only the Middle Span.*

We find that $M_1=M_2=22096$ ton feet.

$$M_0=M_3=0.$$

At any point of the first span

$$M=0{\cdot}6x^2-7{\cdot}52x.$$

At any point of the second span

$$M=22096+1{\cdot}95x^2-585x,$$

and the diagram for half the bridge is $OCC'C''C'''$.

4. *The Rolling Load covering the ends Spans only.*

We find that $M_1=M_2=14788$ ton feet,

$$M_0=M_3=0.$$

At any point in the first span

$$M=1{\cdot}6x^2-246{\cdot}06x.$$

At any point in the second span

$$M=14788-0{\cdot}95x^2-285x.$$

and the diagram for half the bridge is shown in $ODD'D''D'''$.

5. $OFF'F''F'''$ is the diagram for half the bridge when only the first span is covered with the rolling load.

6. $OGG'G''G'''$ is the diagram for half the bridge when only the first two spans are covered with the rolling load.

As the depth is supposed to be constant, the moment of inertia of each section is now supposed to be nearly proportional to the greatest bending moment which is ever found at that section during any of the above distributions of load, and the ordinates of the diagram AEE, &c., show the values which have been assumed for the moment of inertia of each section, these values being either 1, 2, 3, 4, 5, 6, or 7. If the girders had not been of constant depth, so that the moment of inertia might have been changed by altering the height of the girder, as well as the areas of the booms, we should have proceeded in a slightly different way to get the diagram of *I*. As it was, our students

found it advisable to use six thicknesses of plates in the booms at the middle of the centre span, and they therefore made I there equal to 6.

It is unnecessary to enter fully into the details of the complete drawings exhibited. On each curve was written the scale to which it was drawn. Employing the methods previously described, a few hours will be sufficient for making all the necessary calculations.

The results obtained for the new distribution of I were as follows :—

In 1st and 3rd span.

With rolling load	Without rolling load
$m_1=63999$ ton feet,	$m_1=24000$ ton feet,
$g_1=21,030,000,$	$g_1=8,462,223,$
$F_1=1,387,000,000$	$F_1=488,910,000.$

$$d_1=1486,$$
$$n_1=185040,$$
$$f_1=107540,$$
$$q_1=9,191,200.$$

In middle span.

With rolling load	Without rolling load
$m_1=175500$ ton feet,	$m_1=85500$ ton feet,
$g_1=122,817,000,$	$g_1=61,975,000,$
$F_1=7,672,800,000,$	$F_1=3,938,100,000.$

$$d_1=1988,$$
$$n_1=299310,$$
$$f_1=310180,$$
$$q_1=24,048,500.$$

Just in the same way the necessary summations may be made for the most general cases, the moment of inertia and the load varying in any way whatever from point to point in a span.

