In daboia poisoning, sanious discharges are the rule; albuminuria is always found, should the victim live any time, and after the nerve symptoms pass over, the subject has to go through a period of blood poisoning little, if at all, less dangerous than the primary symptoms, from which he may die as late as the end of the second week.

Lastly, the physiological properties of daboia poison undergo great change by its being heated to 100° C. in solution, whereas cobra poison remains unaltered.

II. "On Pendent Drops." By A. M. WORTHINGTON, M.A. Communicated by Professor B. STEWART, F.R.S. Received May 16, 1881.

[PLATE 7.]

About two years ago I was led to examine the forms of pendent drops of liquid by a method of great simplicity, which seems capable of being used with considerable accuracy for determining the value of the surface tension.

Previous observers, so far as I am aware, have observed only the weight of drops which fall, and, making this the basis of calculation, have endeavoured to find the influence on the size of such drops, of the rate of influx of liquid, shape of terminal, as in the case of Dr. F. Guthrie,* or to ascertain the value of the surface tension of the liquid as in the case of Professor Quincke† and M. Dupré.‡

Under no circumstances, however, is the weight of the drop which falls exactly the weight of the volume which it is necessary to know in order to ascertain the value of the surface tension, though under certain circumstances it approximates thereto. Hence we find that Prof. Quincke rejects this method of finding the tension, or recommends it only where other methods fail;§ albeit all his results obtained by this method are vitiated by an assumption to which I shall have occasion to draw attention. The principle of the method which I will now describe is simply to project a magnified image of a drop pendent from a cylindrical tube on to a screen, and there to trace its outline at any required stage of its development.

A vertical cylindrical glass tube, A, whose lower end is ground truly flat and with a sharp edge, communicates by means of a bulb or wider tube, B, and a piece of india-rubber or lead tubing, C, with an air-tight syringe, D, some 10 or 12 feet away. For the usual syringe piston a cup of mercury, E, is substituted, which can be gradually

* "On Drops." By Dr. F. Guthrie. "Proc. Roy. Soc.," vol. 13, p. 444.

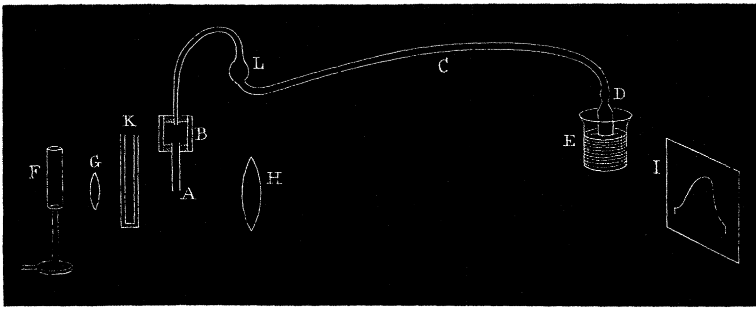
† "Poggendorff's Annalen," vol. cxxxv, p. 621. "Phil. Mag.," 1869.

‡ "Théorie Mécanique de la Chaleur," p. 332.

§ *Loc. cit.*, p. 637, § 12.

raised or lowered by a rack and pinion-screw, so as to serve as a perfectly air-tight piston.

FIG. 1.



By means of an Argand lamp, F, whose light is condensed by the lens G, a magnified image of the end of the tube is thrown by the lens H, upon a piece of white paper, I, fixed on the wall of the room near the syringe.

K is a plate-glass cell filled with water to keep off the heat of the lamp.

A supply of the liquid to be observed is drawn up into the bulb or wide tube B, by means of the syringe, or more conveniently by means of an india-rubber bulb, L, interposed for the purpose between B and D.

The tube A is then carefully wiped on the outside with a clean linen cloth, and the observer standing by the paper gradually expels the liquid by raising the mercury cup, and a drop is extruded which wets the whole of the bottom of the tube, the outline of which he traces on the white paper with a finely-pointed lead pencil, at any stage of which he desires a record, up to the point of rupture.

This tracing affords all the information necessary for calculating the value of the surface tension, the amount of magnification being known from the ratio of the diameter of the tube to that of its image on the screen.

As the first few drops expelled are apt not to wet the whole of the base of the tube thoroughly, and therefore to be rather smaller than those which follow, it is well to let one or two fall off before taking a tracing if a series of outlines at different stages is required.

The tube A can be exchanged at pleasure for others of different diameters, and when it is wished to observe the form of a drop of one liquid pendent in another, the latter is placed in a plate-glass cell with parallel sides, and the tube A dips into it.

The figures of the Plate are tracings (reduced one-half) obtained in this way with different liquids and tubes of various diameters, and

show the manner in which the form of the surface alters with the growth of the drop.

The instability of the figure close to the point of rupture makes it difficult to trace the outline correctly quite to that point.

The short horizontal line below the drop shows the maximum depth attained before rupture, and the area enclosed by the last outline that could be drawn correctly, and the one which is nearest the limit of stability is shaded for the sake of distinctness. The dotted outline, generally the innermost, shows the amount of liquid which remained adhering to the tube after the separation of the lower portion.

It may be worth while to mention here that where the liquid wets the whole of the base of the tube the form and dimensions of the outline were found to depend on the external and not at all on the internal diameter of the tube.

One of the first points that strikes anyone inspecting these figures is, that the maximum depth attained varies surprisingly little with the diameter of the tube.

An early series of observations on this point made with insufficient magnification showed so slight a variation that I was led to suppose that the maximum depth might really be constant and independent of the size of the tube, and to seek for an empirical formula involving only the physical constants of the liquid to express this depth.*

Greater magnification, however, shows that this is not the case; that the maximum depth diminishes on the whole with the size of the tube, but is rather less in a drop pendent from a plane surface of indefinite extent than in one hanging from a tube just narrow enough for the drop to attach itself at the edge.

