On examining the equations resulting from the eliminations of the variables, it turns out that they can be rationally transformed into expressions such as $UP'-U'P=0$, where U and U′ are quadrics, and P and P′ linear functions of the variables remaining after the eliminations. The forty-eight co-ordinates then consist of the twenty-four coefficients of the four functions of the form U (say the U-co-ordinates), together with the twenty-four coefficients of the functions of the form U′ (say the U′-co-ordinates), arising from the four eliminations respectively: viz., $4\times6+4\times6=48$. And it will be found that the coefficients of the forms P, P′, are already comprised among those of U, U′; so that they do not add to the previous total of forty-eight.

The number of identical relations established in the present paper is thirty-four. But it will be observed that the equations $UP'-U'P=0$ are lineo-linear in the U-co-ordinates and in the U′-co-ordinates; and as we are concerned with the ratios only of the coefficients, and not with their absolute values, we are, in fact, concerned only with the ratios of the U-co-ordinates *inter se*, and of the U′-co-ordinates *inter se*, and not with their absolute values. Hence the number of independent co-ordinates will be reduced to $48-34-2=12$, as it should be.

The thirty-four identical relations arrange themselves firstly in two sets: one set belonging wholly or principally to the U-co-ordinates, and the other set wholly or principally to the U′-co-ordinates. In each set there are four groups: one of four, one of eight, one single, and again one of four equations; seventeen in all. In the course of the paper, the two groups of eight are obtained in two forms: first, by a purely algebraical method in a rational form; and secondly by a method partly geometrical and partly algebraical, in an irrational form.

## II. "How do the Colour-blind See the different Colours? Introductory Remarks." By Frithiof Holmgren, Professor of Physiology, University, Upsala. Communicated by W. Pole, Mus. Doc., F.R.S. Received December 6, 1880.

That the colour-blind do not see colours in the same way with normal-eyed persons we may know from the fact that they confuse rays of objective light which, to the normal eye, give quite different impressions.

When, for instance, a red-blind person is confused in his perception of those different sorts of light that to the normal eye appear as red and green, we may conclude that he sees them both as one and the same colour, but not what that colour is, as to its quality—whether it is one of those just mentioned or a third—and whether, on the latter supposition, that colour exists in the colour-system of normal-eyed

persons, say as yellow, or, on the other hand, is a colour of which we in general have not the slightest conception.

If a theory of colour-blindness has for its object to explain the different links in the chain of causes and effects, of which the first is the objective light, and the last the subjective perception thereof, we must first know this last.

From the points of view of the different theories, different opinions have been entertained on this subject, but practical proofs have hitherto been wanting for them all, and, what is worse, the hope of ever gaining a solid basis for such proofs has been given up, for the reason that they must be found in the subjective perception of another man.

Since it is impossible for one person to make himself master of that conception in another, we cannot even objectively prove that all normal-eyed persons see the different colours in the same way. Still it may be assumed as an axiom that at least the quality of the different principal colours is the same for all persons who show the same conception of colours in general. Else we should not talk of colour, as all spiritual communication would be impossible between persons whose sensuous impressions were quite at variance.

If that axiom is accepted, and we take for granted that all normal-eyed persons see colours in the same way, as do also the colour-blind of the different sorts in their own particular manner, I will endeavour to show that the question which stands at the head of this article is not only not impossible to answer, but that I can already give that answer, furnished with scientific and objectively binding proofs.

There is apparently but one way to establish such an answer. A normal-eyed person must, while retaining his own colour-sense, be put in position to see with a colour-blind eye, and compare the impressions on that with those on his own normal eye. He cannot use either the one or the other unless they stand in a living organic communication with his own brain. He must, so to speak, grow into the colour-blind and his chromatic sense without losing the use of his own.

Such a phenomenon, which here is a *conditio sine quâ non*, it is, of course, impossible to procure in an artificial way; but still I will show that an organic combination of a normal and a colour-blind eye with the same brain is not impossible to be realised in a perfectly natural manner.

Congenital colour-blindness is, as is known, in most cases inherited, and inherited according to definite laws. (*Vide* Frithiof Holmgren: "La Cécité des Couleurs en Suède," p. XIII.) One of these laws is, that all the children of the same parents are not afflicted with this defect; some are free. Nor is the colour-blindness of the same degree in those affected. Accordingly it would seem not to be contrary to the laws of inheritance that a person should be born with one normal eye, while the

other was more or less colour-blind, or that while he was colour-blind with one eye, the other should be imperfectly so, perhaps in so slight a manner that it might be called nearly normal.

Besides the congenital defect, there exists an acquired one, which does not necessarily affect both eyes at the same time.

A combination of a normal and an abnormal eye with the same brain is, as we see, not impossible. A few such cases have been noticed in the literature of the subject as curious exceptions, one recently by Professor Becker, in Heidelberg, which is described by him. Such a case is what is wanted: a colour-blind person who can make his conception of the different colours subjectively clear for a normal-eyed person, as well as this latter can make the same conception objectively clear to all other people with normal sight. We have thus a bridge between the subjective perception of the colour-blind person and objective scientific research.

Led by this idea, I have during the last three years looked for cases of this one-sided colour-blindness in combination with my statistical researches through Sweden. The difficulties that formerly were so enormous, and which have been removed by my method with skeins of Berlin worsted, were at this trial still greater, from different causes. In Sweden I had previously only found one case, and this (found in the summer of 1879) became unhappily useless through an accident.

Since I succeeded in finding a practical way of lessening those difficulties, I have, within a comparatively short time (June to October, 1880), been fortunate enough to examine two such cases, one of one-sided violet-blindness, and the other (for which I have to thank Professor Hippel, of Giessen), a case of one-sided red-blindness.

