III. "Preliminary Report to the Committee on Solar Physics on a Method of Detecting the Unknown Inequalities of a Series of Observations." By BALFOUR STEWART, F.R.S., Professor of Natural Philosophy in Owens College, Manchester, and WILLIAM DODGSON. Communicated to the Royal Society at the request of the Committee. Received May 15, 1879.

1. Our chief reason for suspecting the existence of a connexion between the state of the solar surface (as this is revealed by spots) and the magnetism and meteorology of the earth is derived from the fact that our observational series of sun-spots, on the one hand, and of magnetical and meteorological changes, on the other, are believed to be all subject to a common inequality, whose period (about 11 years) is virtually the same in all. But as it is only of late years that observations of great accuracy have been made in these three branches of inquiry, it is impossible to compare together more than a few series of long-period inequality, and hence some observers are still inclined to doubt the reality of a true connexion between the sun and the earth of the kind above mentioned. We are thus led to ask ourselves whether there may not be other inequalities of shorter period in these various observations, and whether we cannot devise some means of ascertaining the exact periodical times of these as well as their other properties.

We might thus expect to decide the question regarding a connexion between these three branches, for if solar observations and those of terrestrial magnetism and meteorology all exhibit a series of inequalities that are essentially the same in each, it is impossible to call in question the reality of some connexion between them.

2. The researches of Broun, Hornstein, Baxendell, and others have indicated the probable existence of inequalities in magnetism and meteorology, with periods of comparatively short length. Messrs. De La Rue, Stewart, and Loewy have likewise observed indications of a short-period fluctuation in sun-spots; but we are not aware that any attempt has yet been made to ascertain with great precision the exact period or periods of unknown inequalities either in terrestrial or in sun-spot observations.

As our sun-spot records have not yet been rendered sufficiently complete, it may be questioned whether the time has yet arrived for attempting any such investigation with respect to them; but our records of observations in magnetism and meteorology are now so long-continued and complete as to render it desirable in their case to make the attempt. This we have done in the following preliminary investigation.

3. We began first of all by making use of the records of the Kew

Observatory, which one of us had received through the kindness of the Kew Committee. In these the daily ranges of the magnetic declination are given after excluding disturbed observations by the process of Sir E. Sabine. The daily ranges are given in inches, and they denote the differences between the greatest and least values of each day's hourly tabulations from the curve, disturbances (as already mentioned) being excluded. These records extend from the beginning of 1858 to the end of 1873, embracing in all 16 years' observations.

4. Let us now proceed to describe the method which we pursued in our search for the natural inequalities of magnetic declination. We may illustrate this by a well-known case. Suppose that we had in our possession extensive records of the temperature of the earth's atmosphere at some one place in middle latitudes, and that, independently of astronomical knowledge, we were to make use of these for the purpose of investigating the natural inequalities of terrestrial temperature. We should begin by grouping the observations according to various periods taken, say, at small but definite time-intervals from each other. Now, if our series of observations were sufficiently extensive, and if some one of our various groupings together of this series should correspond to a real inequality, we should expect it to exhibit a well-defined and prominent fluctuation, whose departures above and below the mean should be of considerable amount. Suppose, for instance, that we have 24 points in our series, and that we group a long series of temperature observations in rows of 24 each, the time-distance between two contiguous members of one row being one hour. The series would thus represent the mean solar day, and we should no doubt obtain from a final summation of our rows a result exhibiting a prominent temperature fluctuation of a well-defined character, which we might measure by simply adding together all the departures of its various points from the mean, whether these points lie above or below; in fine, by obtaining the area of the curve which is the graphical representation of the inequality above and below the line of abscissæ taken to represent the mean of all the points. Suppose next that, still keeping to rows of 24, we should make the time interval between two contiguous members of a row somewhat different from one hour, whether greater or less, we should now in either case obtain a result exhibiting, when measured as above, a much smaller inequality than that given when the interval was exactly one hour, and it is even possible that if our series of observations were sufficiently extensive, we should obtain hardly any traces of an inequality whatever. In fine, when each row accurately represented a solar day the result would give an inequality of large amount, but when each row represented a period either slightly less or greater than a day the result would be an inequality of small amount.

5. We should by this process, after bestowing enormous labour in

grouping according to a great number of periods taken at small intervals from each other obtain definite results. These results might be graphically represented in the following manner: the line of abscissæ might be taken to denote the exact values of the various periods, forming a time scale in fact; while the ordinates might represent the final observed inequalities found by employing these various periods. There would thus be in the case now used for illustration a very prominent peak, corresponding to 24 hours, which would fall off rapidly on either side.

In this particular instance, having obtained as a result a period of exactly 24 hours, there would probably be no occasion to do anything more, because we have no reason to suppose the existence of any other temperature period very near 24 hours in addition to the one exactly corresponding thereto. We might, therefore, proceed finally to evaluate the obtained inequality, which would represent the mean daily variation of temperature.

6. It would be different, however, should there prove to be a number of inequalities having periods very close to one another on the time-scale. In this case, even when we had obtained a graphical representation of our results in the manner just now mentioned, it might be supposed that the various inequalities to some extent interfered with each other, affecting not only the position in the time-scale of the points of maximum inequality, but also the extent of range and the form of these inequalities. We should, therefore, have next to attempt to eliminate the effect of one inequality upon another.

The whole process would thus consist of two parts. In the first place, by enormous labour, we should have to obtain a graphical result showing the exact positions in the time-scale of the points of observed maximum inequality. Secondly, we should have to eliminate the effect of the various inequalities upon each other, provided it be found that there are several such inequalities very close together. In the present preliminary report we exhibit a method by which the great labour of the first of these two processes is materially abridged.

We have not, however, as yet advanced sufficiently far in our accurate estimation of observed maximum periods to proceed to the elimination of the effect of the various inequalities upon each other.

7. In testing our method, we began by grouping the Kew declination ranges in such a manner as to represent a period of 24·25 days. It is unnecessary to describe the details of the method by which a series of daily observations may be grouped so as to represent a period that is not an exact number of days; suffice it to say that we at length obtained a long series of upwards of 240 rows, each embracing 24 horizontal figures. Nor need we give the reasons which induced us to select the precise period of 24·25 days, since for all practical purposes this may be regarded as a period chosen at random.

Having grouped the whole 16 years' observations according to this period, we next broke up these into yearly sets. Each of these sets might thus be expected to be freed from the influence of the well-known annual inequality of declination range. These yearly sets embraced generally fifteen, but sometimes sixteen rows of twenty-four each. Our *starting-point or epoch* in all these calculations was January 1, 1858, that is to say, in all our groupings the declination range for January 1, 1858, formed the first member of a horizontal series.

The next operation was to sum up these 15 or 16 sets for each of the 24 horizontal figures. It might naturally be supposed that we should then divide each of the sums so obtained by 15 or 16, as the case might be, and then find the difference of each of the 24 quotients from the mean of all the quotients, such differences when placed together representing the inequality for that year.

There appears to be, however, reason to believe that (leaving aside all speculation as to causes) on those occasions when the daily range of the declination magnet is greatest, the variations of this daily range are greatest likewise, and possibly in nearly the same proportion (see paper by Balfour Stewart on "The Variations of the Diurnal Range of Magnetic Declination as recorded at the Prague Observatory," "Proc. Roy. Soc.," May 2, 1878). We have thus considered it as on the whole safest and best to adopt this plan with respect to our yearly results; that is to say, we have regarded the mean of the 24 points for each year as equal to 1,000, and we have represented each individual point upon this proportional scale.

