seen to be more advanced than at the part which was free from pigment. The pigment was packed in the tubes around and between the spores; but, by focussing, it could be seen that the substance of the spore was free from it. The free spores and short rods were free from pigment.

The bacteria in which it was observed showed no other peculiarities, and were of about the same calibre as the rod bacteria usually observed.

The fact is noted as affording proof that bacteria can take up minute solid particles through their walls.

V. "On Toroidal Functions." By W. M. Hicks, M.A. St. John's College, Cambridge. Communicated by J. W. L. Glaisher, F.R.S. Received February 21, 1881.

(Abstract.)

This paper contains the development of a theory for functions which satisfy Laplace's equation, and are suitable for conditions given over the surface of a circular anchor ring, and which therefore seem important in the possibility of their application to the theory of vortex rings, as well as other physical problems. From the nature of the case, it will not be easy to give an intelligent and full abstract of the results without making it unduly long, but it may be possible to give some idea of its scope and the method of development.

Curvilinear co-ordinates are employed, the orthogonal surfaces being those formed by the revolution of a system of circles through two fixed points, and the circles orthogonal to them, whilst the third system are planes through the axis of revolution. Calling these v, u, w, it is shown that any potential function can be expanded in the form—

$$\phi=\sqrt{\cosh u-\cos v}\,\Sigma\Sigma\,(\mathrm{AP}_{m\,.\,n}+\mathrm{BQ}_{m\,.\,n})\cos(nv+\alpha)\cos(mw+\beta)$$

where $P_{m.n}$, $Q_{m.n}$ are particular integrals of a certain differential equation, and which are the toroidal functions whose discussion forms the principal part of the paper. They are in fact the same as spherical harmonics of the first and second kinds of imaginary argument, and of orders of the form $(2p+1)/2$. It is shown how the P can be expressed in terms of the first and second complete elliptic integrals F, E; and the Q in terms of the complementary integrals F′, E′. Several interesting results are arrived at, amongst others on relations between the P and Q functions, *e.g.*, between the zonal functions $(m-v)$ $P_{n+1}Q_n-P_nQ_{n+1}=\frac{\pi}{2}/(2n+1)$. The P_n serve for expansions in the space outside a tore, whilst the Q_n serve for space within.

Section ii is devoted to the consideration of zonal functions, *i.e.*, functions suitable when the conditions are symmetrical about an axis. Section iii deals with the general case, whilst in the last section illustrations of the use of the functions are given by application to several problems, such as the potentials of tores under the action of an electrified ring, or point of electricity, and the velocity potential when a tore moves parallel to itself in a fluid. Among the results obtained, which may be mentioned here, is the electrical capacity of an anchor ring. When the section of the ring is not very large compared with the central opening, a very close approximation is given by the formula—

$$q=\sqrt{R^2-r^2}\left\{\frac{F'}{F}+\frac{F'}{E}-\frac{E'}{E}\right\}$$

where

$$k^2=2\frac{\sqrt{R^2-r^2}}{R+\sqrt{R^2-r^2}},$$

R, r being the radii of the circular axis, and generating circle of the ring respectively.

The approximation is so close that the formula only makes an error of about ·3 per cent. when r is so large as $\frac{1}{3}$ R.

If a tangent be drawn from the centre to the anchor ring, and a sphere be described with this for radius, the capacity of the tore measured in terms of that of the sphere is

$$\frac{F'}{F}+\frac{F'}{E}-\frac{E'}{E},$$

when $R=10r$ this is ·36049,

when $R=5r$ this is ·43405.

VI. "Microscopical Researches in High Power Definition. Preliminary Note on the Beaded Villi of Lepidoptera-Scales as seen with a Power of 3,000 Diameters." By Dr. Royston-Pigott, F.R.S. Received January 15, 1881.

In carrying out the investigation of the molecular structure of insect scales, under the finest attainable amplification, the discovery has been made that the striated surfaces of these scales, though appearing approximately beaded, are really covered with villi, chenille or velvet pile, terminating in a spherule.

The recognised object of these striæ regarded as corrugations is to give strength to a most delicate tissue, which are again supported by *cross striæ.* Upon these transverse striæ I have discovered villi erected