Another point that attracts immediate attention is, that whereas the form of drops of different liquids hanging from the same tubes differs materially (compare, for example, water and turpentine), yet drops of (say) turpentine hanging from narrow tubes are similar in form to water drops hanging from wider tubes.

That this would be the case has been already shown from theoretical considerations by M. Dupré,† who has also pointed out the ratio between the diameters of the tubes that corresponds to complete similarity of the drop form for two different liquids.

* A very close approximation is given by the formula—

$$\text{Maximum depth} = \tfrac{3}{4}\pi \sqrt{\frac{\text{tension}}{\text{density}}},$$

which was suggested by the equation to the generating curve.

† "Théorie Mécanique de la Chaleur," p. 329.

The following method of finding the ratio of tube-diameters, necessary for the symmetry of drops of different liquids, being rather simpler in form than that given by M. Dupré, may be welcome to some readers.

It will be observed that with wide tubes the inclination which the extreme element of the curve makes with the horizontal base of the tube gradually increases as the drop is expelled, till it separates; but with narrower tubes it passes through a maximum value, which may be greater than 90°, after which it diminishes again.

Professor Quincke has shown that the angle of contact between a given liquid and solid in air is definite and constant. This, however,

Taking the vertex as origin, and the axis of revolution as the axis of y, the equations to the generating curves of the two liquids may be written—

$$T_1\left(\frac{1}{\rho_1}+\frac{1}{\rho'_1}\right)-\frac{2T_1}{R_1}=D_1y_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{i}),$$

$$T_2\left(\frac{1}{\rho_2}+\frac{1}{\rho'_2}\right)-\frac{2T_2}{R_2}=D_2y_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{ii}),$$

where T_1 and T_2 are the surface tensions.

$\rho_1\rho'_1$ and $\rho_2\rho'_2$ are the principal radii of curvature at any point,

R_1 and R_2 are the radii of curvature at the vertices,

D_1 and D_2 are the densities of the liquids.

If these curves are symmetrical, the ratio between all corresponding lines must be the same, and remembering that ρ'_1 and ρ'_2, the radii of the curvature due to revolution, are the normals drawn from the curve to the axis, and therefore corresponding lines in the plane of the paper, we have—

$$\frac{R_1}{R_2}=\frac{\rho_1}{\rho_2}=\frac{\rho'_1}{\rho'_2}=\frac{y}{y'}=K,$$

whence, writing $R_1=R_2K$, $\rho_1=\rho_2K$, &c., in (i), and dividing by (ii), we have—

$$\frac{T_1\left(\frac{1}{\rho_2K}+\frac{1}{\rho'_2K}-\frac{2}{R_2K}\right)}{T_2\left(\frac{1}{\rho_2}+\frac{1}{\rho'_2}-\frac{2}{R_2}\right)}=\frac{D_1y}{D_2y},$$

or
$$\frac{T_1}{T_2}\times\frac{1}{K}=\frac{D_1}{D_2}K,$$

whence
$$K^2=\frac{T_1D_2}{T_2D_1},$$

or
$$K=\sqrt{\frac{T_1D_2}{T_2D_1}}.$$

The empirical formula of the note to page 364 contains the experimental verification of this result, and shows in addition that the limit of stability is reached at corresponding stages by drops that develop similarly, or that this limit does not depend on the *absolute* dimensions of the curve. M. Dupré's tacit assumption that this will be the case seems to me hardly legitimate.

M. Dupré makes extremely sagacious use of the relative weight of drops whose forms while hanging are similar, for finding the surface tension.

My observations, however, go to show that the depths of residual liquid are not usually proportional to the other linear dimensions, and that any such use is not certain to lead to results that are more than approximately correct.

is in no way inconsistent with the variation just noticed, for the angle at which the liquid leaves the tube is practically indeterminate at the sharp edge of the glass where the liquid can select and attach itself to some elementary portion inclined at the required angle. But this constancy of the real angle of contact accounts for the fact that if a very narrow tube be used, the drop will spread up it on the outside before separating. As soon as the angle between the extreme element of the curve and the vertical side of the tube is equal to this definite angle of contact the spread will take place.

It will be noticed, especially with wide tubes, that the base of the drop suffers but little change after a certain point.

The concavity, however, of the upper portion is seen to deepen before the liquid separates, but the drop remains suspended after this deepening has begun.

It is very difficult to tell precisely where separation takes place, since it is accomplished generally with great rapidity, but I am inclined to think that it occurs at the place of greatest concavity.

Sometimes the small secondary drop (observed by Dr. Guthrie) is seen to follow the main drop. This was very noticeable in the case of water dropping through petroleum, where separation is effected comparatively slowly. The point at which this small drop is first seen serves, in this case, to determine fairly accurately the place of separation. There can be no doubt that this secondary drop is due to the spontaneous segmentation of the cylindrical neck of liquid, which joins the upper and lower portions up to the last moment before complete separation takes place; and that it is the same phenomenon that was first observed and explained by Mr. Plateau in his experiments on cylinders of mercury.*

It is obvious that if drops are extruded in rapid succession there will be a considerable influx of liquid through this residual neck into the lower portion after it has begun to separate, and that the drops which fall will on this score be larger.

Since, however, the velocity of the inflowing liquid will produce a pressure on the lower portion of the drop, tending to hasten separation, it is to be expected that a definite increase in the rate of influx will produce a greater effect on the size of the drop which falls off when the rate of dropping is slow than when it is fast.

These considerations go far to explain the results of Dr. Guthrie's experiments on the influence of the rate of dropping on the drop size. It is further evident that an influx of liquid, after separation has begun, only complicates the phenomenon, and that it is from a study of the curve, when the influx is zero, that we are most likely to obtain an insight into the manner and cause of separation.

* "Statique Expérimentale des Liquides."

Perhaps a statement of the physical circumstances under which the drop develops and separates, in the light of the information afforded by the tracings, may not be out of place here.