8. The yearly inequality is, therefore, represented by the series made up of the differences of each of the 24 points from this normal value. As it is desirable in carrying out our programme to represent each yearly inequality by means of a curve, we have to some extent smoothed or equalised this series of differences. The primary series (A) has been converted into another series (B), also of 24 points, each point of (B) being the mean of four consecutive points of (A), and the series (B) has then been converted by a similar process into a series (C), each point of which is a mean of four consecutive points of (B). This amount of equalisation, by getting rid of what we may term accidental fluctuation is quite sufficient to enable us to draw a curve, well representing each yearly inequality. The equalised yearly inequalities corresponding to the period 24·25 days are represented in Table I.

Table I.—Equalised Yearly Inequalities of Kew Declination-range corresponding to period 24·25 days.

| Year | (0) | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1858 | −38 | −38 | −35 | −21 | −15 | 0 | +13 | +23 | +40 | +44 | +50 | +41 | +31 | +21 | + 3 | − 1 | −11 | −10 | 0 | 0 | − 7 | −18 | −31 | −41 |
| 1859 | −20 | −24 | −27 | −14 | − 2 | + 5 | + 6 | + 2 | + 4 | + 8 | +22 | +26 | +23 | +16 | + 7 | + 1 | − 1 | − 2 | − 3 | − 3 | − 4 | − 3 | − 5 | −12 |
| 1860 | +34 | +24 | + 6 | −10 | −27 | −28 | −20 | −24 | −22 | −29 | −31 | −22 | −19 | − 8 | + 3 | + 7 | +19 | +22 | +20 | +21 | +16 | +18 | +22 | +28 |
| 1861 | +25 | +25 | +25 | +25 | +28 | +30 | +33 | +29 | +26 | +13 | − 3 | −19 | −36 | −46 | −53 | −50 | −42 | −34 | −20 | −12 | 0 | +12 | +18 | +26 |
| 1862 | + 9 | +13 | +18 | +24 | +22 | +14 | + 4 | − 5 | −17 | −20 | −19 | −20 | − 7 | − 4 | − 5 | − 4 | −10 | − 7 | − 7 | − 1 | + 3 | + 4 | + 9 | + 6 |
| 1863 | +10 | + 2 | − 7 | −18 | −28 | −35 | −34 | −26 | −15 | + 5 | +17 | +21 | +19 | + 7 | − 2 | − 6 | − 5 | − 3 | + 4 | +11 | +20 | +24 | +21 | +18 |
| 1864 | − 1 | −12 | −15 | − 9 | + 9 | +21 | +18 | +11 | + 5 | + 6 | +13 | +23 | +26 | +26 | +20 | +10 | + 2 | −14 | −29 | −40 | −42 | −26 | − 7 | + 5 |
| 1865 | −11 | − 7 | − 2 | − 7 | − 3 | − 4 | + 1 | +13 | +11 | + 1 | −14 | −18 | −11 | 0 | + 9 | − 1 | − 7 | − 7 | + 2 | +15 | +21 | +17 | + 5 | − 3 |
| 1866 | −17 | − 8 | + 3 | +26 | +35 | +32 | +28 | + 9 | − 1 | − 1 | + 2 | +12 | +20 | +15 | − 3 | −22 | −43 | −46 | −32 | −15 | + 6 | + 9 | + 2 | −11 |
| 1867 | +14 | +25 | +29 | +33 | +28 | +24 | +22 | + 9 | 0 | − 9 | −22 | −20 | −24 | −26 | −16 | −14 | − 4 | + 5 | + 2 | − 4 | −14 | −19 | −15 | − 4 |
| 1868 | +24 | +24 | +21 | +22 | +18 | +17 | + 6 | −10 | −20 | −28 | −22 | −13 | −10 | − 9 | − 6 | − 3 | + 8 | + 6 | − 7 | −14 | −19 | −11 | + 8 | +18 |
| 1869 | +28 | +28 | +24 | +19 | + 3 | −13 | −23 | −36 | −37 | −33 | −30 | −24 | −20 | −17 | −14 | − 8 | 0 | + 6 | +12 | +22 | +25 | +30 | +31 | +27 |
| 1870 | − 2 | − 7 | −10 | − 9 | −14 | −17 | −26 | −29 | −31 | −32 | −21 | −14 | − 5 | + 4 | +16 | +24 | +32 | +37 | +30 | +28 | +20 | +15 | +11 | 0 |
| 1871 | − 4 | −13 | −17 | −14 | + 1 | +21 | +34 | +44 | +47 | +42 | +34 | +20 | + 4 | − 6 | −13 | −18 | −17 | −25 | −33 | −31 | −31 | −19 | − 5 | − 1 |
| 1872 | +12 | − 1 | − 9 | −13 | −18 | −17 | −18 | −16 | −12 | −10 | − 5 | −10 | − 6 | 0 | + 7 | +25 | +27 | +20 | +12 | + 1 | − 3 | + 8 | +13 | +13 |
| 1873 | +27 | +14 | + 6 | − 2 | − 3 | − 2 | 0 | + 8 | +17 | +24 | +24 | +13 | 0 | −17 | −34 | −39 | −46 | −40 | −27 | −16 | + 4 | +21 | +32 | +36 |
| | +90 | +45 | +10 | +32 | +34 | +48 | +44 | + 2 | − 5 | −19 | − 5 | − 4 | −15 | −44 | −81 | −99 | −98 | −92 | −76 | −38 | − 5 | +62 | +109 | +105 |
| | | | | | | | | | | | | | | | | | | | | | | | (sum=1162) | |

9. A glance at the sums of this table for the whole 16 years will suffice to show that 24·25 days does not correspond to the exact period of any marked inequality. The sums are small, and we conclude therefore that we have not been fortunate in our chance selection of a period to begin with; but the peculiarity of our method is, that it will enable us to ascertain the true position in the time-scale of the neighbouring prominent inequalities by means of the results of Table I. The method of doing this can easily be rendered evident. Each horizontal row of Table I consists of 24 numbers, and there are 16 years, beginning with 1858. We may, therefore, call the numbers of the first row $(0)_{58}$, $(1)_{58}$, $(2)_{58}$, &c., $(23)_{58}$; those of the second row $(0)_{59}$, $(1)_{59}$, $(2)_{59}$, &c., $(23)_{59}$, and so on for each row.

