

$$= -\frac{v^2}{A_2} \frac{d}{dr} \left( \frac{A_1 A_2 - B^3}{A_2} \right),$$

where  $v$  is the "velocity of mean square" of  $(A_1)$ .

When the distance of the spheres is so large that we may neglect twelfth and higher inverse powers of the distance this attraction is

$$\frac{18v^2}{2\rho+1} \cdot \left( \frac{a}{r} \right) \frac{71}{a} \left\{ \left( \frac{r^2}{r^2-b^2} \right)^4 - \frac{3}{2\rho+1} \right\},$$

$a$ ,  $b$ , being the radii of the vibrating and free sphere respectively. From this it may be shown that if the density of the sphere is greater than the fluid it is attracted, whilst if less it will be attracted or repelled, according as its distance is less or greater than a certain critical

distance, which is given by  $r = \frac{b}{\sqrt[4]{1 - \sqrt[4]{\frac{2\rho+1}{3}}}}$ . For instance,

if  $\rho = .9$ , this distance  $= 7.648$ , the radius of the free sphere. These formulæ are obtained on the supposition that the spheres are not so close that  $\frac{dA_1}{dr}$  are too great to be neglected, for at contact  $\frac{dA_1}{dr}$  are infinite.

When the sphere is held fast, the mean force required to do so is  $-v^2 \frac{dA_1}{dr}$ , which, when we neglect twelfth powers of  $\frac{1}{r}$ ,

$= \frac{v^2}{g} \frac{9a^3 b^3 r}{(r^2 - b^2)^4} \times \text{weight of fluid displaced by the vibrating sphere; for example, for two oxygen atoms at } 0^\circ \text{ C., at a distance four times their radius, the force is about } 78 \times 10^6 \text{ weight of fluid displaced by one atom, thus while the force decreases indefinitely, the effective force increases indefinitely.}$

For two spheres ( $a=b=\frac{1}{4}$  inch  $r=4a$ ) vibrating through a distance  $\frac{1}{10}$ th of an inch 256 times a second, in water, the force is equal to the weight of 12.8 milligrams.

VI. "Microscopical Researches in High Power Definition." By G. W. ROYSTON-PIGOTT, M.A., M.D. Cantab., F.R.S. Received May 23, 1879.

(Abstract.)

In its general scope the paper is intended to deal with difficulties in microscopic research, usually found insuperable—such, for instance, as the invisibility of minute *closely packed* refracting spherules, existing in double rouleaux, or promiscuously aggregated, when their individual diameter varies between the 1-80,000th to the 1-200,000th of an inch.

These difficulties are principally created by overlapping images—due partly to residuary aberrations both spherical and chromatic—partly to the effects of diffraction, caused by brilliant illuminations of spurious disks of light—partly to the constant development of *Eidola* or false images, which vary the loci of their development according to the nature of underlying structures and according to the object-glasses being over or under corrected; and they are partly, and indeed very considerably, created by the use of excessively large angular apertures.

The paper discusses also, the relative effects on visibility, of large and small angular apertures in objectives.

It shows that the black margins or black marginal annuli of refracting spherules, constantly displayed by low aperture glasses, are attenuated gradually to invisibility as the glasses employed are endowed with the largest apertures. That the black margins also of cylinders, tubules, or semi-tubules suffer similar obliterations. And that, in consequence, innumerable minute details are concealed or destroyed till the aperture is sufficiently reduced.

That minute refracting bodies obey the laws of their refrangibilities and display beautiful phenomena, discoverable by transcendent powers of definition; but totally unseen by inferior compensations. And that, in consequence, the so-called achromatism of modern glasses is an illusory approximation to correct vision.

Examples are given of molecular structures, varying in form, translucency, and refrangibility, in which natural pencils are caught and displayed in the order in which, as in a rain drop, iridescent rays are emitted by the decomposed light.

Several examples are also introduced, in which a high order of lenticular correction beautifully discovers structure hidden, according to Dr. Carpenter, F.R.S., from the great bulk of observers.

As the paper deals so often with magnitudes very much less than the 1-100,000th of an inch, a method is introduced of readily estimating roughly such magnitudes between the 1-80,000th and the 1-500,000th of an inch, by means of a micrometer gauge. The writer has been emboldened to grapple with these difficult minutiae, in consequence of the sharp and clear definition he has attained of spider lines miniaturized down to the fourteenth part of a hundred-thousandth of an inch. The eye, accustomed to contemplate this subtlety of form, readily appreciates the one-fourth or sixth of this size, *i.e.*, 1-400,000th or 1-600,000th.

The writer has also ventured to bring before the notice of the Royal Society, a new test for the microscope, displaying bright lines of uniform thickness less than the 1-100,000th, and sharp black lines of much less tenuity than those given by Nobert's celebrated lines ruled on glass, and incomparably more easy of illustration.

The employment of various fluids for immersion lenses is carefully considered; and the singular property of castor oil discovered by the writer is referred to.

As the Society honoured the writer by inserting in their "Transactions" a paper on "A Searcher for Aplanatic Images," he now introduces a new form which offers some advantages: by its extended traverse, by its simplicity and economy of light with increase of magnifying power.

Finally, some examples are given of producing transcendent definition in cases found hopeless by a numerous body of observers; as the papers written in "The Monthly Microscopical Journal" during the last ten years abundantly demonstrate. The means also of its attainment are minutely described.

VII. "Note on 'Spectroscopic Papers.'" By G. D. LIVEING, M.A., Professor of Chemistry, and J. DEWAR, M.A., F.R.S., Jacksonian Professor, University of Cambridge. Received May 29, 1879.

In a recent communication to the Royal Society, Mr. Lockyer has criticised our statement of Young's wave-length identifications of certain chromospheric lines. As to the wave-length, we have throughout our table omitted all figures after the decimal point merely for the sake of not cumbering the table. The numbers, Young tells us, are not his own, but taken from Ångström's catalogue. Moreover, as to Young's identifications with metallic lines, he states expressly that they were taken from the maps of Kirchhoff, Ångström, and Thalén, and Watts's "Index of Spectra." But our object was not to criticise Young's work, but only to use it for the purpose of comparing the behaviour of certain metals on the earth and in the sun, and the conditions under which certain lines appear, or do not appear, or are reversed.

We should perhaps have made our meaning clearer if we had given another column with the wave-lengths of the metallic lines referred to side by side with Young's numbers for the chromospheric lines. We mentioned, in relation to aluminium, the two lines with wave-lengths 6245·4 and 6237·3 seen by Young, not because we thought their identity with the aluminium lines proved, but because they are the only two lines in Young's table which are at all close to aluminium lines, and if they be not due to that metal, then we have the remarkable fact that aluminium in the sun gives no lines either dark or bright except the two which have been reversed on the earth. A somewhat similar remark applies to the potassium lines, only in that case Young's line has a wave-length very nearly the mean of the two