escape into the body cavity and make their exit by several pores placed close together, and symmetrically, on each side in the pallial groove; oviducts apparently being absent. I have not any specimens of the species he mentions as possessing this peculiarity (*e.g.*, *Chiton marmoreus* and *ruber*), so have not been able to test his observations by means of sections.

I hope to be able to give a fuller account of these and other points in the anatomy of Chiton at some future period, for the preparation of which it will be necessary to obtain some fresh specimens.

VI. "The Action of Cutting Tools." By A. MALLOCK. Communicated by Lord RAYLEIGH, F.R.S. Received November 4, 1881.

The action of cutting tools has not often been treated from a theoretical point of view; in fact I only know of two papers on the subject, one by Professor Willis and the other by Mr. Babbage. Of these Professor Willis's paper is purely geometrical, showing what angles the edges of tools may make with one another if the cutting angles are to be such as experience shows to answer best. Mr. Babbage, on the other hand, does not enter at all on the question of the shape of the tool, but by making certain assumptions as to the relation between the dimension of the shaving removed by a tool and the work required to remove it, he deduces some results showing how to remove a given amount of material most economically. His conclusions cannot be considered correct, nor do they agree with experience (see Note 1). I do not attempt in the following paper to give any dynamical investigation of the action of tools, in fact it would be almost impossible to do so without a more extended knowledge of the laws which govern the strains in bodies subjected to large forces, but merely to classify the various actions which observation shows to be caused by the progress of the tool, and to quantify approximately the work expended in each. For this purpose, shavings from a great variety of substances were examined both in the course of their formation (by a microscope attached to the toolholder) and after they were removed.

Among the substances examined may be mentioned four or five samples of wrought iron, and as many of steel, cast iron, gun metal, brass, copper, lead, zinc, hard paraffin, soap, and clay.

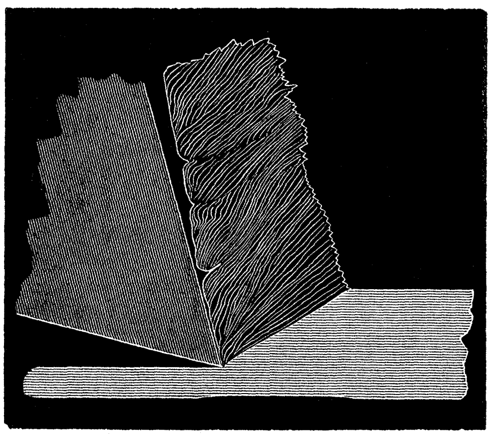
This last-mentioned substance was found extremely useful in examining the formation of the shavings, for by altering the amount of water it contained its behaviour under the tool could be made to

resemble almost any of the others, and at the same time the forces required to take large cuts were not greater than could be conveniently applied by hand.

Sections were made of many of the metallic shavings, and the polished surfaces of these when washed with dilute nitric acid showed their internal structure very well.

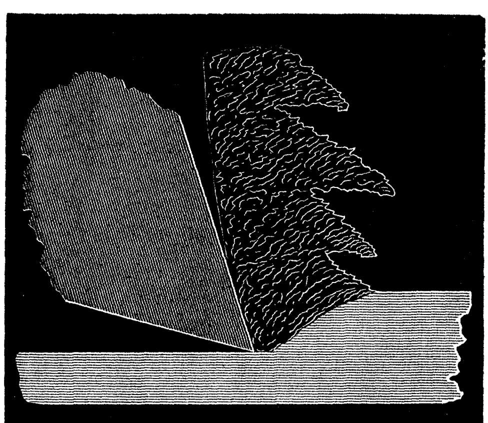
Figs. 1 to 8 show some of these sections enlarged.

FIG. 1.



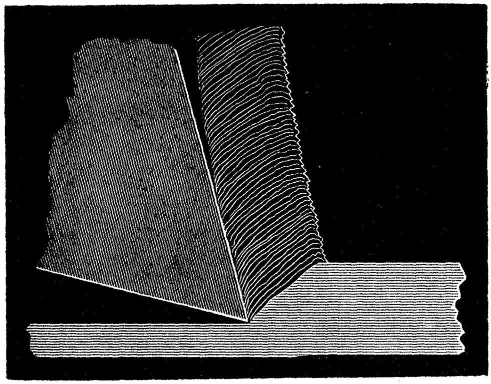
Shaving of wrought iron (armour plate). Actual thickness ·25 inch.

FIG. 2.



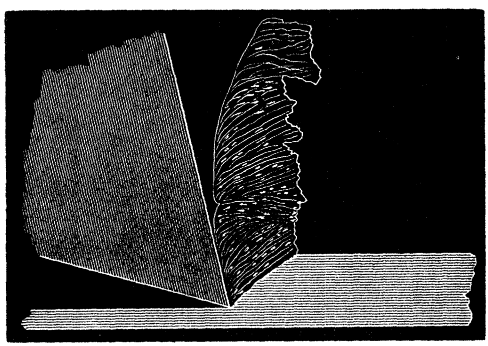
Shaving of cast iron. Actual thicknes ·1 inch.

Fig. 3.



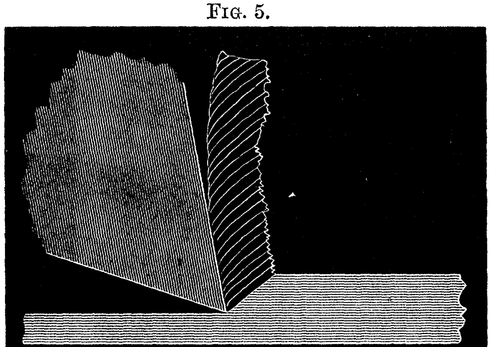
Shaving of hard steel (Whitworth). Actual thickness ·15 inch.

Fig. 4.



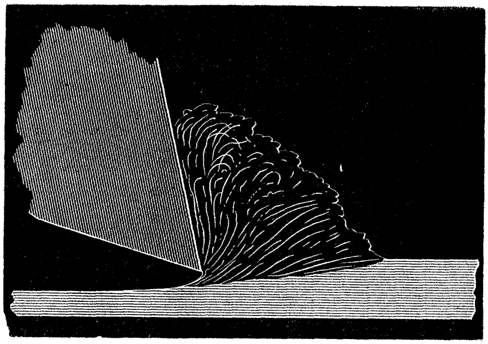
Shaving of gun metal. Actual thickness ·08 inch.

Fig. 5.



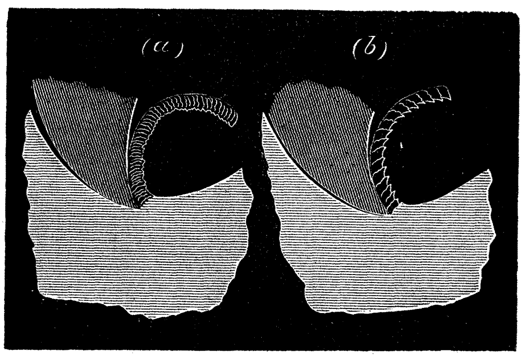
Shaving of brass. Actual thickness ·08 inch.

FIG. 6.



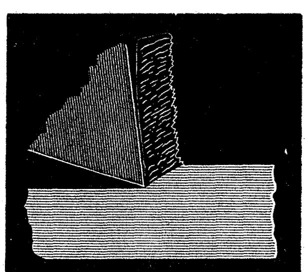
Shaving of copper (unlubricated).

FIG. 7.



(*a*.) Borings, steel. Actual thickness ·005 inch.
(*b*.) „ brass. „ „ ·01 „

FIG. 8.



Shaving of copper (lubricated with soap and water).

It will be seen that there is little difference in any of these, though the materials are of all degrees of hardness, and vary in thickness from three-eighths of an inch, the thickest iron shaving examined, to ·003.

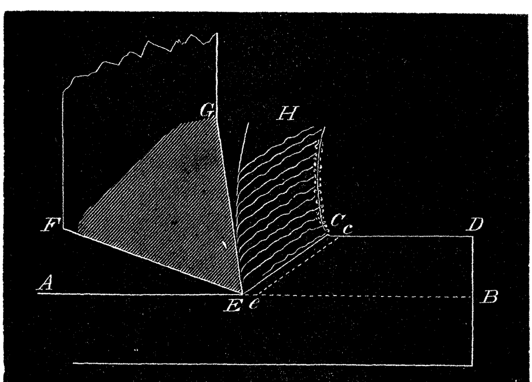
Indeed the action at the edge of the tool seems identical in all cases, such differences as there are being due to the action of the face of the tool on the shaving, while the latter is being pushed out of the way; and this action depends on some of the physical constants of the substance operated on, chiefly its coefficients of friction on the metal of the tool and on itself, but in part also on its ductility, and in some cases, as in lead, on the property which freshly formed surfaces have of reuniting under pressure.

The tools do not act, properly speaking, by cutting but by shearing, and the shaving removed by them may be accurately described as a metallic slate.

This remark does not apply to acute-edged tools, such as razors and penknives.

The difference between cutting and shearing may be defined thus: Conceive the substance to be cut to be divided into an infinite number of cubic elements by parallel planes at right angles to one another; if in a portion of this removed by a tool the elements remain cubes, the removal has been effected by pure cutting. If, however, they are only distorted but are all unaltered in volume, the removal has been effected by pure shearing; if they are both deformed and altered in volume, both cutting and shearing have been called into play.

FIG. 9.



Let ABCD be a section of the substance under the action of the tool, GEF the tool, H the shaving, CD the undisturbed surface of the substance, and AB the direction of the cut.