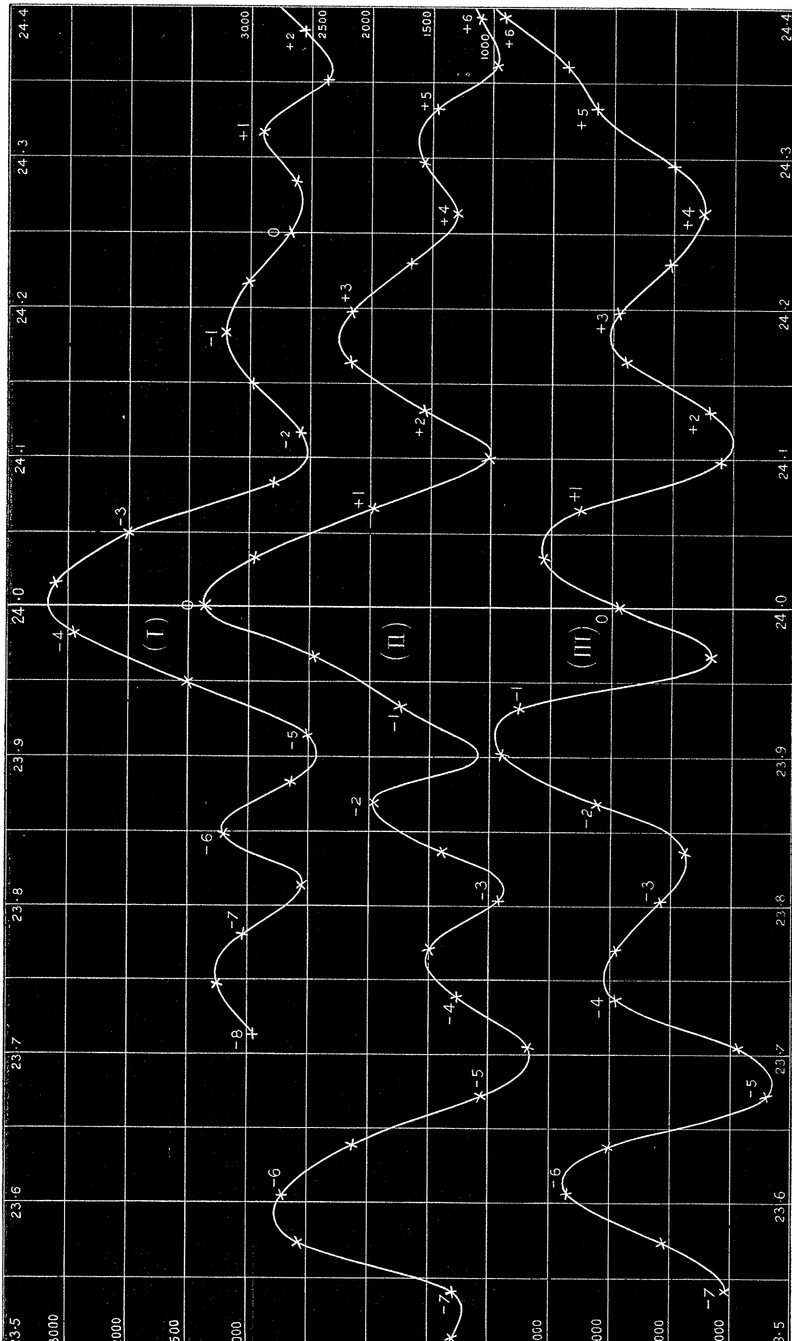
In this table, therefore, each vertical column consists of similar numbers for the various years, adopting the notation now mentioned.

Suppose, however, that we displace these values as follows:—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1858 . . | $(0)_{58}$, | $(1)_{58}$, | $(2)_{58}$ | . . | $(21)_{58}$, | $(22)_{58}$, | $(23)_{58}$. |
| 1859 . . | $(1)_{59}$, | $(2)_{59}$, | $(3)_{59}$ | . . | $(22)_{59}$, | $(23)_{59}$, | $(0)_{59}$. |
| 1860 . . | $(2)_{60}$, | $(3)_{60}$, | $(4)_{60}$ | . . | $(23)_{60}$, | $(0)_{60}$, | $(1)_{60}$. |
| . . | . | . | . | . . | . | . | . |
| . . | . | . | . | . . | . | . | . |
| 1873 . . | $(15)_{73}$, | $(16)_{73}$, | $(17)_{73}$ | . . | $(12)_{73}$, | $(13)_{73}$, | $(14)_{73}$. |

Now, if we add up the various vertical columns of this series the sums will represent an inequality somewhat larger in period than 24·25 days. For it is manifest that if we have a regular series of waves whose values we plot numerically after the manner of Table I, the consequence of adopting too small a time scale will be to throw any salient point of the wave, such as the crest, always further and further to the right, and to correct this we should have to pull the whole series a little to the left each time. Now, this is precisely what we have done in the above process, which will thus give us the representation of an inequality of larger period than 24·25 days. It is easy to find the exact length of period which the above series represents. We pull everything to the left nearly one day, but more accurately the 24th part of 24·25 days in one year. If, therefore, 365·25 days give $\frac{24{\cdot}25}{24}$, what will 24·25 days give? We find from this proportion that the period of the inequality indicated by performing the above process is $24{\cdot}25+\frac{(24{\cdot}25)^2}{24\times 365{\cdot}25}=24{\cdot}317$ days. Again, we may pull things to the left two, three, or four divisions each year, and thus obtain the representation of inequalities with periods of 24·384, 24·451, or 24·518 days.

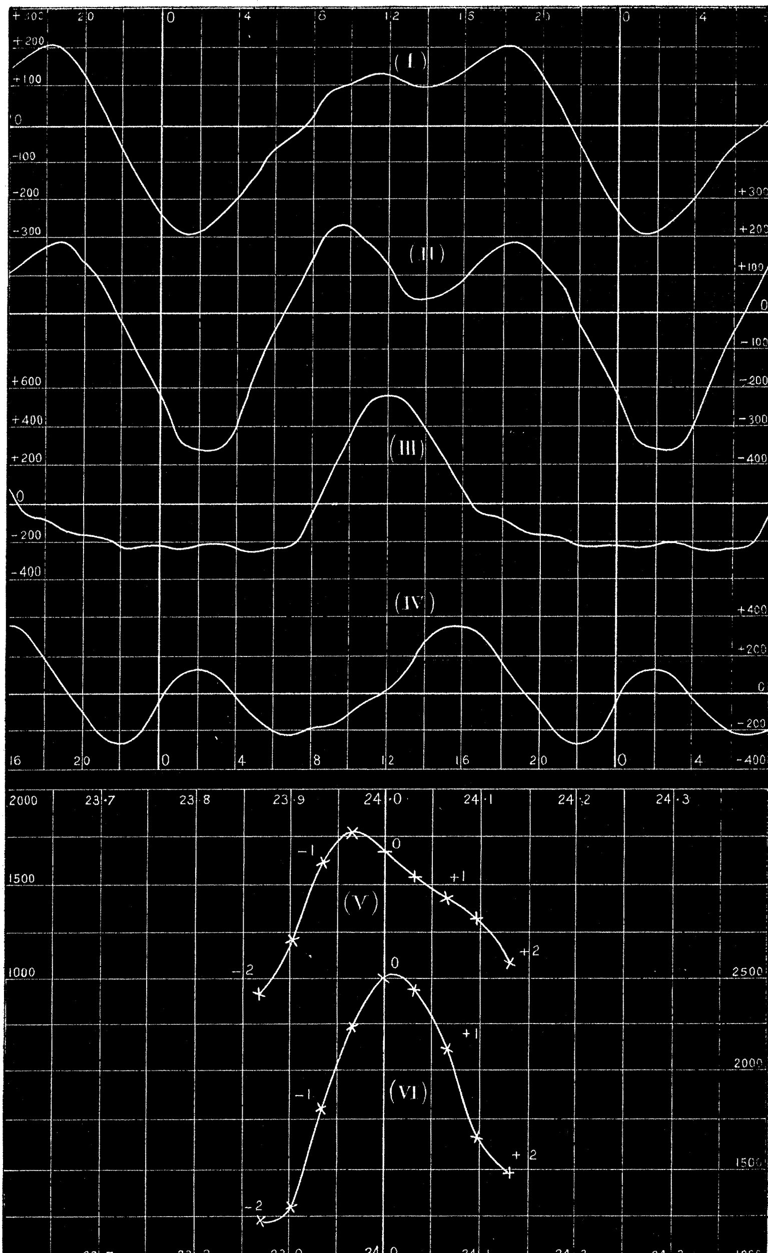
Or we may perform the opposite operation of pushing things to the right one division each year, and thus obtain the representation of an inequality, having a period of 24·183 days, while 2, 3, or 4 such divisions each year would give us periods of 24·116, 24·049, or 23·982 days.