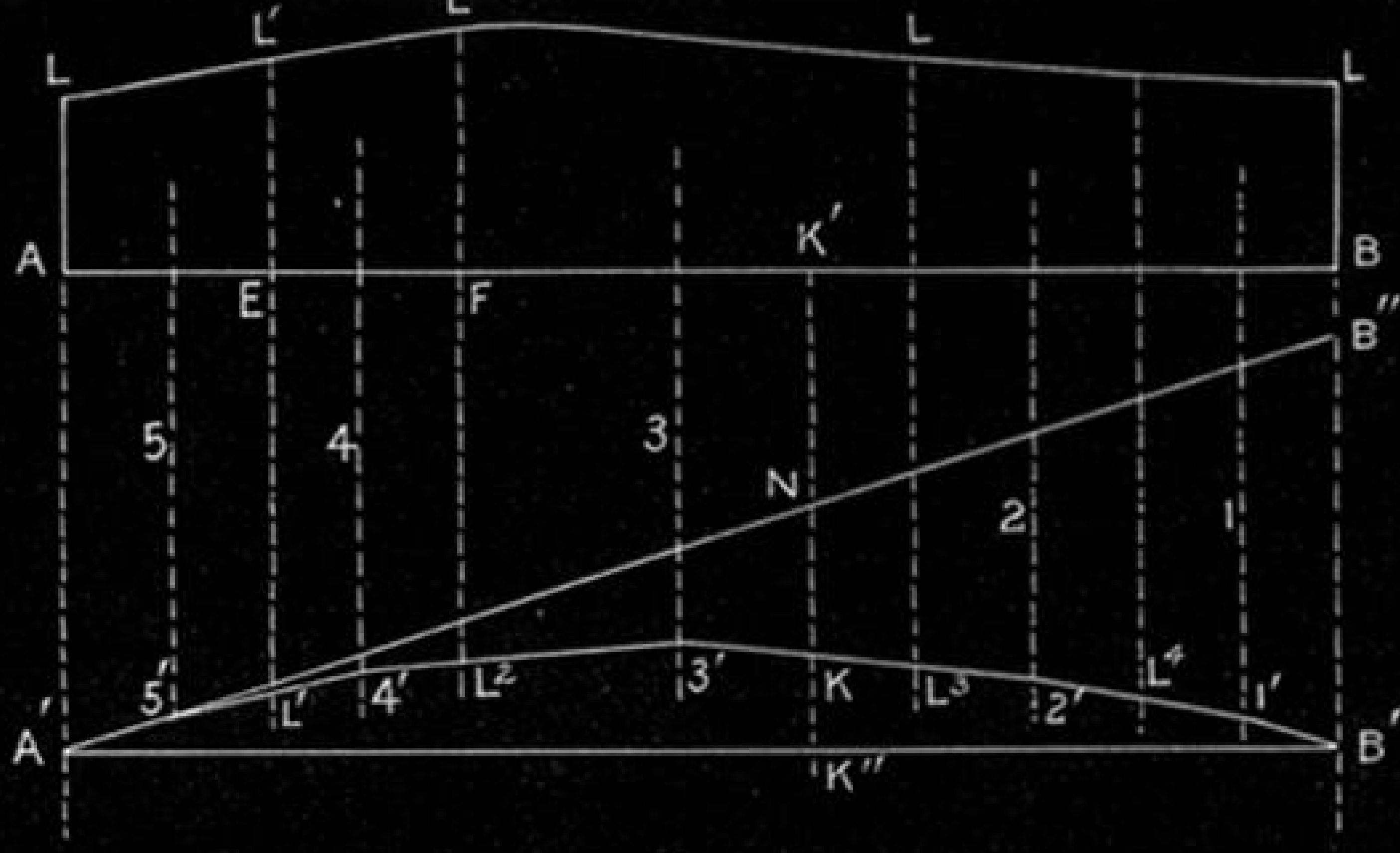


Fig. 1

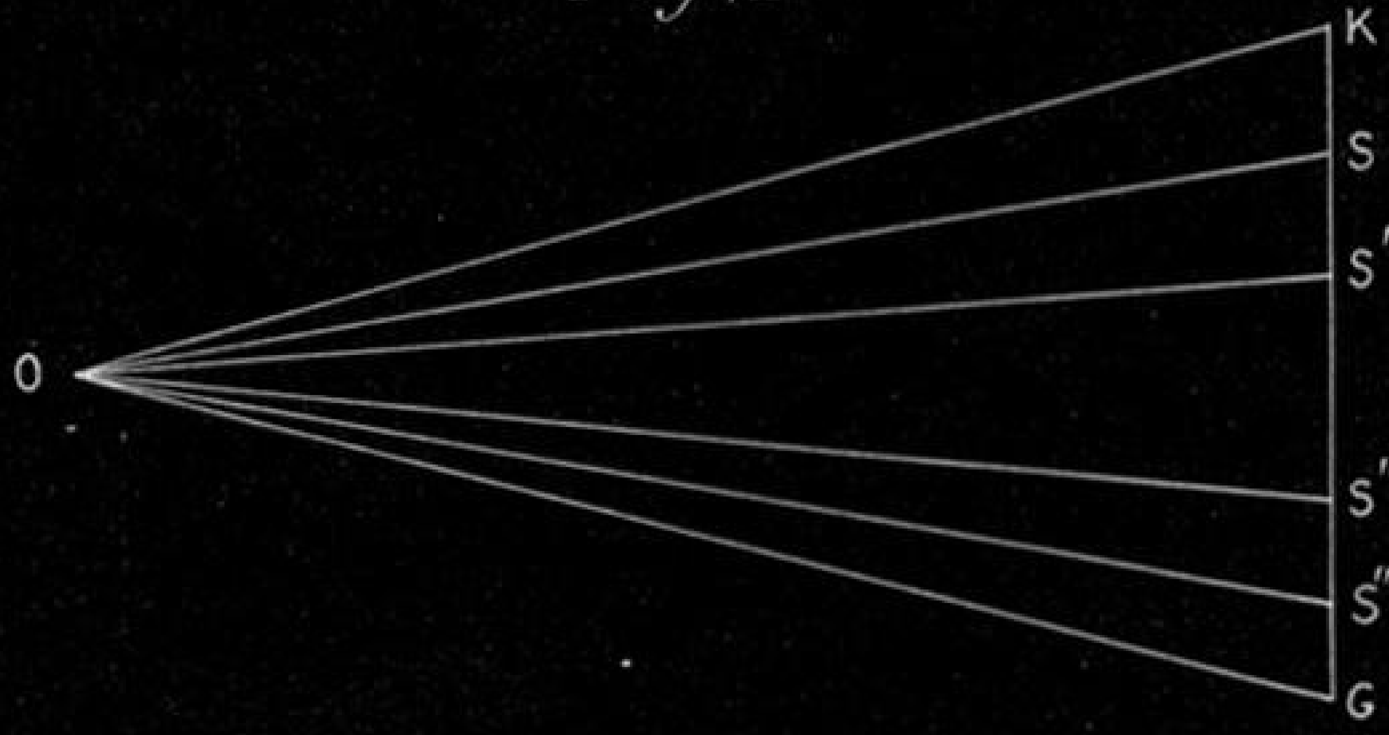