The advance of the tool violently distorts the material in its neighbourhood, and presently along the line *ec* the distortion becomes too great for the substance to preserve its continuity (Note 2), the lamina ECec then begins to slide on *ec*, and its base E*e* to move up the face of the tool, while the point of the tool is repeating the distortion and separation on fresh material ahead.

This in all the cases I have examined is the manner in which all tools, except those with very acute angles, act.

The curvature of shavings appears to be due to the crushing of the base of the laminæ while passing over the face of the tool, thus making them thicker at that end than at the outer surface.

The effect of the friction between the laminæ and the tool has the opposite tendency of thinning out the ends of the laminæ and preventing the curvature, so that when from want of lubrication or the nature of the material the friction becomes excessive, the shavings are nearly straight.

The shaving is generally shorter than the path of the tool, which shows that BED is less than 45°.

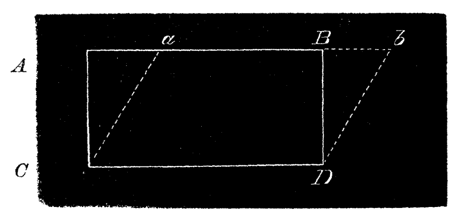
I will now attempt to take account of the forces which are brought into play by the action of the tool.

These are due to (1) elastic distortion, (2) elastic bending, (3) permanent distortion, (4) permanent bending, (5) internal friction, *i.e.*, the friction of the laminæ sliding over one another, (6) the friction of the material on the tool, and (7) if the tool is not considered as being perfectly sharp, the radius of curvature of the edge will appear in a term giving the limit to the rate of distortion in its neighbourhood.

If the tool is perfectly sharp the rate of distortion at the edge is infinite, and the material at the edge can offer no resistance to its progress unless capable of infinite distortion without rupture.

This is easily seen to be the case by the following considerations:—

FIG. 10.



Let ABCD (fig. 10) be the section of a parallelopiped of any material, and let it be distorted as shown by the dotted lines until rupture takes place. The work expended in bringing it to its distorted state depends on B*b*, *i.e.*, the distance AD must be moved before the limit of distortion is reached, and this is simply proportional to BD, the

thickness of the parallelopiped; hence when BD is nothing the work is also nothing. The rate of distortion, *i.e.*, the distortion produced by a definite motion of AD, is inversely as BD, and is therefore infinite when BD=0.

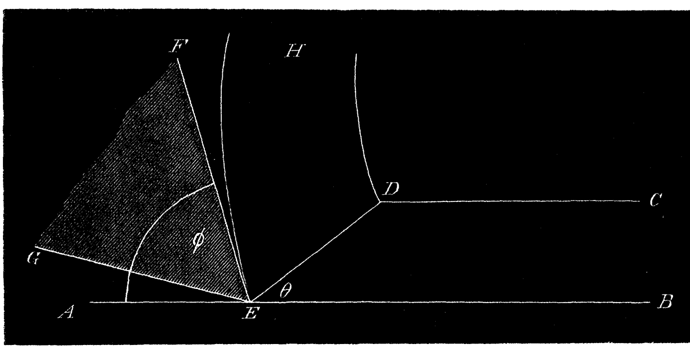
Thus, when the rate of distortion is infinite and the limit of distortion is finite, no work is required to effect the rupture.

In the case of the tool it is, of course, only actually at the edge that the rate of distortion is infinite; but if it were possible to apply suitable forces simultaneously at every point of a solid through which a surface of separation was desired to pass, that separation could be effected with no expenditure of work whatever.

The resistance due to elastic and permanent distortion is $\frac{Q}{\cos\theta}$, θ being the angle DEB (fig. 11), Q the integral of all the reaction due to distortion along the line EC at the moment sliding begins.

The work done in internal friction is EC × distance through which the lamina slides per unit travel of tool × by the pressure under which the sliding takes place × by the coefficient of friction of the material on itself.

Fig. 11.



Let t=BC, the thickness of the cut,

t'=thickness of shaving,

μ=coefficient of friction of the material on itself,

μ_1=coefficient of friction of material on the tool,

θ=DEB,

ϕ=FEA, viz., the angle which the face of the tool makes with AB,

then
$$t'=t\frac{\sin(\phi+\theta)}{\sin\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

$$\mathrm{ED}=\frac{t}{\sin\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

The sliding along ED per unit advance of tool is—

$$\cos\theta+\cot(\phi+\theta)\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

The pressure under which the sliding takes place is to the normal pressure on the face of the tool as

$$\cos(\theta+\phi)+\mu_1\sin(\theta+\phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Thus the work expended in internal friction is proportional to

$$\mu\frac{t}{\sin\theta}\{(\cos\theta+\sin\theta\cot(\phi+\theta))(\cos(\phi+\theta)+\mu_1\sin(\theta+\phi))\} \quad . \quad (5).$$

The work done in friction against the face of the tool is for the same travel proportional to

$$\mu_1 t\frac{\sin\theta}{\sin(\phi+\theta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Collecting these results, the total resistance will be made up as follows:—

(1.) Bluntness $=A\rho$ where $\rho=$radius of edge and $A=$constant.

(2.) Elastic and permanent distortion $=\frac{Q}{\cos\theta}$.

(3.) Internal friction $=\frac{\mu t}{\sin\theta}\{(\cos\theta+\sin\theta\cot\overline{\phi+\theta})$

$(\cos\overline{\phi+\theta}+\mu_1\sin\overline{\phi+\theta})\}$.

(4.) Friction against tool $=\mu_1\frac{t\sin\theta}{\sin\overline{\phi+\theta}}$.

(5.) Elastic bending $=Bt^2$.

Considering these terms in order:—

The first ought always to be small if the tool is sharp.

Q in the second term is proportional to t, and is probably a function also of ϕ and θ, but as observation shows that θ is independent of ϕ, that is to say, that for a given material any form of tool that can be employed causes sliding to begin in the same plane, it seems likely that the reaction due to distortion should not vary much with ϕ, and that for the present purpose Q may be regarded as $t\times$constant.

The internal friction vanishes when

$$\cos\overline{\phi+\theta}=-\mu_1\sin\overline{\phi+\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A),$$

that is, when the resultant force through the face of the tool is parallel to EC.

When this is the case, however, there is a large component tending to make the tool dig into the substance, so that the form indicated by A is not one which can be used in practice without certain precautions.

The same objection applies to making the fourth term a minimum by putting

$$\phi+\theta=90^\circ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B).$$

Perhaps as good a value as any for ϕ is that which makes the resultant force through the face of the tool parallel to the direction of motion. This value is given by the equation

$$\cot\phi=\mu_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C).$$

The values obtained for ϕ for this expression agree fairly with those in common use, or at least are quite compatible with them, considering the uncertainty of the values of μ_1 for high pressures. The elastic and permanent bending is small for such values of ϕ as can be used in tools for metals, the transverse strength of the shaving (which is some hundred times less than that of a piece of the substance of the same dimensions in its natural state) not permitting it to transmit large bending forces.

In true cutting tools, where ϕ is very small, nearly the whole force acting on the blade arises from friction under the pressure on the sides of the tool caused by the bending.

It is evident, therefore, that as ϕ passes from large values to small, the importance of the term in t^2 will continually increase.

All the sources of resistance above mentioned, except the first and last, which should be small, are proportional to the thickness of the shaving and, of course, also to its breadth.

Thus to remove a given volume of material will require the same expenditure of work, whether it be effected by one thick cut or several thinner ones.

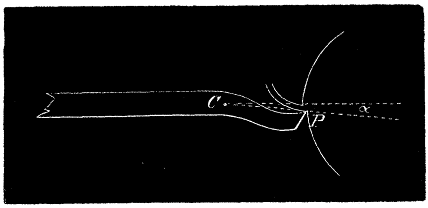
In practice, however, the constant friction of the machinery employed always make thick cuts the most economical. The construction of machines, and the character of work generally, confines within rather narrow limits the thickness of the cut which it is possible to take, for the force required must not be large enough to sensibly bend or distort the substance as a whole.

It would be a great advantage if tools could be so held or shaped that their accidental vibrations should not be sustained but extinguished by the reactions which they call into play; but to make this possible the phase of the vibration of the tool must precede that of the reaction which it causes by some time less than half the period of the tool, or, in other words, the resistance experienced by the tool must increase with its velocity. Now friction between solids being rather greater at low than at high velocities, the sliding of the shaving on the tool tends to keep up a vibration once started, and the same may be said of the forces due to the distortion of the substance, which are at a maximum just before a fresh lamina (such as $e \ldots . c$, fig. 9) begins to slide, and

then suddenly drop to a minimum. It is plain also that a tool with a tendency to dig will call into play forces of a like character.

Vibrations are in some degree neutralised, and digging entirely avoided, by so shaping the shanks of tools that the centre about which they vibrate is in advance of the normal to the direction of motion through the cutting edge.

FIG. 12.



Let ρ be the distance of the centre of flexure (s) from the cutting edge (P), α the angle which the line joining the centre of flexure and edge makes with the normal, and $\delta\alpha$ the angular distance of the tool from its mean position, the thickness of the shaving removed is $t+\rho d\alpha \tan\alpha$, and if τ be the period of the vibration of the tool, $\delta\alpha$ is proportional to $\sin c\tau$, or $\delta\alpha=\kappa\sin c\tau$, say, κ and c being constants; and since the pressure exerted by the shaving on the tool is proportional to its thickness, $\rho\kappa\sin c\tau\tan\alpha$ also expresses the variable part of the reaction. The effect of this variable pressure is neither to sustain nor extinguish the vibration, but to increase in effect the rigidity of the tool by a quantity proportional to $\tan\alpha$.

In tools designed for rough work α is usually small, but when the quality of the surface left by the tool is of more importance than the thickness of the shavings which it can remove, it may be largely increased with advantage.