(I)
(II)
(III)
23·5
23·6
23·7
23·8
23·9
24·0
24·1
24·2
24·3
24·4
3000
2000
1500
1000
5000
4000
3000
2000
2500


Diagram I.

## Diagram II.


(I)
(II)
(III)
(IV)
(V)
(VI)
+200
+100
0
-100
-200
-300
+600
+400
+200
0
-200
-400
+300
+200
+100
0
-100
-200
-300
-400
+400
+200
0
-200
-400
16
20
0
4
8
12
16
20
0
4
2000
1500
1000
23·7
23·8
23·9
24·0
24·1
24·2
24·3
2500
2000
-2
-1
0
+1
+2

10. We have found it necessary to push things not merely by the multiple of a whole division—right or left each year—but by the multiple of half a division.

To accomplish this we must obtain for every alternate year a series of half-way points, which is best done by converting the series of such years into curves, and using these to give us the half-way points.

For instance, should we wish to pull things to the left half a division each year, we should adopt the following arrangement.—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1858 . . | $(0\frac{1}{2})_{58}$, | $(1\frac{1}{2})_{58}$, | $(2\frac{1}{2})_{58}$ | . . | $(21\frac{1}{2})_{58}$, | $(22\frac{1}{2})_{58}$, | $(23\frac{1}{2})_{58}$. |
| 1859 . . | $(1)_{59}$, | $(2)_{59}$, | $(3)_{59}$ | . . | $(22)_{59}$, | $(23)_{59}$, | $(0)_{59}$. |
| 1860 . . | $(1\frac{1}{2})_{60}$, | $(2\frac{1}{2})_{60}$, | $(3\frac{1}{2})_{60}$ | . . | $(22\frac{1}{2})_{60}$, | $(23\frac{1}{2}{}_{60})$, | $(0\frac{1}{2})_{60}$. |
| 1861 . . | $(2)_{61}$, | $(3)_{61}$, | $(4)_{61}$ | . . | $(23)_{61}$, | $(0)_{61}$, | $(1)_{61}$. |
| . | . . | . . | . . | . . | . . | . . | . |

A further extension of the same method would enable us to go from halves to quarters of a division, but we have not yet met with any inequality that is not sufficiently well indicated by half divisions.

11. We have found the following plan very useful in abridging the labour of these reductions.

Sixteen perfectly similar strips of thick paper are taken, one for each year. Each of these strips is divided into 48 small compartments, a vertical black line being ruled at the beginning, the middle, and the end. Upon the first of these strips the inequality, say for 1858, is written in duplicate, so that the second 24 figures are a repetition of the first. The same is done for the other years. These strips are then attached to a frame which allows them to slide along with regard to each other, and at the same time only exposes 24 lines of figures at a time. By this method they can easily be arranged according to any given order, and the sums made with very little trouble.

It is further desirable to exhibit all the positive values, say in black, and all the negative values in red, so that the eye may easily distinguish between them.

The following will exhibit the nature of the arrangement—only to save space we take an inequality consisting of only four terms.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | $(0)_{58}$ | $(1)_{58}$ | $(2)_{58}$ | $(3)_{58}$ | $(0)_{58}$ | $(1)_{58}$ | $(2)_{58}$ | $(3)_{58}$ |
| | | $(0)_{59}$ | $(1)_{59}$ | $(2)_{59}$ | $(3)_{59}$ | $(0)_{59}$ | $(1)_{59}$ | $(2)_{59}$ | $(3)_{59}$ | |
| | $(0)_{60}$ | $(1)_{60}$ | $(2)_{60}$ | $(3)_{60}$ | $(0)_{60}$ | $(1)_{60}$ | $(2)_{60}$ | $(3)_{60}$ | | |
| $(0)_{61}$ | $(1)_{61}$ | $(2)_{61}$ | $(3)_{61}$ | $(0)_{61}$ | $(1)_{61}$ | $(2)_{61}$ | $(3)_{61}$ | | | |

The vertical portion between the brackets is here all that is exposed.

12. Before concluding this description of our method we ought to remark that it can only be considered as correct for periods not far removed on either side from that for which the series or book (as we call it) was originally framed. Thus, in the present instance, while the results of Table I will readily indicate inequalities that lie six divisions (of 0·067 day each) either to the right or left of the normal series, it is apparent that +12 divisions from the normal must necessarily exhibit the same result as −12; it will not therefore do to push the method nearly so far.

13. When this method is applied to Table I we obtain the following results:—

Table II.—Exhibiting the results of the above method applied to the numbers of Table I.

| Divisions from normal. | | Exact period in days. | | Magnitude of inequality. |
|---|---|---|---|---|
| −8·0 | ...... | 23·7133 | ...... | 1438 |
| −7·5 | ...... | 23·7469 | ...... | 1757 |
| −7·0 | ...... | 23·7804 | ...... | 1546 |
| −6·5 | ...... | 23·8140 | ...... | 1047 |
| −6·0 | ...... | 23·8475 | ...... | 1707 |
| −5·5 | ...... | 23·8810 | ...... | 1141 |
| −5·0 | ...... | 23·9146 | ...... | 1002 |
| −4·5 | ...... | 23·9481 | ...... | 2009 |
| −4·0 | ...... | 23·9817 | ...... | 2954 |
| −3·5 | ...... | 24·0152 | ...... | 3109 |
| −3·0 | ...... | 24·0487 | ...... | 2504 |
| −2·5 | ...... | 24·0823 | ...... | 1289 |
| −2·0 | ...... | 24·1158 | ...... | 1068 |
| −1·5 | ...... | 24·1494 | ...... | 1473 |
| −1·0 | ...... | 24·1829 | ...... | 1700 |
| −0·5 | ...... | 24·2165 | ...... | 1513 |
| Normal | ...... | 24·2500 | ...... | 1162 |
| +0·5 | ...... | 24·2835 | ...... | 1115 |
| +1·0 | ...... | 24·3171 | ...... | 1398 |
| +1·5 | ...... | 24·3506 | ...... | 867 |
| +2·0 | ...... | 24·3842 | ...... | 1062 |

14. The results of Table II are exhibited graphically by means of a curve in Diagram I, fig. I, which accompanies this paper.