Fig. 2

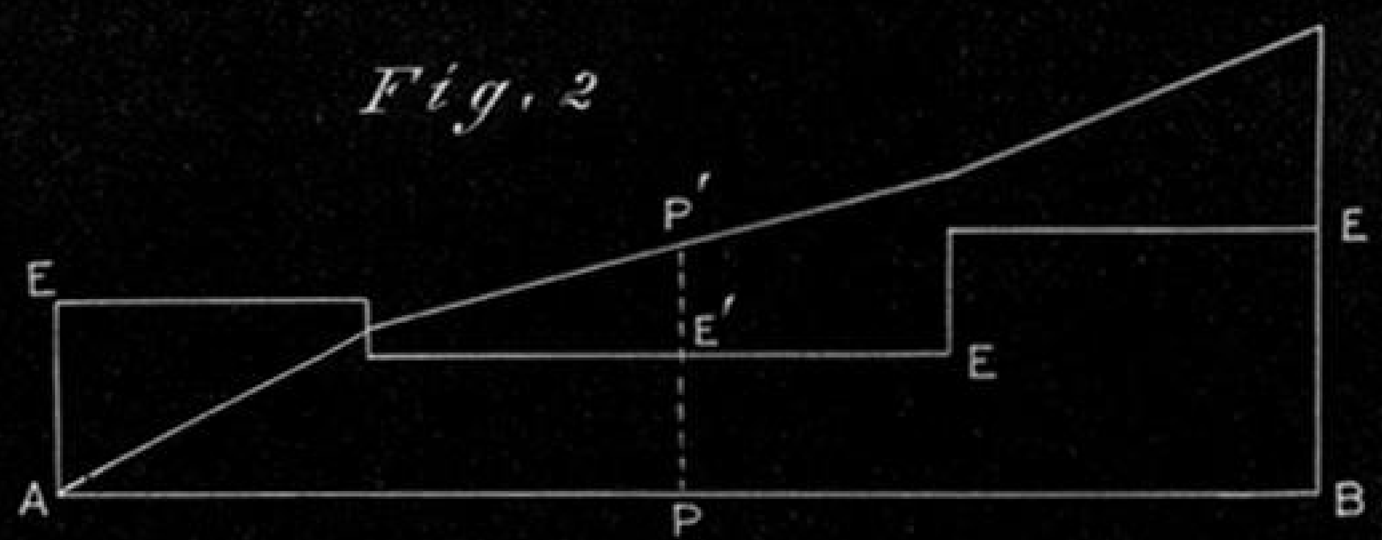


Fig. 4