The cohesion of a liquid, as is well known, manifests itself in a uniform internal compression (A) which may conveniently be regarded as transmitted from the surface, and, in addition, a surface tension (T) which produces a further pressure proportional, at any point of the surface, to the sum of the reciprocals of the principal radii of curvature at that point, so that the total pressure at any point of the surface may be written,

$$\mathrm{A}+\mathrm{T}\left(\frac{1}{\rho}+\frac{1}{\rho'}\right).$$

The value of the last term may be positive or negative, according to the nature of the curvature, but the total pressure must always be positive, indeed the term A is always relatively very great.

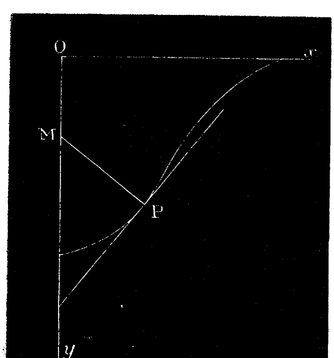
The surface acts in many respects like a membrane stretched uniformly in all directions, and in the case of a pendent drop, the liquid within may be regarded as sustained by the tension of this skin.

On ordinary hydrostatic principles the pressure must be the same at all points in the same horizontal plane, from which condition the equation to the generating curve is at once obtained—

$$\mathrm{T}\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)=c+y\mathrm{D},$$

where the axes are those of the figure, D the weight of the unit volume of the liquid, and c a constant expressing the pressure due to curvature at the base of the drop where $y=0$.

FIG. 2.



If we take the radius of curvature in the plane of the paper as ρ,

then ρ' is the radius of the curvature due to revolution round the axis of y, and is equal to the normal PM.

It is possible to obtain, by inspection of the curve, some notion of the value of the total curvature as measured by $\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$ to which the pressure is proportional.

The length of the normal ρ' is of course capable of exact measurement, while that of the radius of curvature ρ may be obtained approximately by striking the circle of curvature.

At a point of recurvature, $\frac{1}{\rho}=0$, and the normal alone determines the pressure.

If the reader will trace in this way, in one of the figures, the value of the total curvature from the vertex of the drop to the base of the tube at different stages of extrusion, and will, in the case of some one liquid, compare the final values attained before separation, when tubes of different diameters are used, he will perceive that, before the drop has attained its maximum size, the tension of the surface *seems* to press the contained liquid against the end of the tube. But, as the drop increases in size, the tension is more and more exerted in supporting the additional weight of liquid, and this pressure at the end of the tube diminishes; and that this pressure at the base, when the drop separates, is greater with small tubes than with wide ones, and is obviously still positive in the case of water hanging from the narrower tubes.

But, with a wide tube, the final value of the total curvature that is attained before rupture is obviously less than with a narrow tube; and, by the methods of measurement, which will be described in the sequel, it was found that, as the diameter of the tube is increased, the final value of the curvature becomes gradually smaller, zero, and then increasingly negative, and here the term A in the expression for the pressure

$$A+T\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$$

gives meaning to the whole expression which would otherwise be negative and meaningless.

Up to the point at which the curvature at the base is greater than 0 everything would have gone on the same, even though this term A were non-existent,* the tension of the surface sufficing for the support of the drop; but, at this point, the tension ceases to be sufficient, and is aided by the true internal cohesion of layer for layer of the liquid, and the adhesion of each unit of area of the liquid for the solid base

* This is only mathematically, not physically, conceivable. T is really a function of A, and would cease to exist with it.

with which it is in contact. The extent to which this portion of the cohesion is called into play in sustaining the weight of the drop, is measured by the amount of negative curvature above-mentioned.

Direct evidence of this negative curvature is easily obtained by allowing a drop to hang from the end of an open vertical tube of narrow bore, into which a supply of liquid is led by a fine capillary thread; as the drop grows the head of liquid supported may be observed to diminish. Owing, however, to the smallness of the change, the slowness with which the meniscus in a fine tube adjusts itself, and the liability to alteration of the surface tension of the drop through long exposure, the method cannot be used with advantage for measuring the curvature.

It is important, however, to notice that there is not any physical breach of continuity, or critical point reached, when the pressure at the base becomes zero or negative, which can influence the separation of the drop. For there is no reason to suppose that the small negative pressure finally attained at the base of the drop, varying as it does with the diameter of the tube, represents any physical limit of the cohesion, which is probably very great.

Hence, it is clear that the separation of the drop is to be attributed to the surface becoming unstable for small oscillations, rather than to the cohesion being in any true sense overcome by the force of gravity, and that the cleavage of the drop is analogous to the cleavage of the unduloid or cylinder first studied experimentally by M. Plateau.

It is necessary to insist on this point, for it has been apparently misconceived by Professor Quincke, who has endeavoured to deduce the value of the tension by equating the weight of the drop which falls to the tension multiplied by the circumference of the tube or cylinder from which it hangs, the latter being taken so small that the drop hangs with the uppermost element of the liquid vertical. (See fig. 3.)

The nature of his proceeding will be best seen from the following quotation.*

FIG. 3.

* From a paper communicated to the Royal Academy of Sciences at Berlin, May 28, 1868, and translated in "Phil. Mag.," 1869, by Professor Jack. The italics are my own.

"The equation (7) $\frac{W}{2\pi r}=T,$

is true also for drops formed at the mouth of a vertical pipe *on the assumption that in consequence of the gradual accession of new fluid the same pressure is found in the interior fluid at the mouth of the pipe as in a level fluid surface.*" "The drop goes on increasing till $W=0$, or till the highest element of the fluid is vertical, and then it falls off.

"If the radius of the cylinder on which the drop is formed be very small, the weight of the portion of the fluid which remains hanging may be neglected, and the weight of the portion of the drop which falls may be treated as the W in equation (7)."