In this curve the abscissæ denote periods, while the ordinates represent corresponding inequalities. It will be noticed that we have

Table III.—Equalised Yearly Inequalities of Kew Declination-range corresponding to period 24·00 days.

| Year | (0) | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1858 | −34 | −48 | −51 | −46 | −44 | −31 | −15 | − 1 | +19 | +29 | +35 | +36 | +36 | +38 | +31 | +21 | + 7 | − 9 | − 4 | + 9 | +17 | +21 | + 5 | −20 |
| 1859 | −20 | −26 | −22 | −24 | −22 | −18 | −21 | −17 | −13 | − 2 | + 9 | +11 | +17 | +12 | +15 | +23 | +22 | +28 | +25 | +20 | +16 | + 6 | − 5 | −12 |
| 1860 | + 5 | +14 | +18 | +18 | +16 | +17 | +19 | +26 | +39 | +44 | +41 | +22 | − 8 | −34 | −50 | −49 | −35 | −21 | −10 | −10 | −16 | −19 | −17 | − 6 |
| 1861 | −27 | −47 | −56 | −62 | −56 | −45 | −36 | −21 | − 6 | + 8 | +15 | +22 | +24 | +21 | +29 | +26 | +27 | +32 | +30 | +36 | +38 | +30 | +17 | − 3 |
| 1862 | −24 | −30 | −22 | −13 | − 1 | + 6 | + 5 | + 1 | − 1 | 0 | − 2 | − 5 | −11 | −17 | −13 | − 2 | +10 | +20 | +25 | +28 | +27 | +21 | + 8 | − 8 |
| 1863 | −17 | −17 | −23 | −21 | −15 | 0 | +22 | +32 | +33 | +24 | +10 | + 4 | + 6 | + 3 | + 2 | − 2 | − 4 | + 6 | +10 | + 8 | − 4 | −17 | −21 | −21 |
| 1864 | +21 | + 8 | 0 | − 9 | − 7 | 0 | + 5 | + 1 | − 2 | − 5 | − 9 | − 6 | − 8 | − 4 | + 1 | + 6 | +13 | + 9 | + 2 | − 8 | −17 | − 7 | + 2 | +16 |
| 1865 | + 2 | 0 | − 3 | − 5 | − 4 | − 5 | −10 | − 9 | − 9 | − 2 | +14 | +19 | +26 | +21 | + 4 | + 1 | − 3 | + 2 | + 6 | − 6 | −12 | −15 | −11 | 0 |
| 1866 | −34 | −33 | −17 | − 5 | + 6 | + 8 | − 4 | −16 | −19 | −14 | − 2 | +20 | +26 | +24 | +15 | 0 | − 2 | + 2 | +10 | +22 | +22 | +13 | − 3 | −24 |
| 1867 | −55 | −40 | −26 | − 5 | +14 | +24 | +30 | +19 | + 6 | + 2 | 0 | + 8 | +21 | +26 | +31 | +30 | +28 | +30 | +20 | + 6 | −10 | −40 | −51 | −56 |
| 1868 | −27 | −29 | −23 | −16 | −13 | −14 | − 8 | − 2 | + 1 | + 3 | −10 | −19 | −23 | −15 | + 5 | +16 | +28 | +29 | +31 | +39 | +33 | +25 | + 5 | −17 |
| 1869 | 0 | −17 | −20 | −25 | −27 | −33 | −40 | −47 | −48 | −36 | −24 | − 8 | + 6 | +11 | +16 | +23 | +29 | +32 | +37 | +41 | +41 | +43 | +31 | +15 |
| 1870 | + 2 | + 2 | − 3 | 0 | − 8 | −17 | −22 | −31 | −34 | −25 | −18 | − 6 | + 6 | + 9 | +12 | +17 | +22 | +25 | +24 | +21 | +12 | + 6 | + 6 | − 2 |
| 1871 | −12 | 0 | + 2 | − 2 | −11 | −20 | −10 | + 4 | +20 | +41 | +46 | +41 | +34 | +23 | +15 | +18 | +16 | + 6 | − 5 | −30 | −48 | −51 | −47 | −31 |
| 1872 | +14 | + 6 | + 3 | 0 | + 3 | + 8 | +11 | + 8 | + 1 | − 8 | −19 | −24 | −19 | −15 | −10 | − 6 | −10 | − 7 | − 4 | + 1 | +11 | +16 | +19 | +21 |
| 1873 | −27 | −32 | −38 | −34 | −22 | − 5 | +15 | +30 | +39 | +35 | +21 | +11 | − 3 | − 2 | − 3 | − 5 | − 4 | − 5 | + 7 | +13 | +16 | +11 | − 3 | −14 |
| | −233 | −289 | −281 | −249 | −191 | −125 | −59 | −23 | +26 | +94 | +107 | +126 | +130 | +101 | +100 | +117 | +144 | +179 | +204 | +190 | +126 | +43 | −65 | −162 |
| | | | | | | | | | | | | | | | | | | | | | | | (sum = 3364) | |

indications of several maximum points, more especially of one corresponding very nearly to 24 days.

This has induced us to make a new book for 24 days in addition to the one for 24·25 days, the construction of which we have already described. Both have been treated in precisely the same manner.

The results of this book are given in Table III.

15. When our method is applied to Table III, we obtain the following results:—

Table IV.—Exhibiting the results of the above method applied to the numbers of Table III.

| Divisions from normal. | Exact period in days. | Magnitude of inequality. |
|---|---|---|
| −7·5 | 23·5072 | 1310 |
| −7·0 | 23·5400 | 1280 |
| −6·5 | 23·5729 | 2568 |
| −6·0 | 23·6057 | 2710 |
| −5·5 | 23·6386 | 2128 |
| −5·0 | 23·6715 | 1070 |
| −4·5 | 23·7043 | 674 |
| −4·0 | 23·7372 | 1260 |
| −3·5 | 23·7700 | 1500 |
| −3·0 | 23·8029 | 922 |
| −2·5 | 23·8357 | 1394 |
| −2·0 | 23·8686 | 1976 |
| −1·5 | 23·9014 | 1096 |
| −1·0 | 23·9343 | 1754 |
| −0·5 | 23·9671 | 2464 |
| Normal | 24·0000 | 3364 |
| +0·5 | 24·0329 | 2960 |
| +1·0 | 24·0657 | 1974 |
| +1·5 | 24·0986 | 1032 |
| +2·0 | 24·1314 | 1548 |
| +2·5 | 24·1643 | 2174 |
| +3·0 | 24·1971 | 2160 |
| +3·5 | 24·2300 | 1664 |
| +4·0 | 24·2628 | 1278 |
| +4·5 | 24·2957 | 1570 |
| +5·0 | 24·3285 | 1456 |
| +5·5 | 24·3614 | 954 |
| +6·0 | 24·3943 | 1110 |

16. The results of Table IV are exhibited graphically by means of a curve in Diagram I, fig. II.

It will at once be seen from this curve that it exhibits very nearly the same positions for maximum inequalities as those shown in fig. I.

Thus, by selecting at random the period 24·25 days we are by our method referred to nearly the true positions of the various maximum inequalities, and it might then be well to construct a separate book for each of these, as we have done for the large inequality corresponding to 24 days nearly.

17. While this method has succeeded in bringing before us the hitherto unknown inequalities of the Kew declination-range, it might yet be imagined that the results obtained are of a local or semi-local, and not of a truly cosmical nature. To test this we have compared together the daily declination-ranges as recorded at Trevandrum by Mr. Broun during the years 1858–64, with the corresponding portions of the Kew series. Plotting these in series of 24 days each we have obtained by the method already described the following results:—

Table V.—Comparing together by the above method 7 years of Trevandrum and 7 years of Kew Declination-range.

| Divisions from normal. | Exact period in days. | Magnitude of inequality. Kew. | Trevandrum. |
|---|---|---|---|
| −2 ·0 | 23 ·8686 | 925 | 1235 |
| −1 ·5 | 23 ·9014 | 1206 | 1301 |
| −1 ·0 | 23 ·9343 | 1633 | 1809 |
| −0 ·5 | 23 ·9671 | 1774 | 2245 |
| Normal | 24 ·0000 | 1687 | 2507 |
| +0 ·5 | 24 ·0329 | 1546 | 2448 |
| +1 ·0 | 24 ·0657 | 1425 | 2123 |
| +1 ·5 | 24 ·0986 | 1322 | 1662 |
| +2 ·0 | 24 ·1314 | 1085 | 1489 |

The results of Table V are exhibited graphically in Diagram II, figs. V, VI, No. V denoting the Kew, and No. VI the Trevandrum inequality.