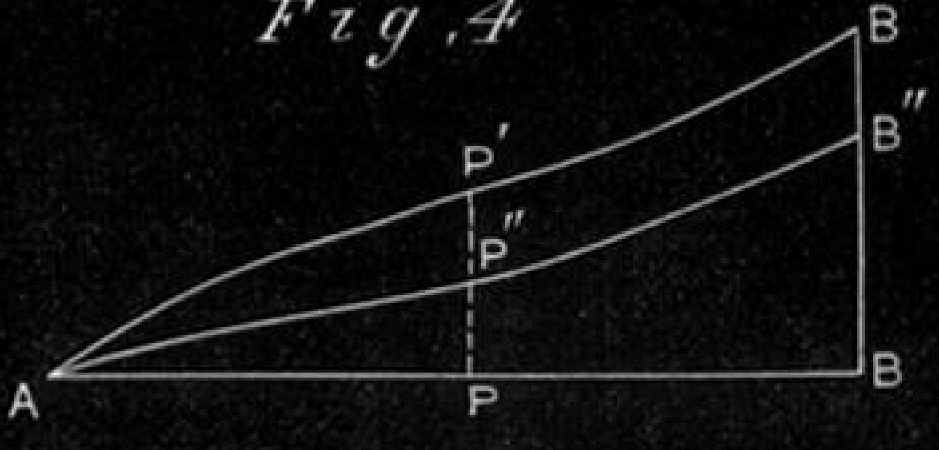
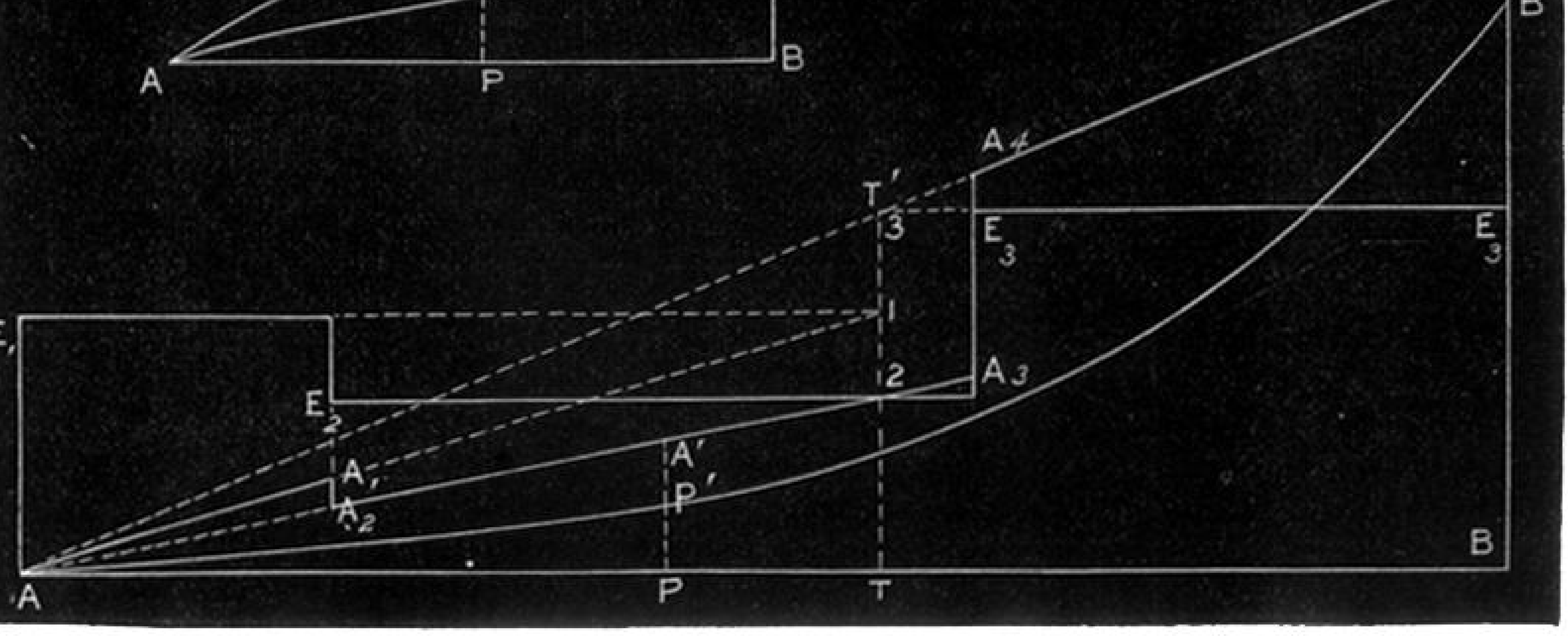