He is here speaking of a drop pendent from a tube so small that it wets the outside and hangs as in fig. 3. It is, however, obvious that, under such circumstances, the surface of the liquid, where vertical, shares the curvature of the tube, and that the equation of equilibrium should be $2\pi r T = W +$ the pressure on the area of the section of radius r,

or
$$2\pi r T = W + \pi r^2 \frac{T}{r},$$

or
$$\frac{W}{\pi r} = T.$$

In other words, Professor Quincke's value of the tension is precisely half what it ought to be. Hence, the very low value that he obtains for the surface tension of water by this method, ·0548 as compared with ·08253 by other methods.

He applies the same equation to Professor Guthrie's experimental data, although indeed it is not applicable to the larger terminals used by the latter, so that the approximate agreement of the results with those obtained by other methods is really due to a fortuitous cancelling of errors.*

I will now explain the manner in which the tracing may be used for evaluating the pressure at the base, and determining the value of the tension.

It is convenient to assume first that this pressure is zero, then to introduce in the value of T the correction which is found from an examination of the curve to be necessary.

Assuming then that the pressure at the base is zero, it is evident that if any horizontal section of the drop be made, the vertically resolved part of the surface tension across the circumference of this section is equal to the weight of the portion of the drop below the section, plus the pressure of the head of liquid reaching from the section to the bottom of the tube multiplied by the area of the section on which it acts.

* "Pogg. Ann.," vol. cxxxi, p. 137.

Thus if—

r be the radius of the section,

θ the inclination of the tangent to the axis of revolution,

H the head of liquid in centims.,

V the volume of the part below the section in cub. centims.,

T the surface tension in grams per linear centim.,

D the weight in grams of 1 cub. centim. of the liquid,

then
$$T \cos\theta \times 2\pi r = (V + \pi r^2 H) D,$$
$$T = \frac{(V + \pi r^2 H) D}{2\pi r \cos\theta}.$$

r and H were measured carefully with compasses and a millimetre scale, which could be read by estimation to tenths of millimetres, and $\cos\theta$ was determined directly by the same means after drawing the tangent to the curve. V was obtained by dividing the area below the section into narrow horizontal strips, whose dimensions were measured with scale and compasses, and the volume of the solid generated by the revolution of each, considered as the frustum of a cone, was calculated. It was found in practice that the bottom part of the curve is generally so nearly circular that the best way is to strike the circle of which it is the arc, for other parts of the curve lines, ruled at a distance of 5 or 10 millims. apart (when the magnification was about fifteen times linear), gave a sufficiently accurate approximation, it being impossible to distinguish from straight lines the small portions of the curve intercepted between each pair.

It was easy, by marking the depth, to take two or more independent tracings at exactly the same stage of extrusion, and superposition of these showed very complete agreement, and gave confidence in the result; moreover, the symmetry of the tracing on the two sides of the axis affords a check on the accuracy of the drawing. The tangent is most easily drawn at a point of re-curvature, or where the curvature in the plane of the paper is small; on the other hand, an error in the value of $\cos\theta$ increases in importance as θ increases.

These considerations should influence the choice of sections.

Treating in this way the curve of a drop of olive oil hanging in air, the following values were obtained for the surface tension at the different sections (1), (2), (3), and (4), which are arranged in order of increasing area.

$$T_1 = \cdot 049401,$$
$$T_2 = \cdot 051215,$$
$$T_3 = \cdot 057775,$$
$$T_4 = \cdot 065063.$$

The discrepancy at once shows that the assumption of no pressure

due to curvature at the base is wrong, and that the pressure is in reality negative, and that each value of the tension requires to be diminished by a correction which, being proportional in each case to the area of the section, will affect the higher values most, the smaller least. If we call the unknown pressure due to curvature at the base c, and introduce this in the equation, it becomes

$$\mathrm{T}\cos\theta \times 2\pi r = (\mathrm{V} + \pi r^2 \mathrm{H})\mathrm{D} - \pi r^2 c,$$

$$\text{or } \mathrm{T} = \frac{(\mathrm{V} + \pi r^2 \mathrm{H})\mathrm{D}}{2\pi r \cos\theta} - \frac{rc}{2\cos\theta},$$

so that we have now four equations for determining T and c, viz.,

$$\mathrm{T} = \mathrm{T}_1 + \frac{r_1 c}{2\cos\theta_1},$$

$$\mathrm{T} = \mathrm{T}_2 + \frac{r_2 c}{2\cos\theta_2},$$

$$\mathrm{T} = \mathrm{T}_3 + \frac{r_3 c}{2\cos\theta_3},$$

$$\mathrm{T} = \mathrm{T}_4 + \frac{r_4 c}{2\cos\theta_4},$$

any two of which are sufficient.

Combining these equations by the method of least squares so as to obtain the most probable values of the unknown quantities, we get

$$c = -\cdot 09943 \text{ grm. per square centim.},$$

$$\mathrm{T} = \ \cdot 03373 \text{ grm. per linear centim.},$$

and using these values to calculate the most probable values of the observed quantities T_1, T_2, &c., we have

Calculated.		Observed.
$\mathrm{T}_1 = \cdot 04935$	....	·04940
$\mathrm{T}_2 = \cdot 05121$	....	·05181
$\mathrm{T}_3 = \cdot 05777$	....	·05688
$\mathrm{T}_4 = \cdot 06506$	....	·06538

It remains to observe that, as will appear in the table of results, the pressure due to curvature at the base was found to be negative even where the drop hung from a plane surface of indefinite extent which it wetted entirely.

It must be admitted that the surface here (see fig. 1, Plate 7) really passes into a continuous plane where the curvature, and consequently the pressure due to curvature, is zero; and that, nevertheless, at an indefinitely small distance below this, the pressure due to curvature is undoubtedly negative.