18. We see from Table V that the maximum inequality occurs for these 7 years, both for Kew and Trevandrum, at points not far distant from 24·00 days. There are thus exhibited signs of repetition in this Kew inequality, as well as evidences of its cosmical nature, inasmuch as Trevandrum gives results similar to Kew as far as *period* is concerned.

19. We shall now compare together the *forms* of the inequalities in these two places.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Kew, 1858 to 1864. | (0) | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| | −219 | −334 | −357 | −359 | −295 | −162 | −48 | +48 |
| | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| | +158 | +224 | +226 | +192 | +128 | +43 | +34 | +53 |
| | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) |
| | +91 | +149 | +178 | +190 | +139 | +80 | −25 | −123 |
| Trevandrum, as above. | −217 | −224 | −215 | −206 | −226 | −247 | −233 | −213 |
| | −53 | +158 | +345 | +526 | +562 | +539 | +407 | +240 |
| | +91 | −50 | −73 | −128 | −158 | −178 | −219 | −222 |

In the above comparison the inequalities for 7 years are multiplied by $\frac{16}{7}$ in order to bring them to the same scale with that for 16 years exhibited in Table III.

In Diagram II, which accompanies this paper, the Kew declination inequality for 24 days (16 years) is given in fig. I.

The Kew declination inequality for 24 days (7 years) is given in fig. II, while the Trevandrum declination inequality for 24 days (7 years) is exhibited in fig. III. It will be seen that there is a very considerable likeness between all the three curves which are of the same type. The Trevandrum inequality is more marked than the corresponding Kew one—this may possibly be due to the fact that in the Kew observations the disturbances were eliminated.

20. Our method has hitherto been applied to magnetic declination-ranges. But if the cause of these inequalities be cosmical and connected with our luminary, we might suppose that the meteorological elements of the earth would also be affected. We have, therefore, taken the Kew diurnal temperature ranges—in other words, the difference between the daily indications of the maximum and minimum thermometers for the same 16 years for which we have declination-ranges, and we have treated these in precisely the manner already described, that is to say, we have grouped these ranges into series of 24 days each, and dealt with them as we have dealt with the similar declination series. The results are recorded in the following table :—

Table VI.—Equalised Yearly Inequalities of Kew Temperature-range corresponding to period 24·00 days.

| Year | (0) | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1858 | −45 | −17 | + 6 | +15 | +21 | + 9 | + 8 | +18 | +11 | +14 | 0 | − 1 | +18 | +30 | +54 | +50 | +32 | +18 | − 2 | −18 | −38 | −54 | −65 | −64 |
| 1859 | + 1 | + 9 | + 7 | + 7 | − 6 | −11 | −20 | −38 | −37 | −26 | − 2 | +33 | +42 | +22 | + 5 | −12 | − 5 | +27 | +37 | +30 | + 6 | −22 | −26 | −21 |
| 1860 | + 6 | + 3 | −11 | −24 | −34 | −32 | −20 | + 4 | +34 | +53 | +58 | +49 | +32 | +26 | +28 | +19 | +11 | − 9 | −31 | −44 | −48 | −40 | −25 | − 5 |
| 1861 | −71 | −63 | −64 | −78 | −90 | −87 | −78 | −55 | −35 | −13 | +19 | +41 | +61 | +77 | +90 | +112 | +114 | +106 | +94 | +61 | +22 | −21 | −61 | −81 |
| 1862 | −70 | −62 | −49 | −37 | −25 | − 5 | − 6 | − 8 | −14 | −15 | +17 | +44 | +65 | +72 | +58 | +52 | +41 | +43 | +47 | +31 | + 4 | −34 | −66 | −83 |
| 1863 | +14 | +25 | +30 | +36 | +40 | +33 | +28 | +37 | +39 | +57 | +63 | +39 | +21 | − 9 | −30 | −39 | −54 | −65 | −74 | −67 | −59 | −41 | −18 | − 6 |
| 1864 | +65 | +43 | −12 | −42 | −55 | −52 | −21 | −13 | −22 | −36 | −43 | −22 | + 9 | +55 | +74 | +57 | +31 | −19 | −43 | −39 | −28 | + 8 | +46 | +59 |
| 1865 | −109 | −77 | −28 | +18 | +51 | +65 | +62 | +60 | +60 | +43 | +26 | − 1 | −31 | −27 | − 2 | +28 | +52 | +59 | +38 | + 2 | −23 | −64 | −95 | −107 |
| 1866 | +16 | −15 | −30 | −32 | −44 | −18 | + 4 | + 3 | +10 | − 8 | −25 | −20 | −17 | − 4 | + 5 | − 8 | −12 | −12 | + 5 | +33 | +48 | +57 | +40 | +24 |
| 1867 | + 8 | +30 | +63 | +94 | +98 | +68 | +29 | − 8 | −36 | −45 | −47 | −60 | −69 | −82 | −80 | −53 | −29 | − 6 | +10 | +14 | +28 | +35 | +25 | +13 |
| 1868 | −10 | +31 | +40 | +32 | +29 | + 8 | + 9 | +10 | + 5 | + 7 | +10 | +29 | +45 | +47 | +47 | +21 | + 5 | − 7 | −39 | −55 | −73 | −85 | −68 | −38 |
| 1869 | −21 | − 3 | + 3 | +13 | + 9 | + 3 | − 6 | −14 | − 1 | + 8 | +14 | +19 | +23 | +31 | +47 | +47 | +34 | +11 | −29 | −41 | −44 | −50 | −29 | −24 |
| 1870 | − 0 | + 2 | −11 | −26 | −50 | −84 | −98 | −99 | −73 | −43 | −17 | + 4 | + 4 | +19 | +36 | +49 | +71 | +84 | +85 | +73 | +49 | +22 | + 5 | − 2 |
| 1871 | +79 | +101 | +104 | +99 | +67 | +50 | +39 | +16 | − 4 | −41 | −86 | −111 | −128 | −114 | −82 | −51 | −23 | − 1 | + 7 | + 7 | + 7 | + 5 | +19 | +41 |
| 1872 | +70 | +74 | +64 | +35 | − 6 | −33 | −58 | −65 | −69 | −61 | −39 | −28 | − 4 | +16 | +33 | +43 | +34 | +15 | − 9 | −24 | −25 | −12 | +10 | +39 |
| 1873 | +26 | +22 | +21 | − 4 | −34 | −55 | −65 | −68 | −61 | −61 | −68 | −65 | −59 | −45 | −12 | +25 | +50 | +68 | +77 | +77 | +79 | +71 | +47 | +34 |
| | −41 | +103 | +133 | +106 | −29 | −141 | −193 | −220 | −193 | −167 | −120 | −50 | +12 | +114 | +271 | +340 | +352 | +312 | +173 | +40 | −95 | −225 | −261 | −221 |
| | | | | | | | | | | | | | | | | | | | | | | | (sum=3912) | |

21. When our method is applied to Table VI we obtain the following results:—

Table VII.—Exhibiting the results of the above method applied to the numbers of Table VI.