FIG. 13.

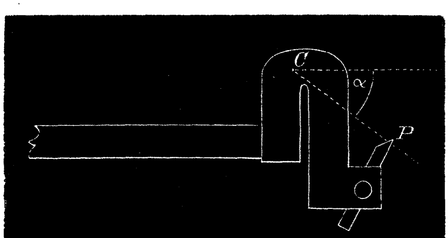


Fig. 13 shows an excellent form of cutter-holder, designed by the late W. Froude, F.R.S., in which α is about 45°. Tools held by such a cutter-holder leave a very smooth surface on the substances

which they cut, and at the same time may have smaller values for ϕ given to them than when held in any other way with which I am acquainted. The general conclusion to which the foregoing remarks point are:—

(1.) Work has to be expended in dividing substances merely because the necessary forces cannot be applied locally enough; the more local the application of the force, the less is the travel, and therefore the work required to effect the separation.

(2.) All ordinary tools act by shearing the substance on which they operate in a plane inclined at an angle of less than 45° to the plane or surface swept out by the edge of the tool.

(3.) To remove a given volume of material requires nearly the same amount of work, as far as the tool itself is concerned, whether it be removed in few cuts or many; but the constant friction of the machinery always makes the thicker cuts more economical in practice.

(4.) Tools for heavy work should be so shaped that the resultant force on them may lie nearly in the direction of motion. In order that this may be the case, ϕ must be determined by equation (C).

If a less value for ϕ than this be adopted, less work will be required to effect the same cut, but the tool will have a tendency to dig.

(5.) In tools which are merely required to leave a good surface and not to take cuts of any appreciable depth, the angles are unimportant.

One curious point connected with the subject of cutting tools is the manner in which their action is facilitated by lubricants. Lubricants seem to act by lessening the friction between the face of the tool and the shaving, and the difficulty is to see how the lubricant can get there, since the only apparent way is round the edge of the tool, and there it might be expected that the contact between the tool and the substance would be too close to admit of its passage. Somehow or other, however, some of the lubricant does find its way between the shaving and the tool, and perhaps also into the substance of the shaving.

Some metals, copper for instance, when unlubricated, actually refuse to slide over the face of the tool, and the metal is then driven before the tool in a growing lump, as stiff mud would be before a board pushed through it (fig. 6). The separation in these cases does not take place at the edge of the tool, but some distance beneath it.

Note 1.—*On Mr. Babbage's Paper on the Principles of Tools for Turning and Planing Metal.*

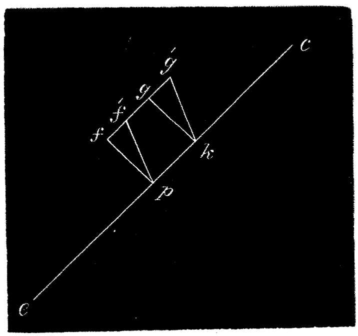
Mr. Babbage, in the paper above referred to, assumes that the force required to remove a shaving of constant width may be expressed in terms of its thickness by the series $A+Bt+Ct^2+$ &c., and this of course is perfectly true. But in his application he reduces this series to two terms only, viz., A and Ct^2; of these he says that A is the constant force "necessary to tear along the whole line of section

each atom from the opposite one to which it was attached." Ct^2 is of course dependent on the bending and material.

As to the first of these terms, I have shown that it is $=0$ when the tool is sharp; and the second must be small, in the first place, because but little true bending occurs, and, secondly, because the resistance which a shaving can oppose to bending is, on account of its laminated structure, very feeble.

Note 2.—Though the general line of shearing is in the direction *e*, *c*, it can hardly be doubted that separation first occurs across the lines of greatest tension.

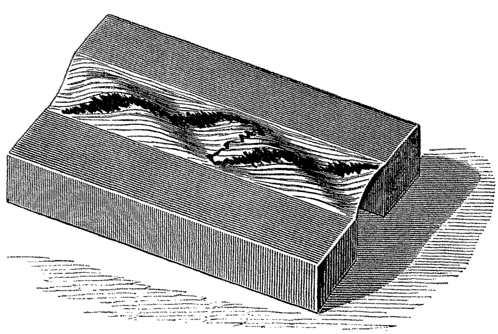
FIG. 14.



Let *f*, *g*, *h*, *k*, be a small cube of substance contiguous to *ec*, and unstrained; let *f′*, *g′*, *h′*, *k′*, be the same substance when strained and just about to shear. The lines of greatest tension are parallel to *p*, *g′*, and rupture will take place in a direction at right angles to this.

Ruptures of this kind will happen all along the line *ec*, and the saw-tooth-edge left will be rubbed down when the lamina begins to slide.

FIG. 15.



Paper ruptured by distortion.

Rupture along the lines of greatest tension in shearing may be well illustrated by pasting a piece of paper over two flat boards, with straight parallel edges, about $\frac{1}{4}''$ apart; if now, preserving this distance, the boards are forced to move past one another in the direction of their edges, folds appear in the paper parallel to the lines of greatest tension, and if the sliding be continued the paper tears at right angles to the direction of the folds (fig. 15).

VII. On Seismic Experiments." By JOHN MILNE, F.G.S., and THOMAS GRAY, B.Sc., F.R.S.E. Communicated by A. C. RAMSAY, LL.D., Director-General of the Geological Survey and of the Museum of Economic Geology. Received November 5, 1881.

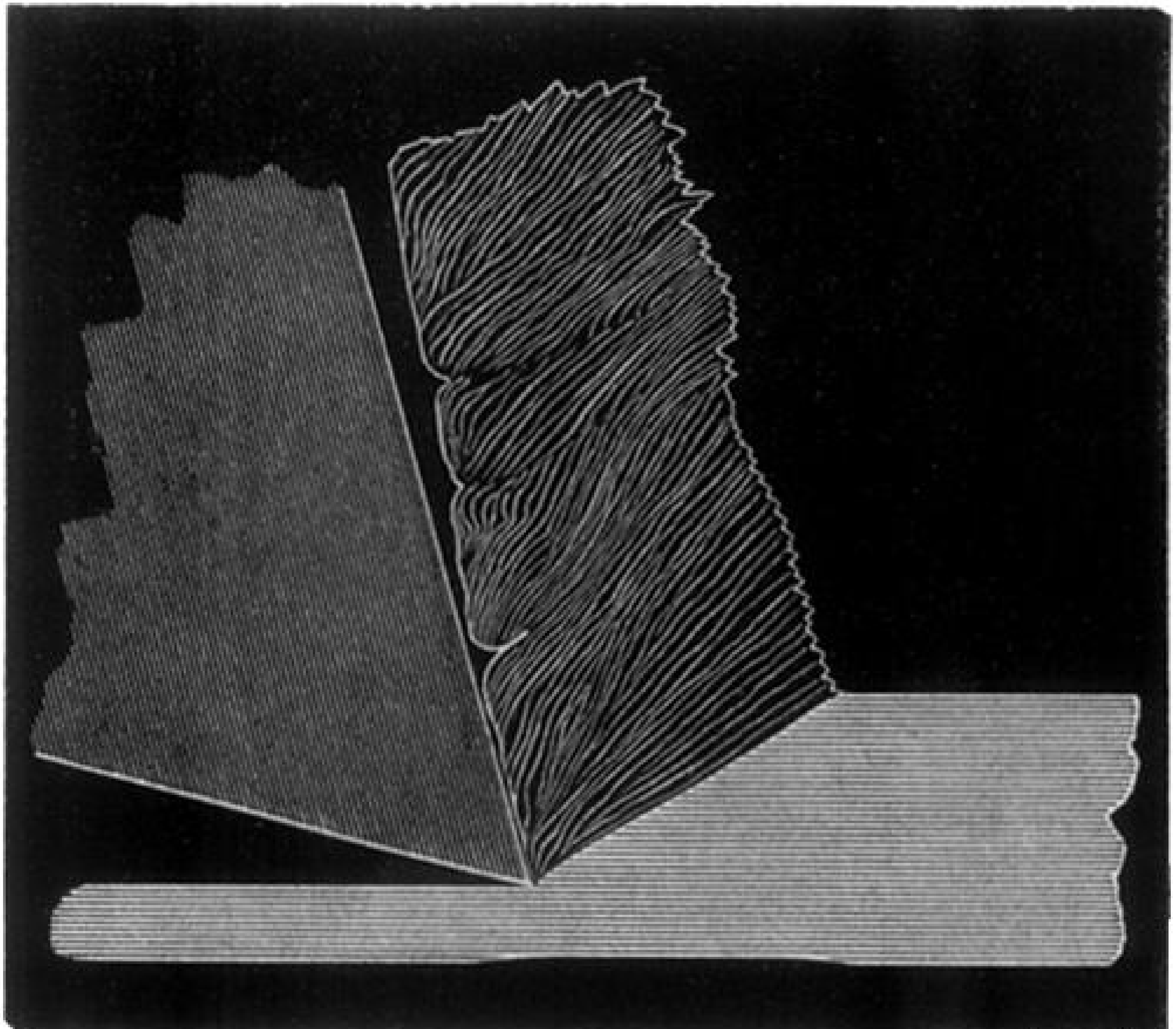
(Abstract.)