The explanation is, I think, to be found in the fact that the liquid film which wets the surface, is, by reason of the contact, in a modified condition, viscous and unable to transmit pressure like the rest of the liquid.* The existence of such modified covering films seems to have been well established by Professor Quincke.†

I have endeavoured to lessen the labour of calculating the volume of liquid supported by the tension at any section, by cutting out of a uniform sheet of card the area by whose revolution round the axis this volume is generated. The magnitude of this area was found by weighing, and the position of its centre of gravity by balancing on a knife-edge, from which, by applying the well known "Property of Guldinus," the volume of the solid was at once obtained. Owing, however, chiefly to the unequal thickness of even the most uniform material, whether card or metal foil, that I could procure, the results obtained in this way were liable to differ from those obtained by the previously described method of integration by 1 or 2 per cent., so that the method cannot be recommended. I have, however, retained in the following table the measure of T obtained in this way for water in a particular case, and have also inserted for comparison the values thus obtained at different sections of a chloroform drop. The table contains the record of results obtained in the way described.

In most cases a tube was used, the diameter of which is recorded, instead of a plane surface of indefinite extent, since it was found difficult to prevent the drop from gliding about and getting out of focus, when pendent from the latter.

The temperature (when recorded) is that indicated by a thermometer placed close to the drop.

The surface tension (T) is expressed in grams per centim.

Olive Oil (density ·92129).

Indefinite plane. Temperature about 18° C.

	Calculated.		Observed.
$T_1=$	·04935	....	·04940
$T_2=$	·05121	....	·05181
$T_3=$	·05777	....	·05688
$T_4=$	·06506	....	·06538

$c = -·09943$

$T = ·03373$

* It sometimes seemed to me that the rest of the liquid started out of this film at a small but definite angle, of which I obtained the following measures in the case of turpentine hanging from a flat iron terminal, which it wetted to an indefinite distance :—18°, 20°, 21°·5, 20°·5, 19°·2 mean 19°·84.

† "Phil. Mag.," June, 1878.

Alcohol (density ·8001).

Diameter of tube ·9525 centim. Temperature about 18° C.

	Calculated.		Observed.
$T_1=$	·03396	....	·03415
$T_2=$	·03617	....	·03583
$T_3=$	·03821	....	·03834

$c = -·05596$

$T = ·02586$

Russian Turpentine (density ·8679).

Diameter of tube 1·153 centim. Temperature about 18° C.

	Calculated.		Observed.
$T_1=$	·040185	....	·040354
$T_2=$	·042123	....	·042088
$T_3=$	·045613	....	·045749

$c = -·081236$

$T = ·028179$

Chloroform (density 1·51294).

Diameter of tube ·9525. Temperature about 15°.

Drop No. 1.

	Calculated.		Observed.
$T_1=$	·037113	....	·037354
$T_2=$	·039542	....	·039239
$T_3=$	·048582	....	·048644

$c = -·04438$

$T = ·03025$

Drop No. 2.

	Observed.		By weighing.
$T_1=$	·02983	....	·02921
$T_2=$	·03034	....	·02949
$T_3=$	·03074	....	·03005
$T_4=$		....	·02958
$T_5=$	·03046	....	·02965

T=mean= ·03034

Here the difference between the values at different sections was so small that c was neglected and the mean taken.

Water.

Diameter of tube ·9525 centim. Temperature —

Calculated.		Observed.
$T_1 =$ ·098829	....	·098804
$T_2 =$ ·092912	....	·09324
$T_3 =$ ·098829	....	·098804

$c = -$ ·07625
T = ·080056

Diameter of tube ·9525 centim. Temperature about 18° C.

$T_1 =$ ·078802
$T_2 =$ ·077588
$T_3 =$ ·078349
$T_4 =$ ·078592

Here the difference was so small that the correction was neglected and the mean taken—

T = ·078327.

In saturated air. Temperature about 15° C. December 4, 1880. Diameter of tube 1·252 centim.

Calculated.		Observed.
$T_1 =$ ·09376	....	·09377
$T_2 =$ ·09677	....	·09648
$T_3 =$ ·010005	....	·010001

$c = -$·12547
T = ·07359

Water (continued).

Indefinite plane. October 15, 1880.

Calculated.		Observed.
$T_1 =$ ·10158	....	·10142
$T_2 =$ ·10999	....	·11023
$T_3 =$ ·11528	....	·11522

$c = -$ ·11600
T = ·07383

By weighing.

Diameter of tube ·9525. Temperature 4° C.

	Calculated.		Observed.
$T_1=$	·07598	....	·07516
$T_2=$	·07685	....	·07797
$T_3=$	·07930	....	·07897

$$c = -\cdot 03030$$
$$T = \cdot 07180$$

It will be observed that the values of T for water vary very considerably. Moreover, on no occasion have I obtained so high a value as the mean ·08253 deduced by Professor Quincke by the method of flat bubbles.

The agreement of the measures is in each case so close as to leave no doubt that the surface tension of the liquid really differed considerably on different occasions. I have at present no better explanation of this to offer than the fact that the surface tension of water is particularly liable to diminution by exposure to the impurities of the atmosphere.

To aid a comparison of the method with that of flat drops or bubbles, on which Professor Quincke seems chiefly to rely, I append a table giving the values of the tensions by the two methods in parallel columns. It is to be observed that the liquids used are often not quite the same, as is indicated by the difference of density.

The name of the observer is signified by the initial letter.

Name of substance.	Density.		Surface tension in grams per centim.	
	Q.	W.	Q.	W.
Water	1	1	·08253	·0718 to ·08005
Alcohol	·7906	·8001	·02599	·02586
Turpentine	·8867	·8679	·03030	·028179
Olive oil	·9136	·92129	·03760	·03373
Chloroform	1·4878	1·5129	·03120	·03025

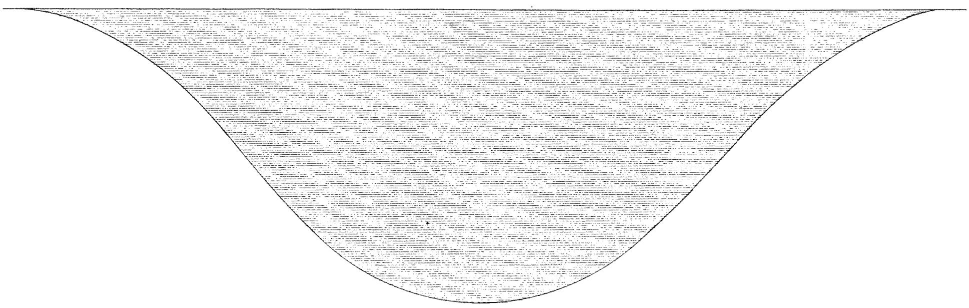
Professor Quincke's result in the case of water is the mean of seven measures ranging between ·0800 and ·0892; that for turpentine is the mean of eight ranging from ·0284 to ·0318; that for olive oil is the mean of seven, ranging from ·03617 to ·04113.