| Divisions from normal. | | Exact period in days. | | Magnitude of inequality. |
|---|---|---|---|---|
| −7·0 | ...... | 23·5400 | ...... | 2100 |
| −6·5 | ...... | 23·5729 | ...... | 3093 |
| −6·0 | ...... | 23·6057 | ...... | 4700 |
| −5·5 | ...... | 23·6386 | ...... | 4025 |
| −5·0 | ...... | 23·6715 | ...... | 1386 |
| −4·5 | ...... | 23·7043 | ...... | 1887 |
| −4·0 | ...... | 23·7372 | ...... | 3910 |
| −3·5 | ...... | 23·7700 | ...... | 3915 |
| −3·0 | ...... | 23·8029 | ...... | 3140 |
| −2·5 | ...... | 23·8357 | ...... | 2771 |
| −2·0 | ...... | 23·8686 | ...... | 4234 |
| −1·5 | ...... | 23·9014 | ...... | 5921 |
| −1·0 | ...... | 23·9343 | ...... | 5518 |
| −0·5 | ...... | 23·9671 | ...... | 2374 |
| Normal | ...... | 24·0000 | ...... | 3912 |
| +0·5 | ...... | 24·0329 | ...... | 5135 |
| +1·0 | ...... | 24·0657 | ...... | 4516 |
| +1·5 | ...... | 24·0986 | ...... | 2157 |
| +2·0 | ...... | 24·1314 | ...... | 2378 |
| +2·5 | ...... | 24·1643 | ...... | 3795 |
| +3·0 | ...... | 24·1971 | ...... | 3926 |
| +3·5 | ...... | 24·2300 | ...... | 3043 |
| +4·0 | ...... | 24·2628 | ...... | 2520 |
| +4·5 | ...... | 24·2957 | ...... | 3004 |
| +5·0 | ...... | 24·3285 | ...... | 4302 |
| +5·5 | ...... | 24·3614 | ...... | 4761 |
| +6·0 | ...... | 24·3943 | ...... | 5824 |

The results of the above table are graphically represented by a curve in Diagram I, fig. III. It will be seen that there is a great, though not absolutely exact, coincidence between the periods recorded in this curve, and those of the same diagram which exhibit the declination results.

The form of the temperature inequality for 24 days, *see* Table VI, is exhibited in Diagram II, fig. IV. It will be seen that there is a considerable likeness in form between the various curves of this diagram; but it must not be forgotten that these curves are in all probability affected by the influence of adjacent inequalities.

22. We have thus given our method a preliminary trial, and the result is we think decidedly hopeful. We are inclined to believe that when more completely worked out it may not only reduce to certainty the existence of periods of short range in terrestrial magnetism and meteorology, but also give determinations of the exact lengths of these periods, and of the forms of the inequalities.

We would remark, in conclusion, that a glance at the yearly inequalities exhibited in Tables I and III, will show us that these are more marked and more regular in those years which correspond to maximum sun-spots, than they are in years of minimum sun-spots.

## Note on the above Paper. By Professor G. G. STOKES.

As the search for periodic inequalities of unknown period must always be more or less laborious, it seems desirable to point out another mode in which the search might be conducted, and which seems to offer great facilities for the object, assuming the possession of the required instrument.

It seems to me that the harmonic analyser of Sir William Thomson is singularly well adapted to this purpose, which, as I have ascertained from him, was one of the applications of his machine that he has had in view.

If $f(t)$ be any function of the time $t$ given by observation, and $2\pi \div n$ a period $p$ assumed at pleasure, then by plotting if necessary the function on a scale adapted to the paper cylinder of the machine, we shall get, by a simple mechanical process, the values of the integrals

$$\int f(t) \sin nt\,dt, \qquad \int f(t) \cos nt\,dt,$$

between any limits. We may take the inferior limit for the origin of the time, and then by reading off the cylinders of the machine for as many values of the superior limit $t$ as we please, we shall get the corresponding values, as many as we like, of the integrals.

Suppose, now, that $f(t)$ contains a small term of the form

$$c \sin (n't+\alpha),$$

where $n'$ is not much different from $n$, so that the period tried approaches closely to the period $p'$ of this inequality. The corresponding part of the integrals will be—

$$\frac{c}{2(n'-n)} \sin \{(n'-n)t+\alpha\} - \frac{c}{2(n'+n)} \sin \{(n'+n)t+\alpha\},$$

and $$-\frac{c}{2(n'-n)} \cos \{(n'-n)t+\alpha\} - \frac{c}{2(n'+n)} \cos \{(n'+n)t+\alpha\},$$

taken between the proper limits. The terms divided by $n'+n$ having but a small coefficient in the numerator, and having a denominator which is not small, may be left to take their chance with casual fluctuations; but the terms divided by $n'-n$ rise into importance from the smallness of the denominator, and express an inequality in the integrals of comparatively large amount and long period.

We may therefore confine our attention to the terms

$$\frac{c}{2(n'-n)}\sin[(n'-n)t+\alpha], \qquad \frac{c}{2(n'-n)}\cos[(n'-n)t+\alpha],$$

occurring in the indefinite sine integral and cosine integral respectively.

If, therefore, the values of these two integrals, obtained from the two cylinders respectively, be plotted, we shall obtain a periodic fluctuation of a period more or less long as we hit on a period more or less near to that of the inequality which we have supposed to exist. The zero points of the fluctuation in the sine integral will correspond to the maxima and minima in that of the cosine integral, and *vice versâ*. The reciprocal of the period of the fluctuation will give the difference of the reciprocals of $p$ and $p'$, and thus $p'$ will be known from $p$ provided only we know which of the two, $p$, $p'$, is the greater. This will be shown by comparing the phases of the fluctuations of the two integrals. If $n'>n$, the fluctuation of the cosine integral will be a quarter of a period behind, if $n'<n$ before, that of the sine integral.

If $f(t)$ be subject to known periodic inequalities with approximately known coefficients, the integrals should be cleared of the terms thence arising, if of sufficient moment, by using their analytical expressions, and the residues only plotted.

Of course $f(t)$ is not necessarily a function of the time given by *direct* observation; it might be a function deduced from one so obtained. For example, $f(t)$ might be the coefficient of the principal term in the daily fluctuation of the element when each day's record is separately subjected to harmonic reduction.

*May* 29, 1879.