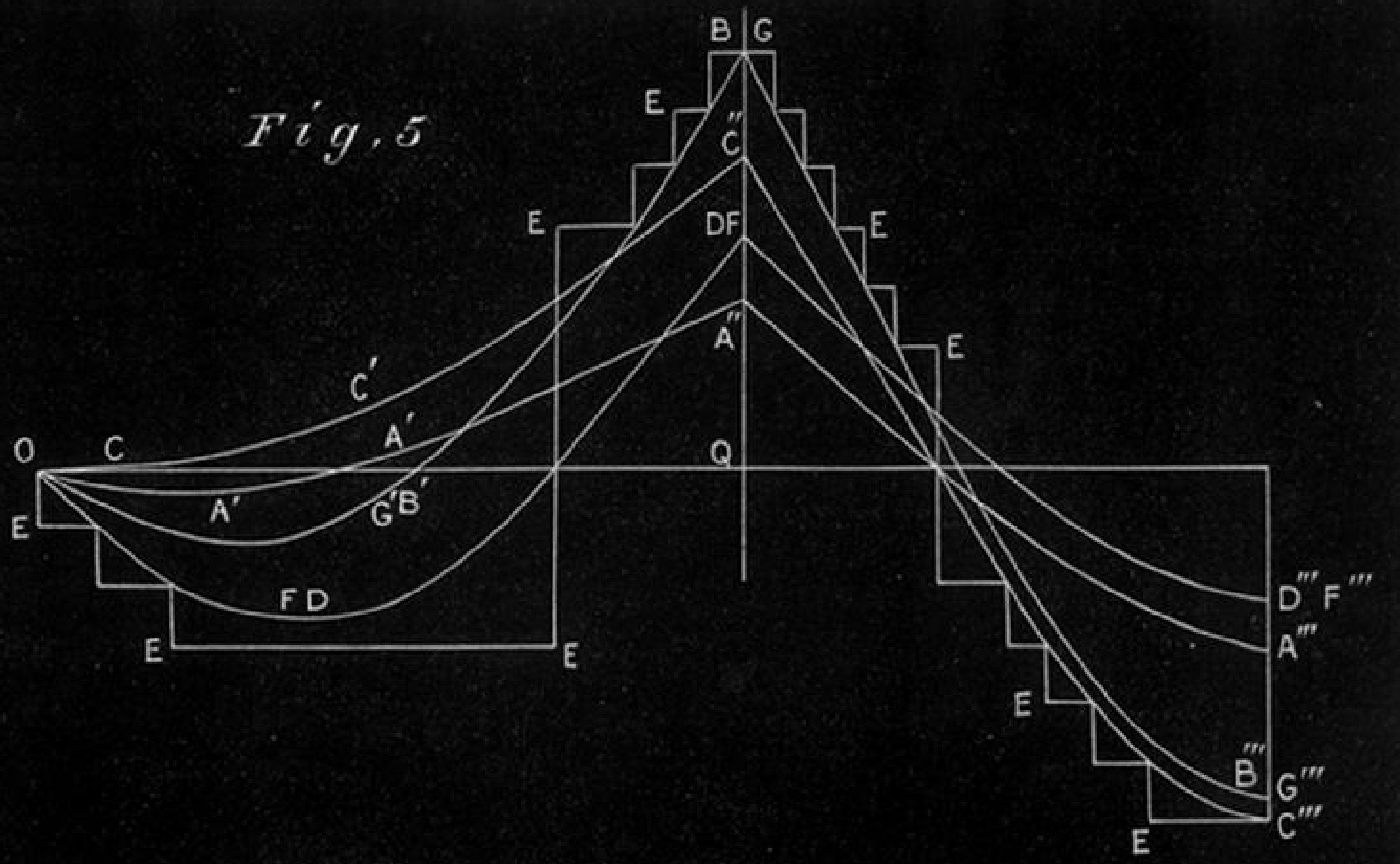


Fig. 3



Fig, 5



Fig, 6