The method of flat drops and bubbles is, as he himself points out, only approximate, since the curvature due to revolution is neglected, and is liable to give results that are rather too high.

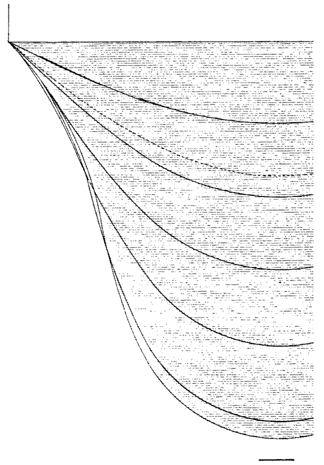
Worthington.

WATER.

Indefinite plane (glass)

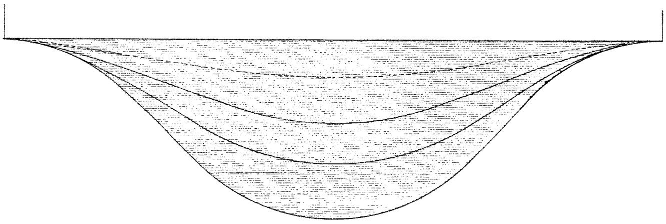


Diam. = ·

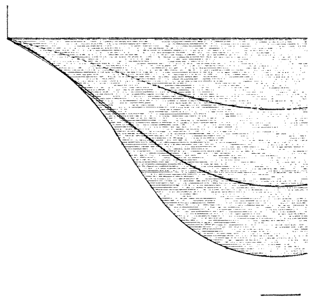


TURPENTINE (*Densit*

cm.
Diam. = 1.153.



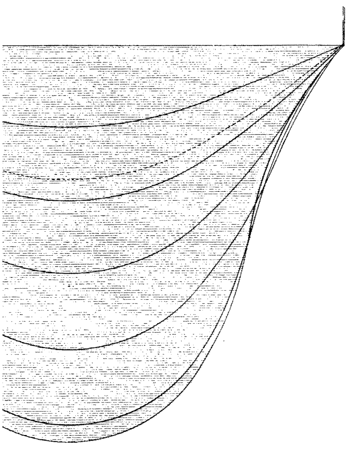
Diam. = ·



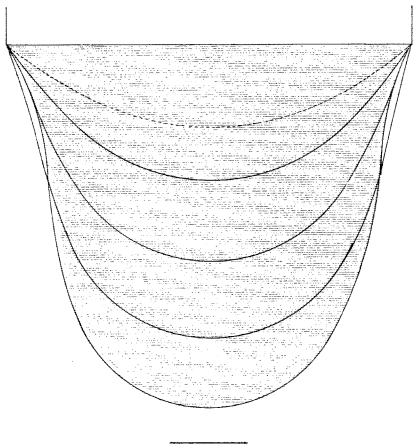
West Newman & C? lith.

R.

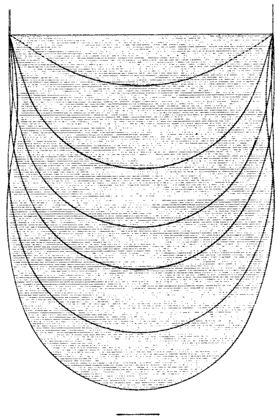
cm.
Diam. = ·9525.



cm.
Diam. = ·724

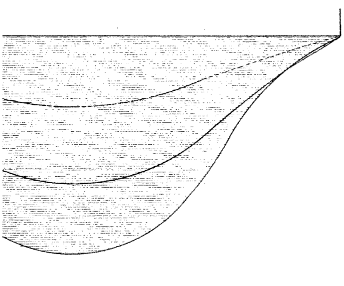


cm.
Diam. = ·448.

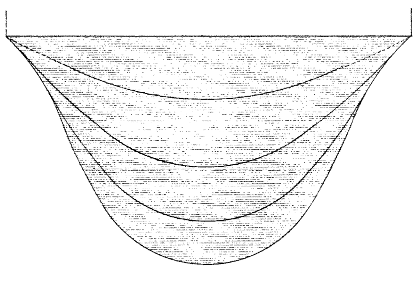


(Density. ·8679.)

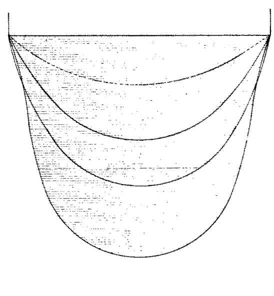
cm.
Diam. = ·9525.



cm.
Diam. = ·7238.



cm.
Diam. = ·448.



ith.

The method which I am advocating is open to no such objection; it is moreover rapid, allowing little time for the alteration by exposure of the surface under consideration, and it is independent of the very difficult determination of the contact angle of liquid and solid, which enters into some other methods.

At the same time the results are probably not so accurate as could be obtained by the use of photography for recording the form of the drop.

I am in hopes of obtaining, by its means, results which may, in some measure, serve to verify the fundamental assumptions of the theory on which the value of the so-called surface tension is calculated.