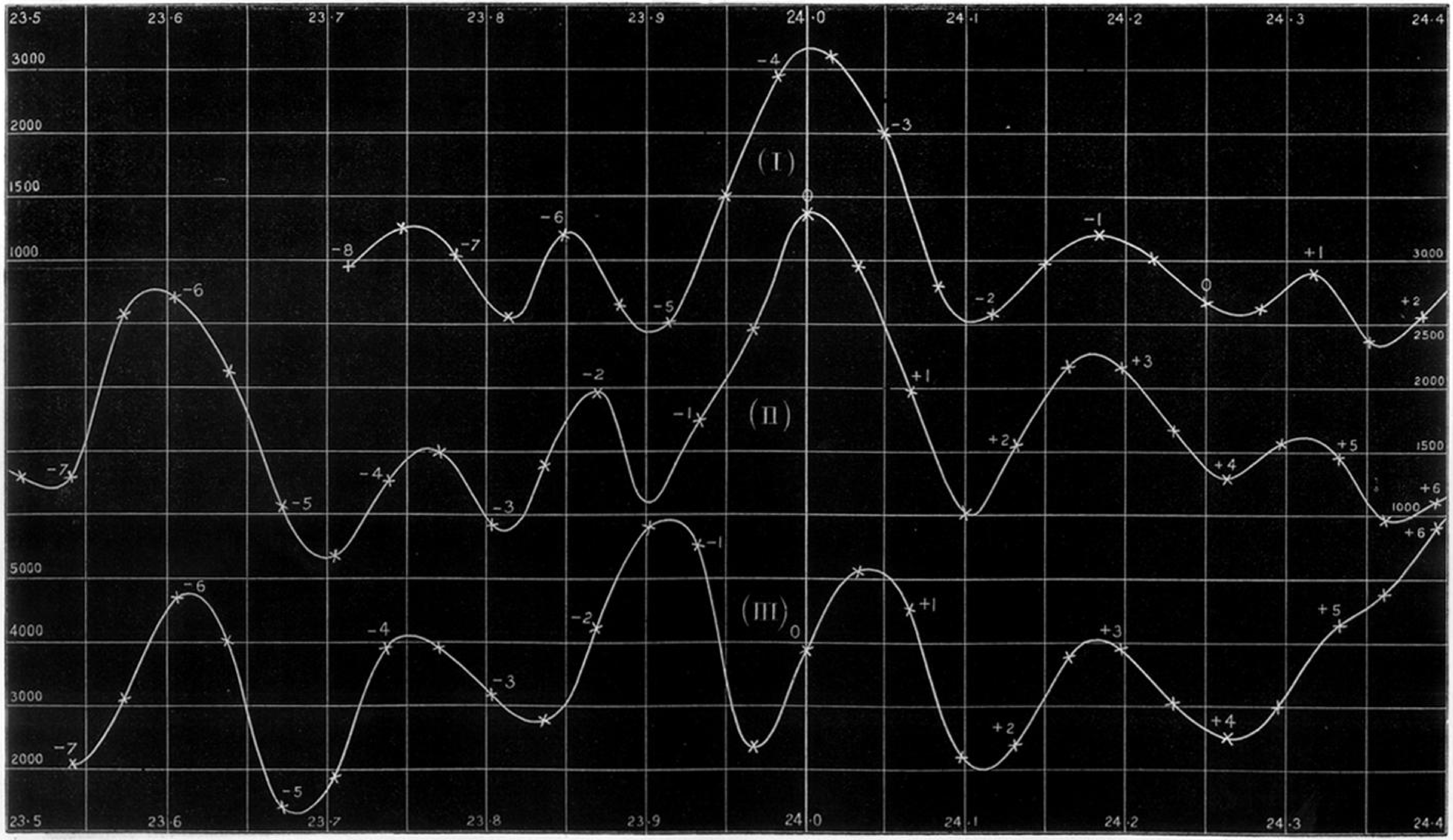
## IV. Researches on Explosives. No. II. (Fired Gunpowder.).*

By Captain NOBLE, late R.A., F.R.S., F.R.A.S., F.C.S., and F. A. ABEL, C.B., F.R.S., V.P.C.S. Received May 21, 1879.

(Abstract.)

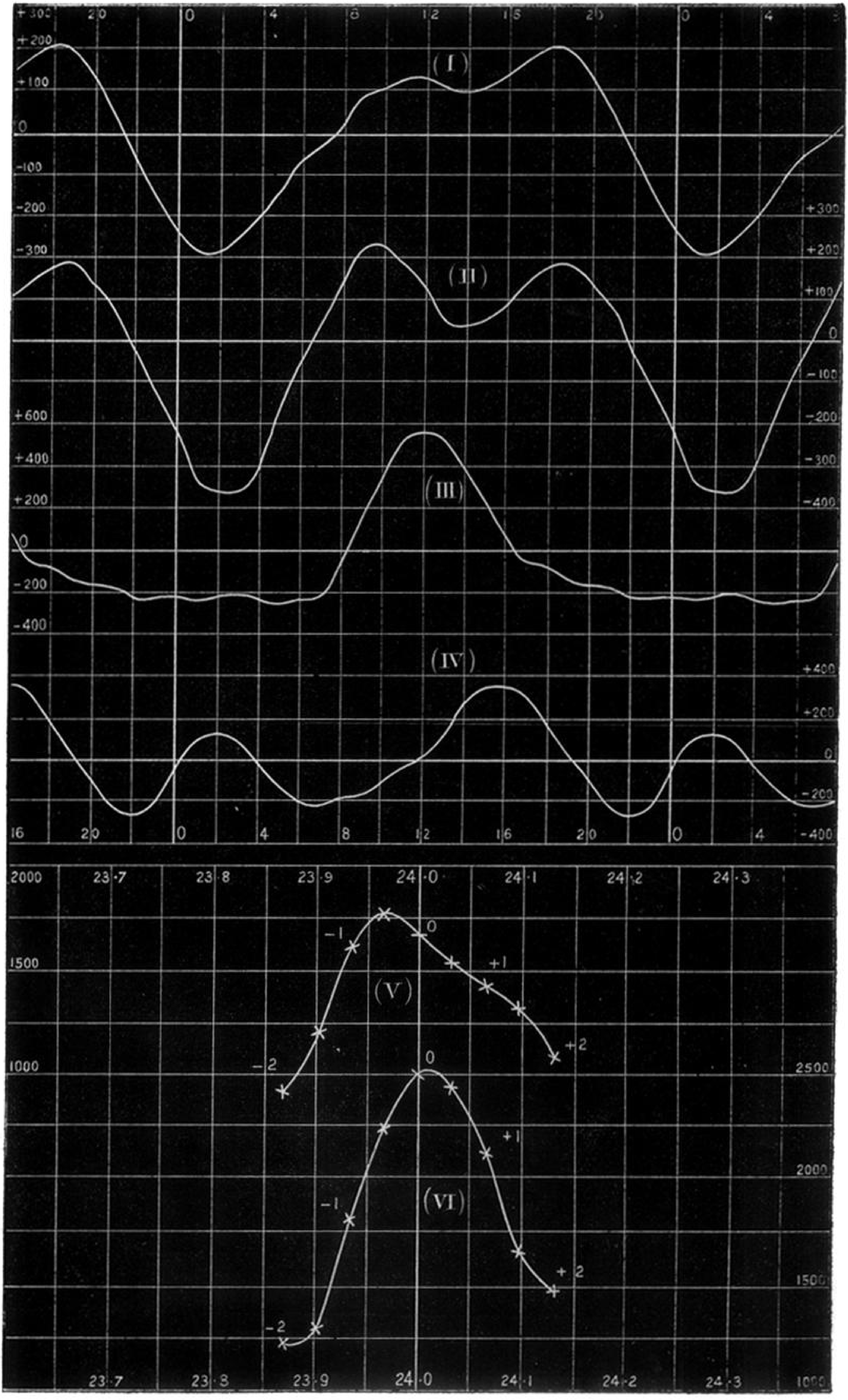
The authors preface this memoir by two tables: one of these gives the results of some analyses of products of explosion of the three service powders of Waltham Abbey manufacture, pebble, R.L.G. and F.G., which are required to complete the series of results obtained by

Diagram I.



23·5
23·6
23·7
23·8
23·9
24·0
24·1
24·2
24·3
24·4
3000
2000
1500
1000
5000
4000
3000
2000
3000
2500
2000
1500
1000
(I)
(II)
(III)

Diagram II.


(I)
(II)
(III)
(IV)
(V)
(VI)
+300
+200
+100
0
-100
-200
-300
+600
+400
-400
2000
1500
1000
2500
23·7
23·8
23·9
24·0
24·1
24·2
24·3
-2
-1
+1
+2