This paper is an account of a series of experiments made at the Akabane Engineering Works, Tokio, for the purpose of investigating some points connected with earthquake motion. The mode of experiment consisted in creating a disturbance at a point on the earth's surface by allowing a heavy block of iron (1,710 lbs.) to fall from a height (35 feet), and observing the resulting motion produced in the earth at points variously situated relatively to the centre of disturbance. The centre of disturbance was situated near to one corner of a pond about 10 feet deep, and close to the foot of a small steep hill, the remaining ground being very nearly level, and composed of hardened mud, which extended to a depth of from 20 to 30 feet. The configuration of the ground here briefly described is clearly shown by means of a map accompanying the complete paper. In the earlier experiments a number of similar vessels of mercury were placed at the different points, and the vibrations produced on the surface taken as a rough indication of the intensity of the disturbance at the point. This method of observation showed with considerable definiteness where the motion became insensible. These preliminary experiments showed that the disturbance could be distinctly propagated to a distance of 650 feet (which was the greatest distance available); that the pond cut off the disturbance from points beyond its distant side if these points were sufficiently removed from the corner, but that the hill did not cut off the vibrations.

In subsequent experiments more definite observations were made by using seismographic apparatus, and by this means the following conclusions were reached.

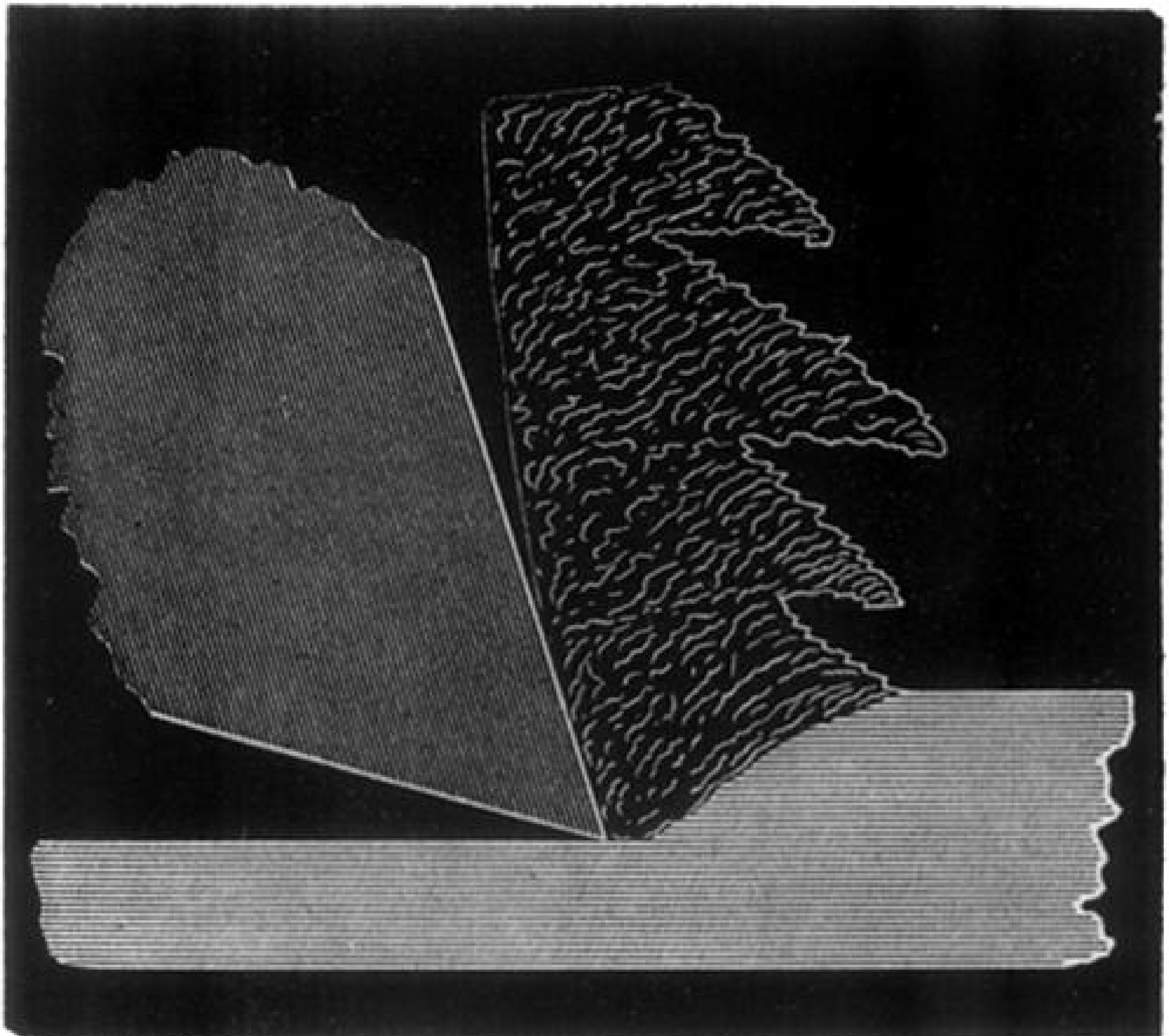
A disturbance emanating from a centre as above described, produced at least two distinct sets of vibrations. One of these sets has

FIG. 1.



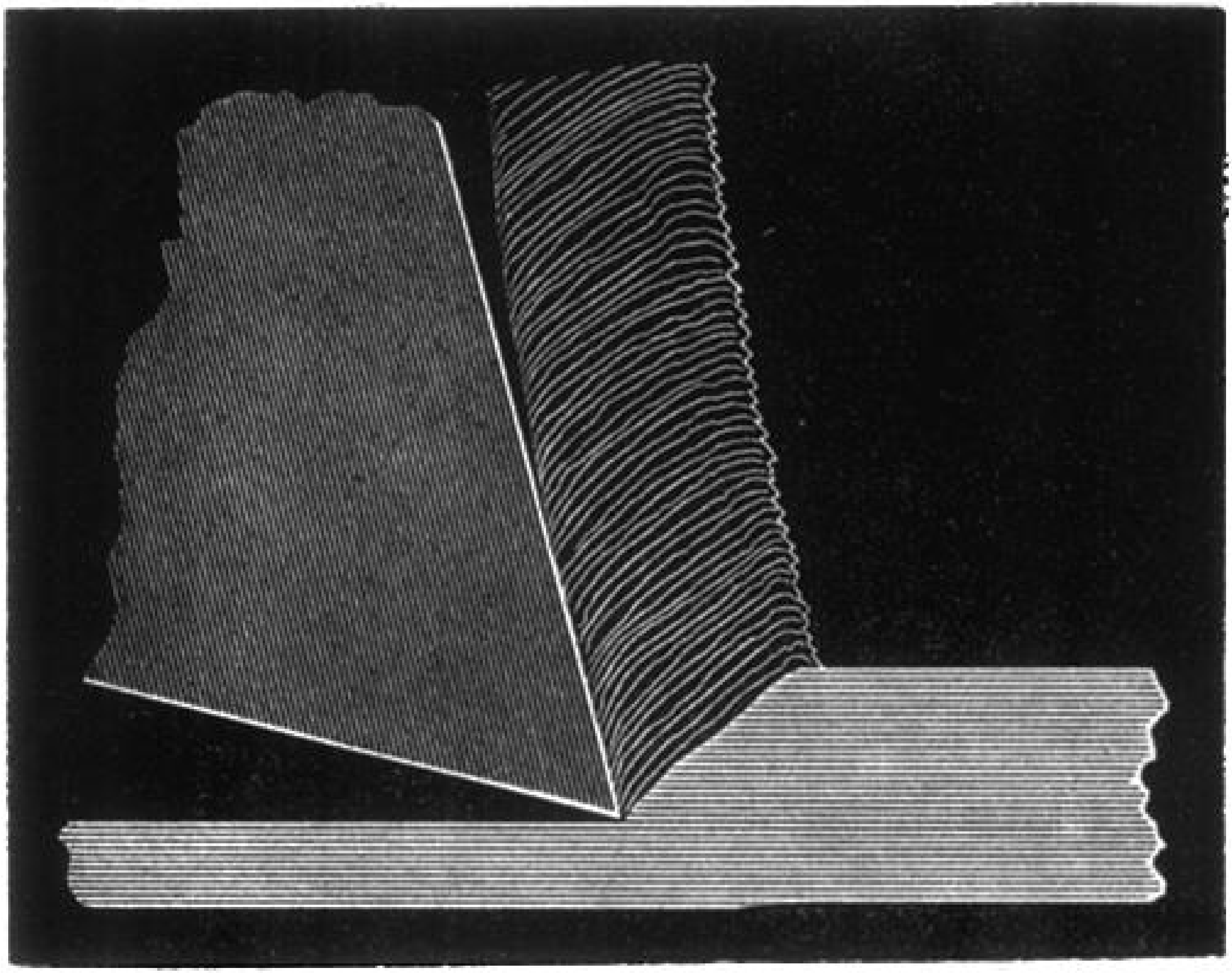
Shaving of wrought iron (armour plate). Actual thickness ·25 inch.

FIG. 2.