Of the phenomena, to which I have ventured incidentally to call attention, that of the approximate constancy of the drop-depth, requires closer investigation than I have been able to give it, as it seems to offer a very satisfactory means of obtaining a first approximation to the value of the surface tension. In all experiments on this subject the scrupulous cleanliness so absolutely necessary renders progress slower than many would suppose, which is my excuse for presenting an investigation, which I trust may be found useful, though obviously incomplete.

III. "Postscript to the Chronological Summary of Methods of Computing Logarithms in my Paper on the Potential Radix." By ALEXANDER J. ELLIS, B.A., F.R.S., F.S.A. Received May 18, 1881.

After the publication of my "Chronological Summary of Methods of Computing Logarithms," in the "Proceedings" for 3rd February, 1881 (vol. 31, pp. 401–407), Mr. Isaac Todhunter, F.R.S., kindly pointed out to me an almost unknown pamphlet by George Atwood, F.R.S., inventor of "Atwood's Machine," which he informs me is not mentioned even by Dr. Thomas Young, in his account of Atwood. A copy of the pamphlet is in the library of the Royal Astronomical Society, and at the request of Mr. Peter Gray, F.R.A.S., was kindly lent to me for inspection. The following account of this important and little known work should be inserted in my summary, on p. 403, between 1771, *Flower*, and 1802, *Leonelli*:—

1786. *Atwood*, George, F.R.S. "An Essay on the Arithmetic of Factors, applied to various computations which occur in the practice of numbers." This is essentially a method, when any number A is given, of finding factors, M, p, q, r, s, t, such that $A \times M$ being very nearly $=1$, of the form $\cdot 9 \ldots$ or $1\cdot 0 \ldots$, and p, q, r, s, t being of the form $1 \pm \cdot 0_n k$ (where 0_n means n successive zeroes, as in the notation of my

FIG. 1.

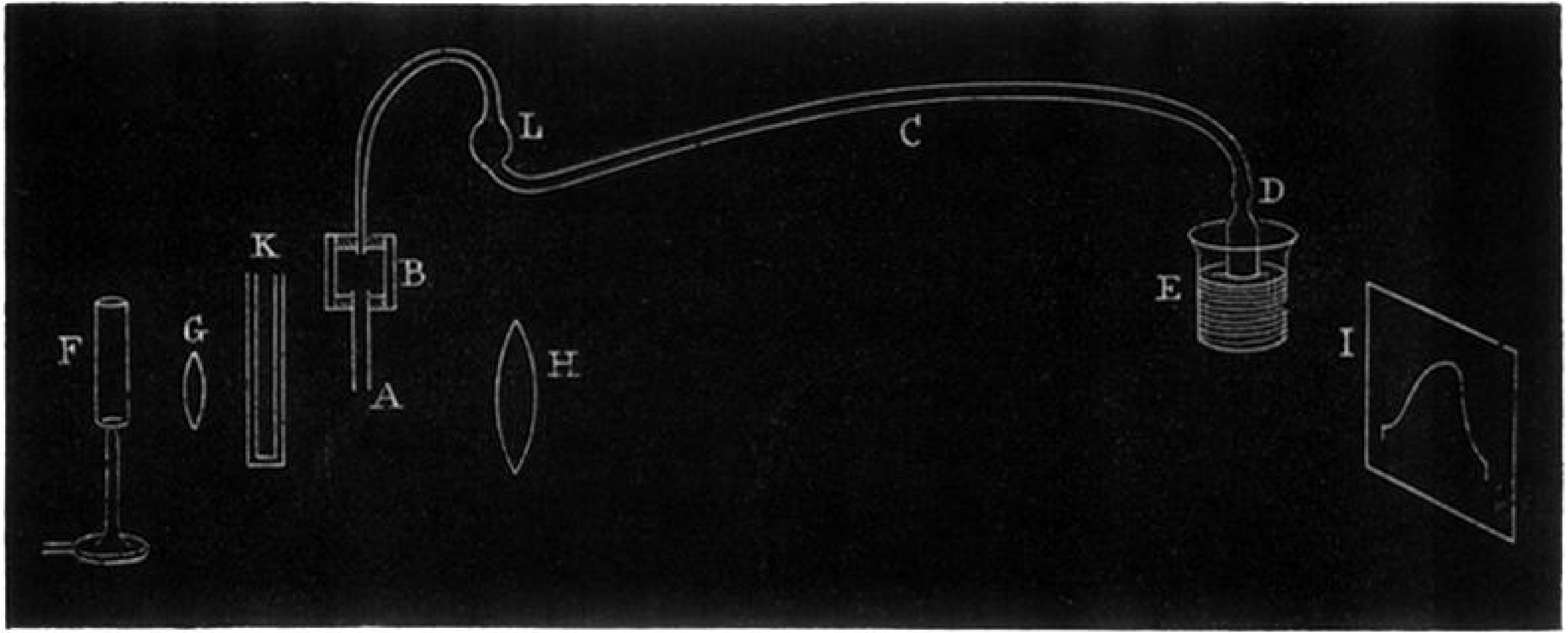


FIG. 2.