Experience will probably show that such cases are not so rare as we have hitherto thought, and we have every reason to hope for a speedy and perfect solution of the problem in this way; but the results of these two cases have been so remarkable that I will here give a short description of them.

The plan, principle, and result were as follows:—

First the diagnosis of both eyes was carefully made. In both cases there was found a perfect, typical, partial colour-blindness on one of the eyes (the violet-blind on the left, and the red-blind on the right eye); the other eye had a weak colour-sense, but still so nearly normal that the principal colours were ascertained with perfect ease. A slight hesitation was only shown in distinguishing the lightest and darkest shades of those colours. Both the cases were thus perfectly fit for the purpose.

The principle of the trial was exactly this:—To let the normal eye control the perception of the abnormal one, and bring the result into a form that was perfectly plain to other normal-sighted persons.

A one-sided colour-blind person has through his normal eye a per-

fectly clear conception of the normal-eyed people's different colours, and can tell his conception, by the aid of his other eye, for other normal-sighted persons. His definitions are thus—in opposition to persons colour-blind on both eyes—perfectly reliable.

As words are always less reliable than actions (*vide* Frithiof Holmgren: "De la Cécité des Couleurs dans ses Rapports avec les Chemins de Fer et la Marine," p. 116), and a description in this case is always inferior to a shown colour, I have in every instance let the person in question point out an objective colour with his normal eye for every one of his conceptions with his abnormal one.

Indirectly we find in this way which qualities of perception are wanting in the abnormal eye in comparison with the normal one. The same result is directly arrived at by letting the colour-blind eye control the subjective perception of the normal.

It is my intention to explain the application of this principle in my more detailed work on this subject, as well as the particulars of the result.

I will now only give the chief points of this result, which perhaps is done in the shortest and easiest way by giving the details of the colour-blind person's subjective spectrum. If we take the objective solar spectrum for a starting point, and choose our own definition of the different colours from the subjective spectrum of the normal eye, we come to the following results:—

As we have long supposed, for good reasons, a colour-blind person sees only two colours in the spectrum. These are his two subjective principal colours.

The principal colours in the spectrum of a *violet-blind* person are, as to their fundamental tone, *red* and *green*. Towards the red end his spectrum has quite the same extension as that of a normal-eyed person, and is thus, in comparison with the latter, "unshortened." Reckoned from the red end, his first fundamental colour stretches over that part of the spectrum which is generally seen as *red*, *orange*, and *yellow*. First in the yellowish-green (a little on the other side of Fraunhofer's line D) he sees a narrow, uncoloured ("paper-white") belt, from which his other colour, *green*, commences, and is continued with at first more and more saturated, and afterwards darker and darker shades, over the place where we see *green*, *greenish-blue*, *cyan-blue*, and *indigo* to the commencement of the violet, where his spectrum absolutely ends with a sharp limit (about Fraunhofer's line G). His spectrum is thus at this end considerably "shortened." The fact that violet-blind persons confuse the pigment colours (such as *green* and blue, *purple* and red, orange and yellow, *violet* and yellowish-green and grey) is thus explained of itself.

All this is in the main consistent with the Young-Helmholtz theory. Respecting the tone of the violet-blind person's subjective funda-

mental colours, it may be said that his red is not quite identical with the common spectral red of the normal-eyed (something like cinnabar), but rather a clearer red, having a shade of carmine, about the same as the red towards the end of the subjective spectrum of the normal-eyed. His other fundamental colour, *green*, is also a clear green that for the normal eye has a shade of blue in it.

The two principal colours in the spectrum for the *red-blind* are as to their fundamental tone *yellow* and *blue*. This yellow commences a little later, reckoned from the end, than the red of the normal-eyed (about Fraunhofer's line C), and stretches over the rest of the *red*, *orange*, *yellow*, *yellowish-green*, and ends in the *blue-green* (between Fraunhofer's lines *b* and F, nearer to the latter), where a narrow, neutral, colourless belt forms the limit against the other principal colour, *blue*, which stretches through the remaining part of the spectrum, corresponding with our *cyan-blue*, *indigo*, and *violet*. At this end there is no "shortening." The red-blind person's confusing of pigment-colours (*green* and yellow, orange and red, *purple* and blue and violet *red* and blue-green and grey) is equally well explained by this.

All this, as we see, is objectively taken in perfect accordance with the Young-Helmholtz theory. Regarded from a subjective point of view, we should perhaps have expected green instead of yellow as one of the fundamental colours. But that yellow, and not green, is that colour (as I have already for some time supposed, *vide* Upsala "Läkareförenings Förhandlingar," vol. vii, 1871, p. 119, and "Centralblatt f. d. med. Wissenschaften," 1872, p. 826) does not shake the basis of that theory, as is shown by Fick ("Zur Theorie der Farbenblindheit," 1873) and by myself ("Om Färgblindhetens Theori," 1878). Besides, the tone of the red-blind person's first fundamental colour is not perfectly golden-yellow, but seems for the normal eye to have a shade of greenish-yellow, perhaps best defined as citron-yellow in the lighter, and as olive-green in the darker, shades. His other fundamental colour does not seem to be purely cyan-blue or indigo, but is rather a blue with a perceptible shade of violet. It might be called indigo-violet.

Perfect clearness in the theory will not perhaps be gained until we shall have had opportunity of studying more cases of different kinds and degrees, and especially a case of typical perfect *green-blindness*.

Still the path is opened, and a more definite starting-point has been found for the treatment of the theoretical problem of colour-blindness, of which it is my intention to speak more explicitly in my larger work on the same subject.