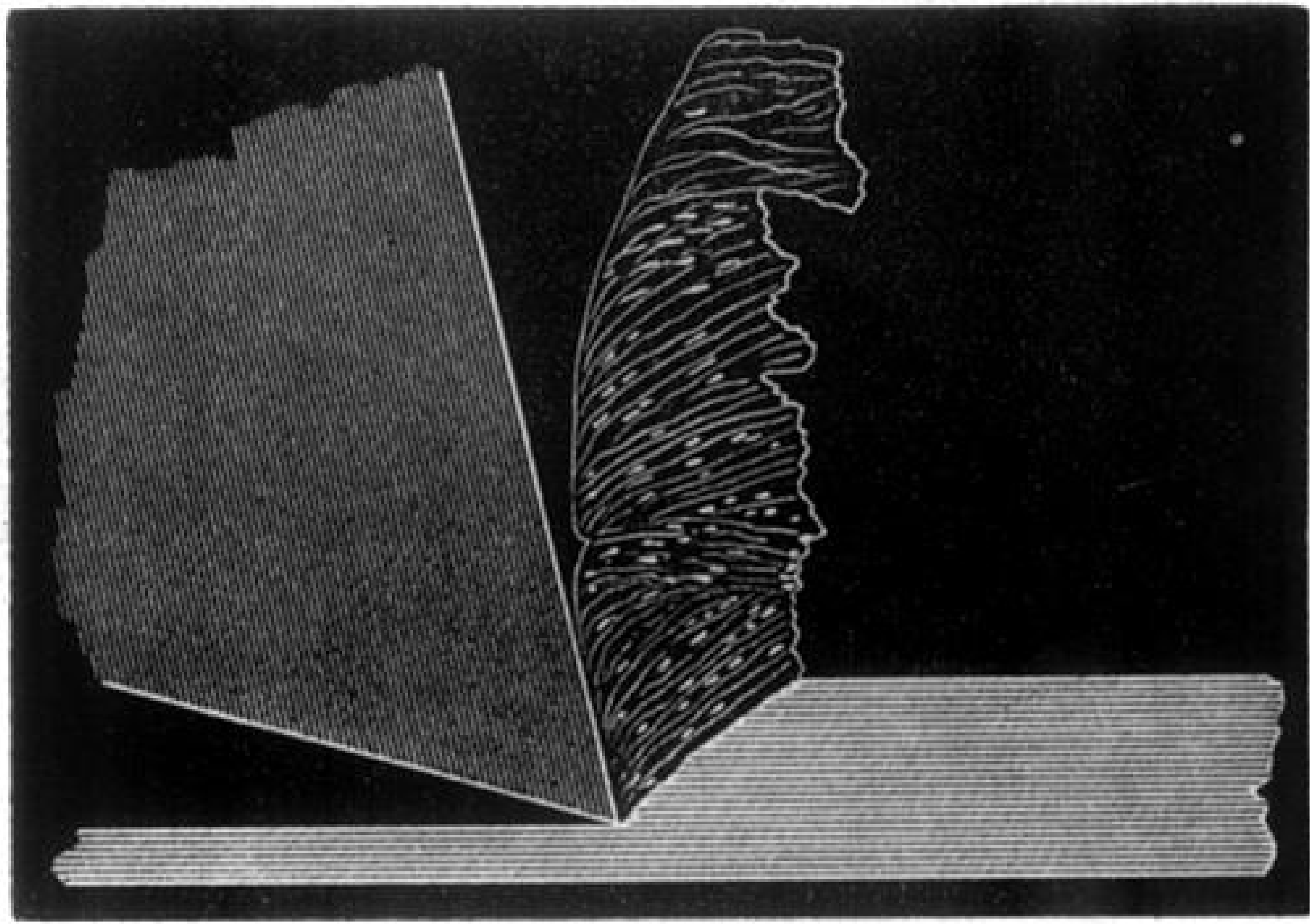
Shaving of cast iron. Actual thicknes ·1 inch.

FIG. 3.



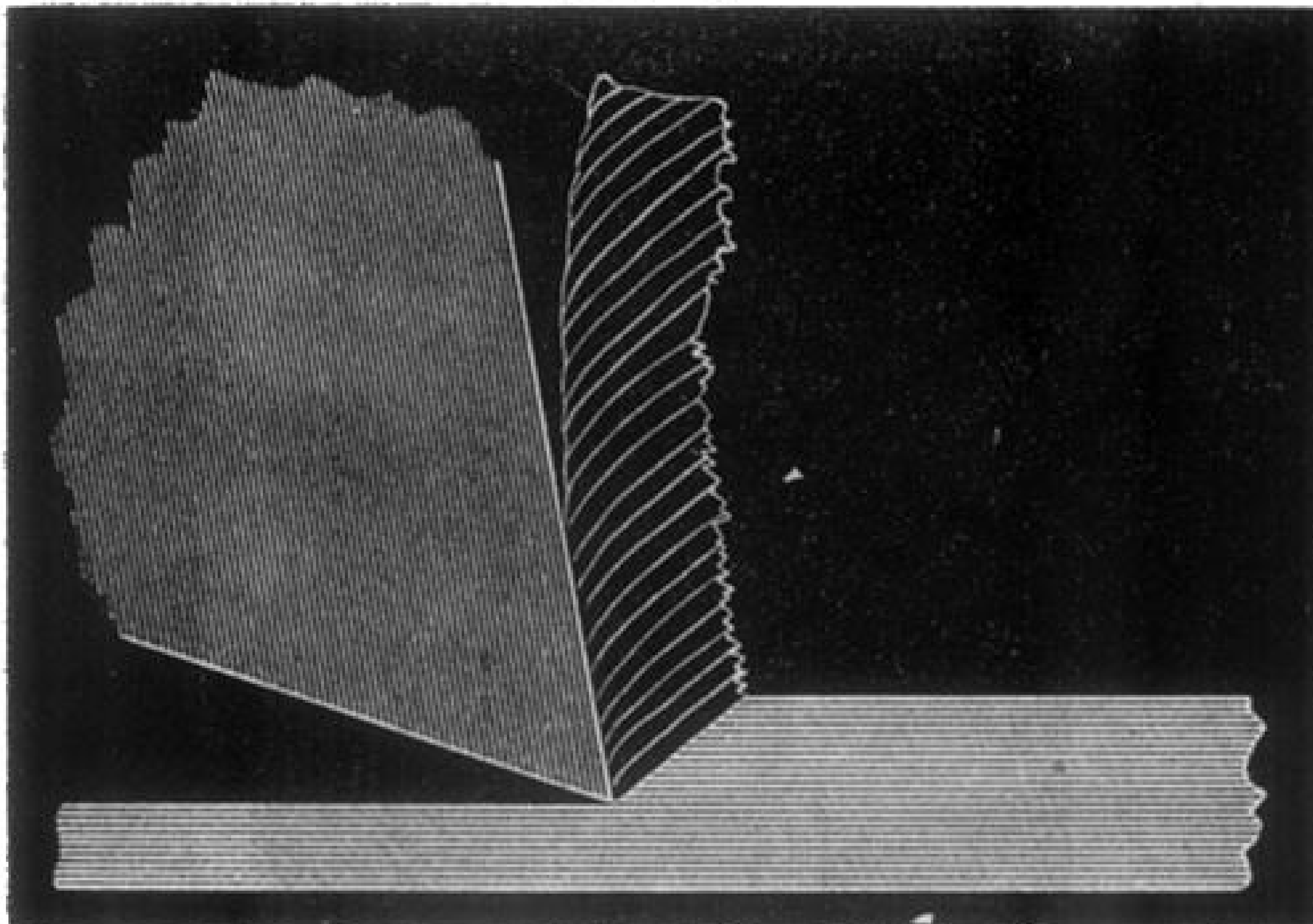
Shaving of hard steel (Whitworth). Actual thickness ·15 inch.

Fig. 4.



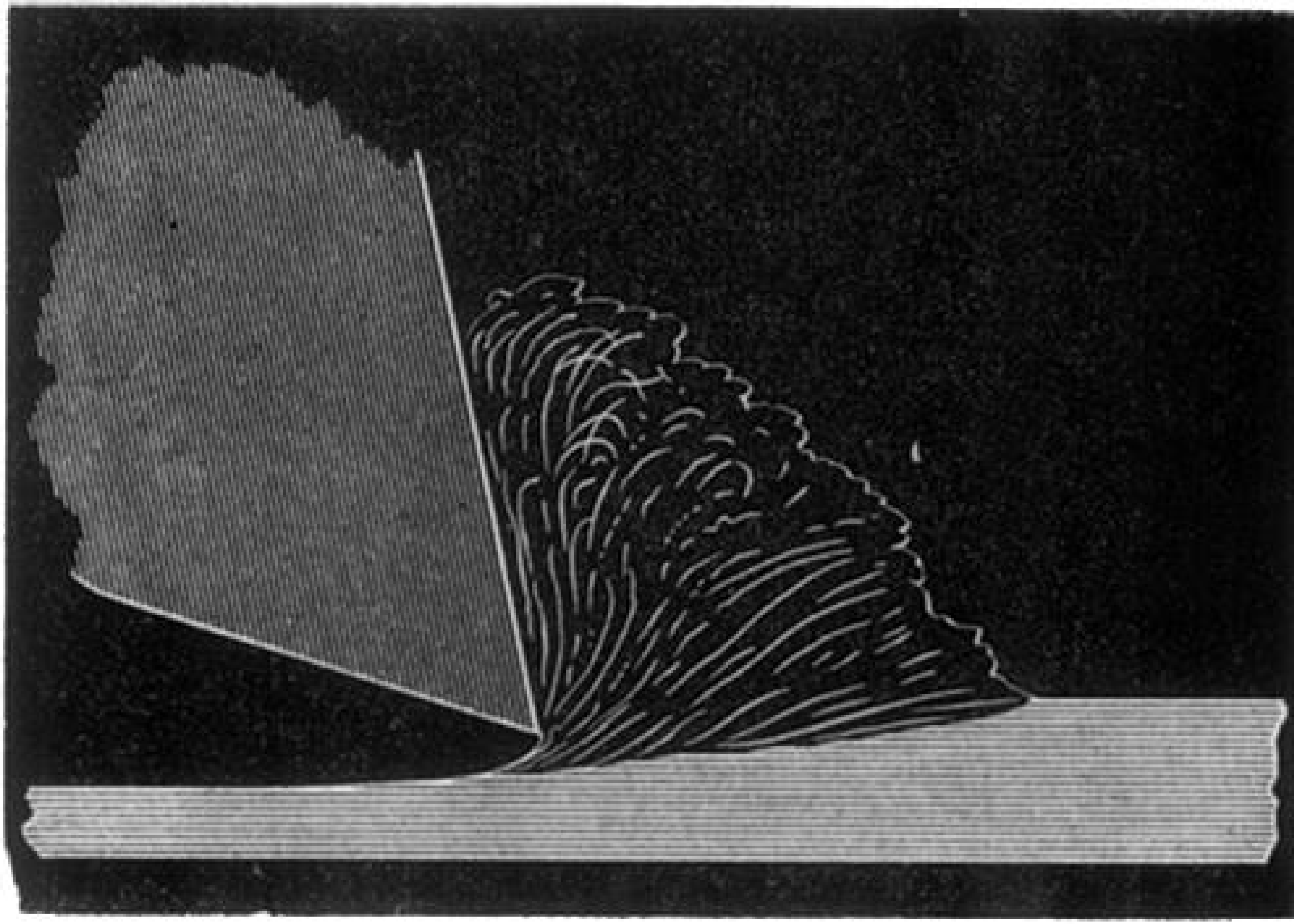
Shaving of gun metal. Actual thickness ·08 inch.

FIG. 5.



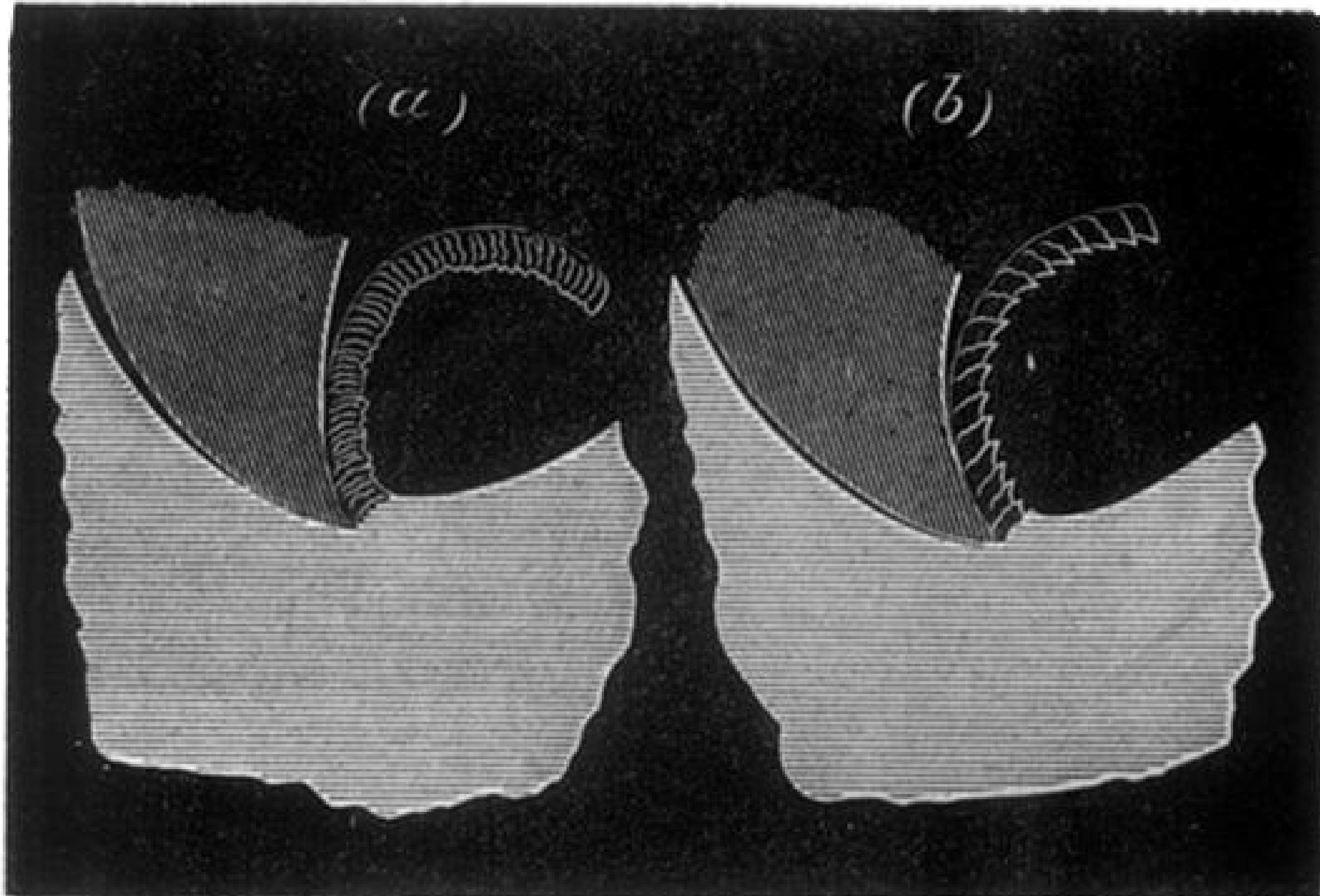
Shaving of brass. Actual thickness ·08 inch.

FIG. 6.



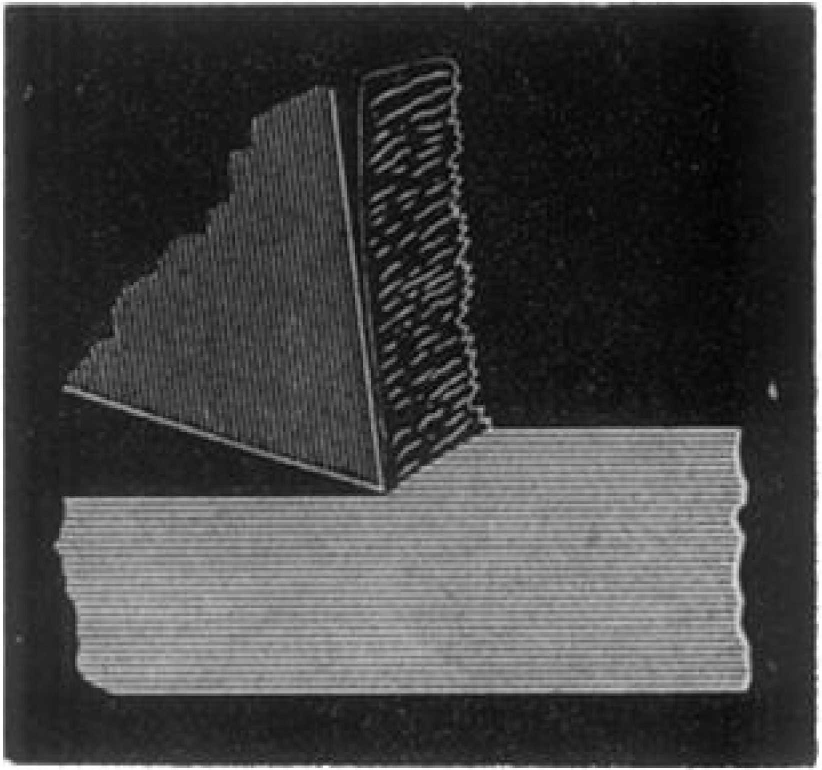
Shaving of copper (unlubricated).

FIG. 7.



(*a*.) Borings, steel. Actual thickness ·005 inch.
(*b*.) „ brass. „ „ ·01 „

FIG. 8.



Shaving of copper (lubricated with soap and water).

Fig. 9.

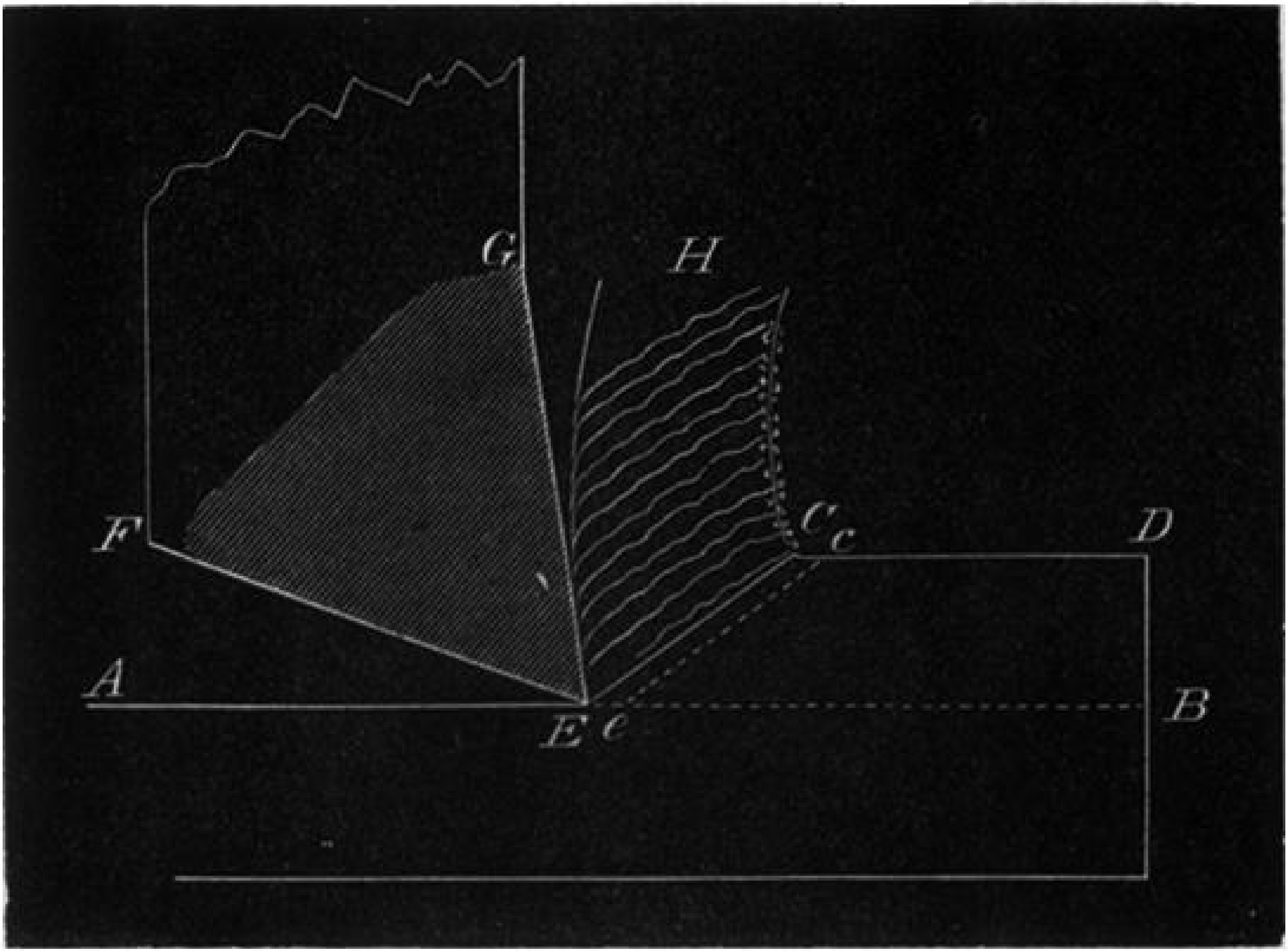


FIG. 10.

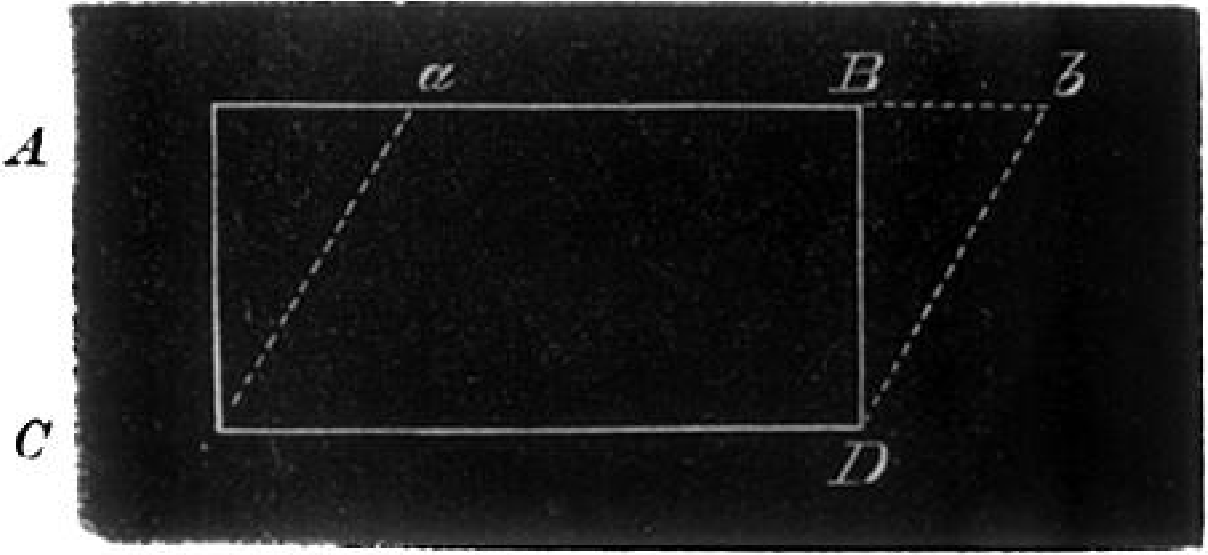


Fig. 11.

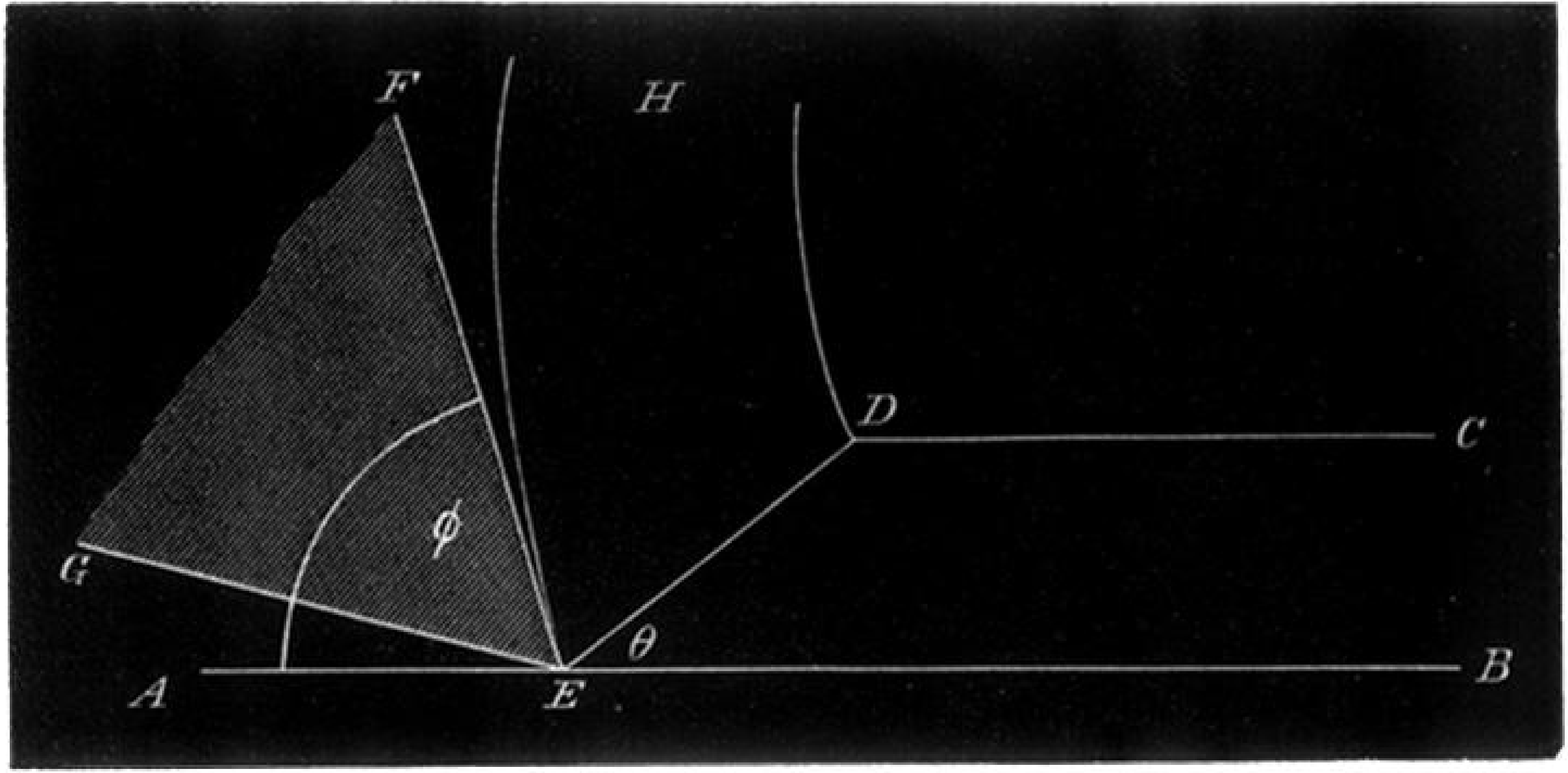


FIG. 12.

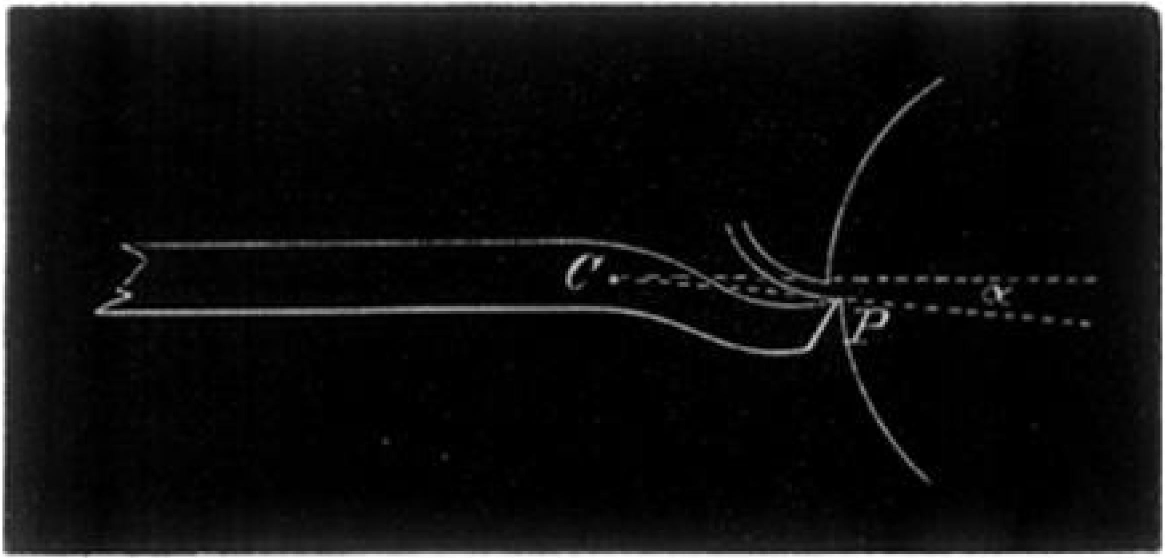


Fig. 13.

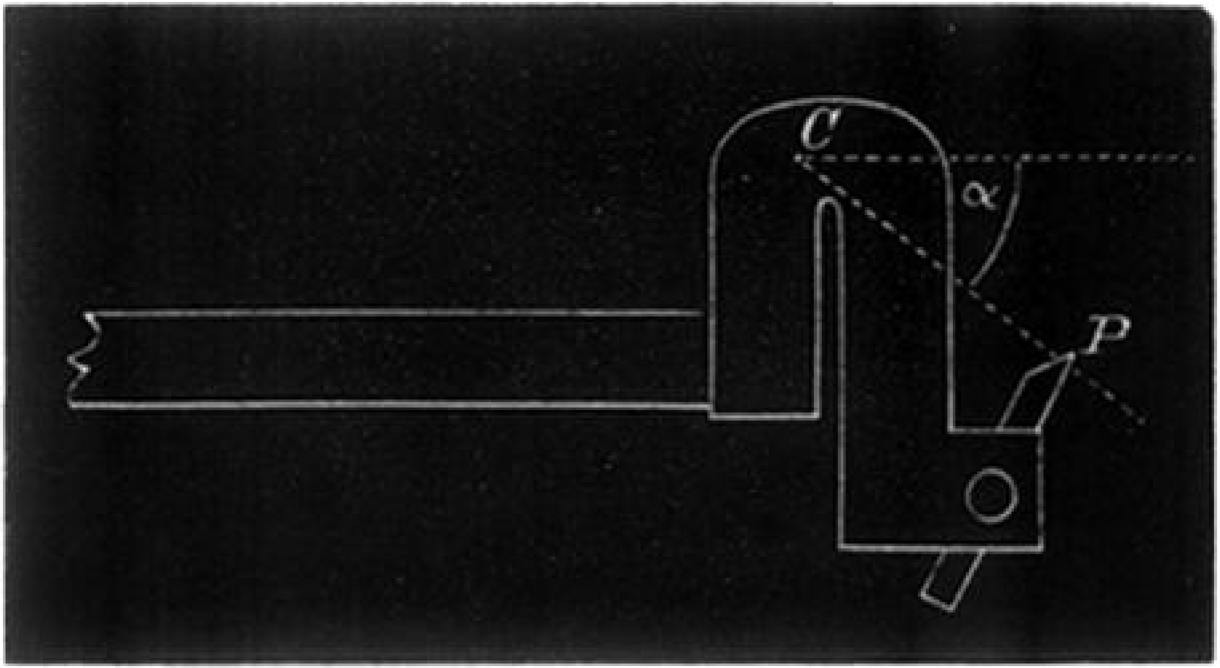


FIG. 14.

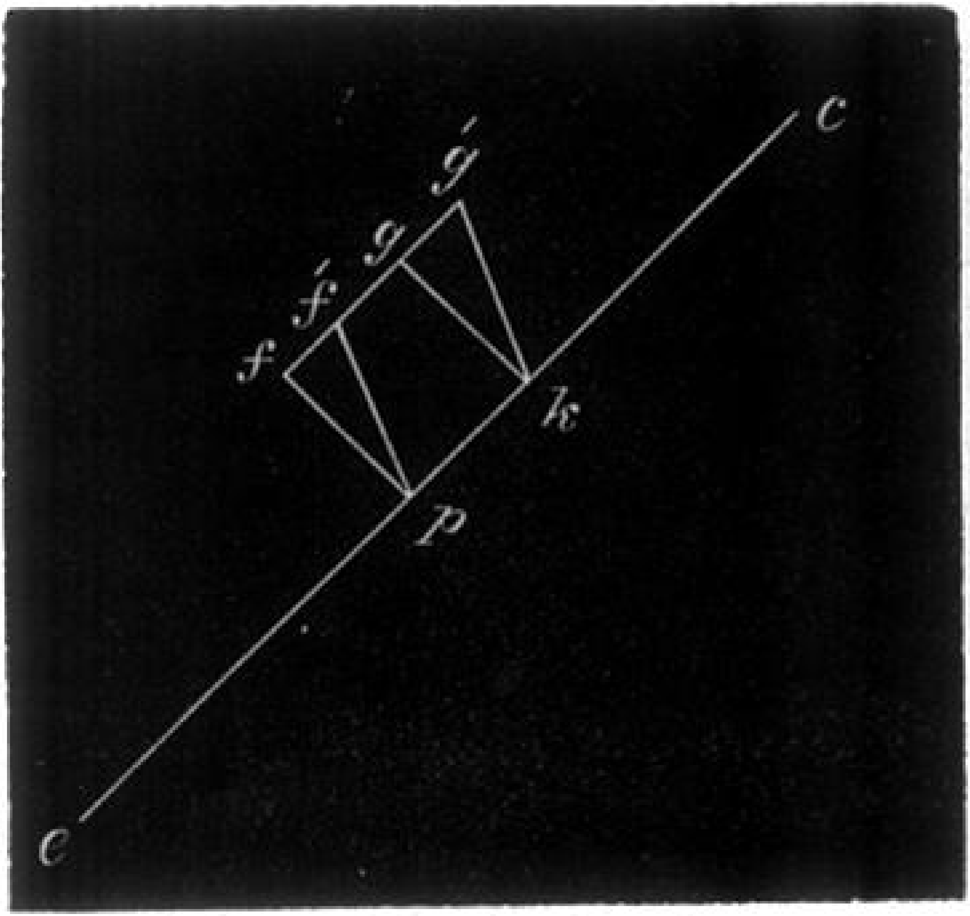
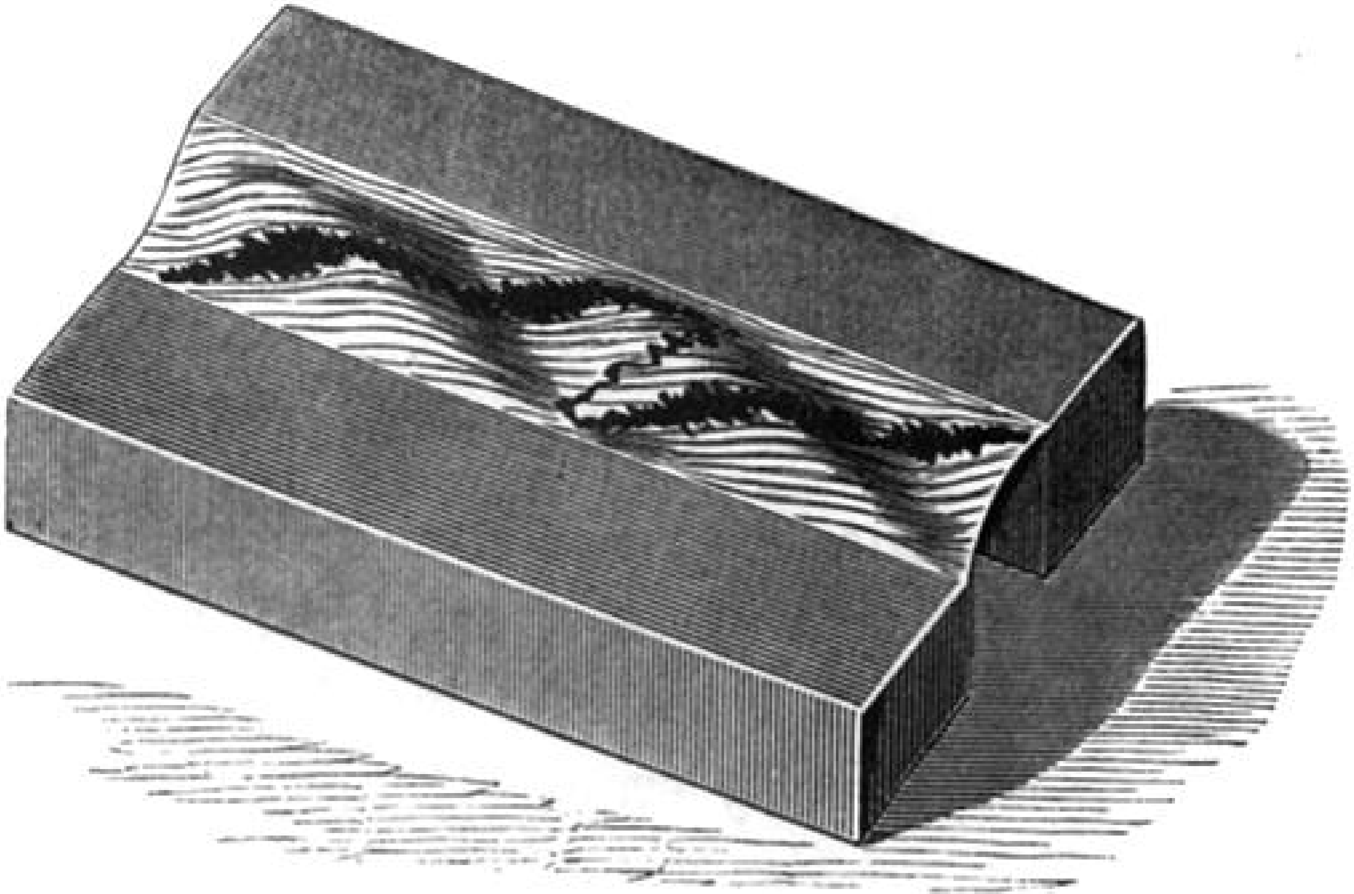


FIG. 15.



Paper ruptured by distortion.