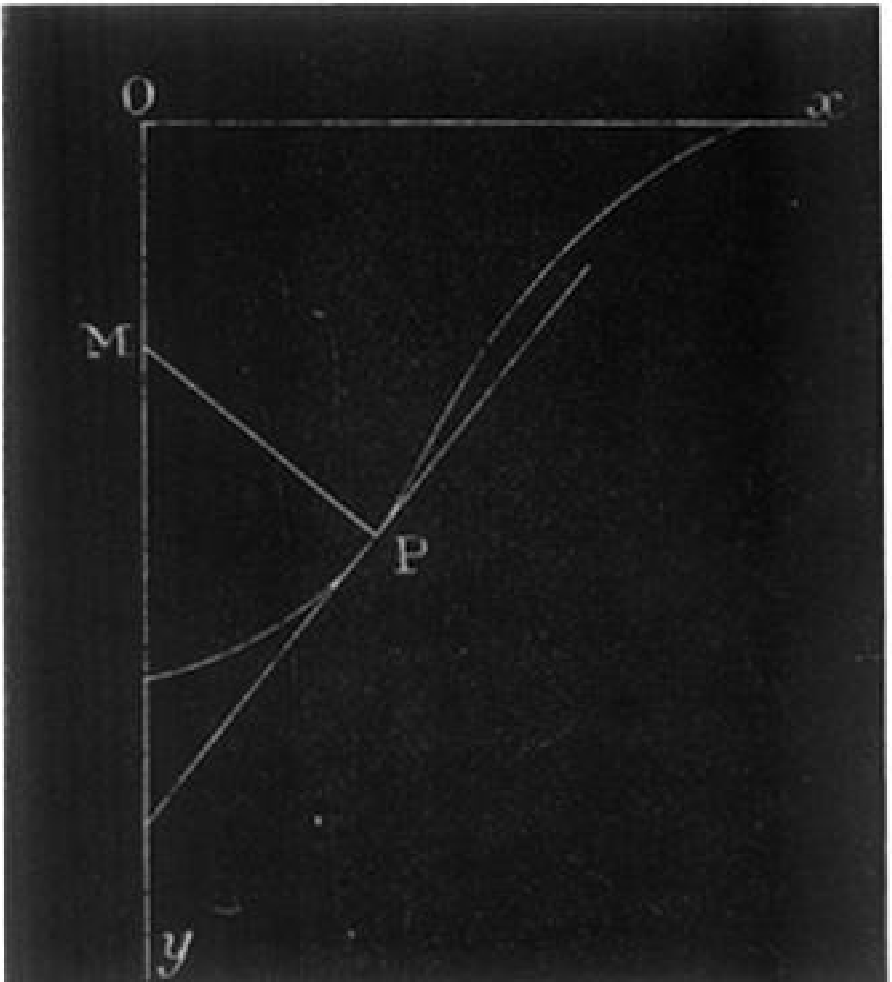
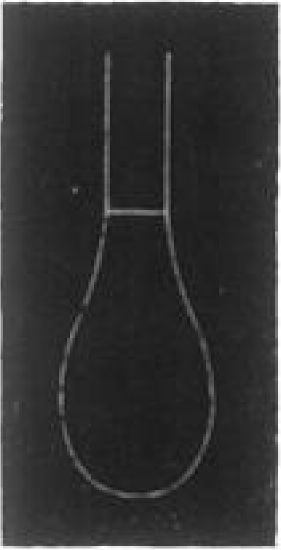
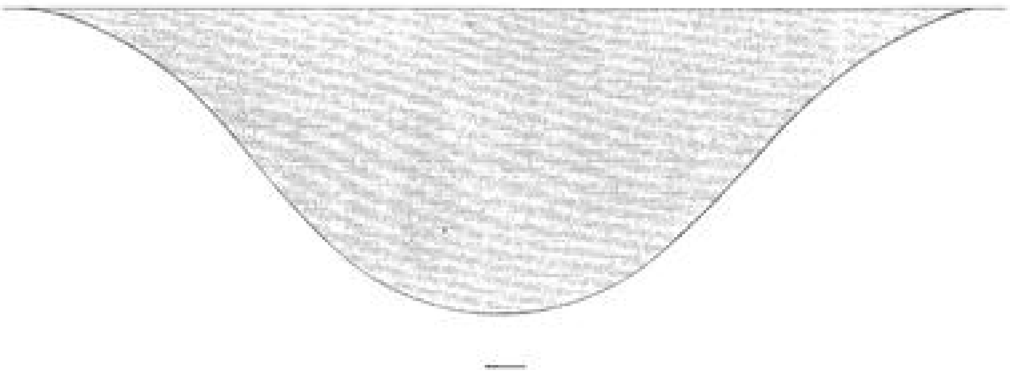


FIG. 3.

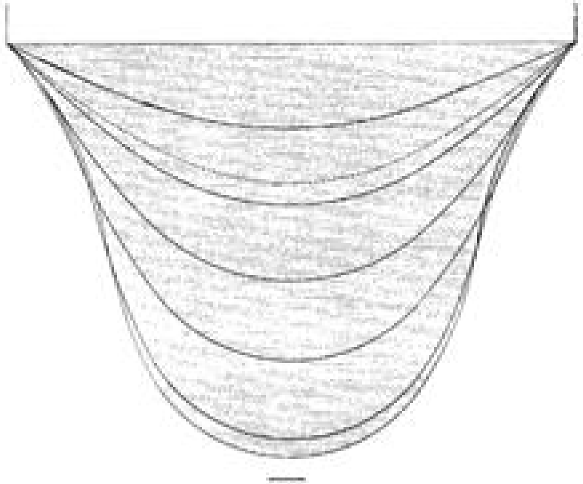


WATER.

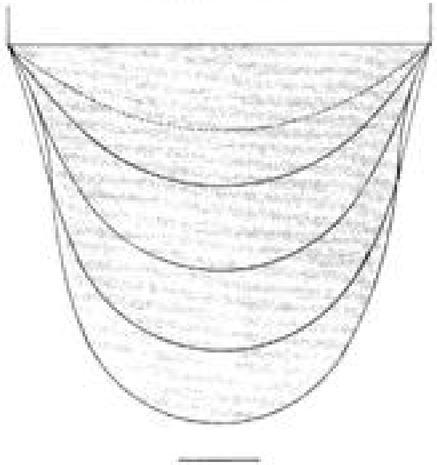
Indefinite plane (glass)



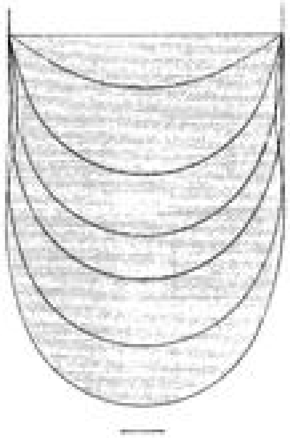
Diam. = ·9525 cm.



Diam. = ·724 cm.

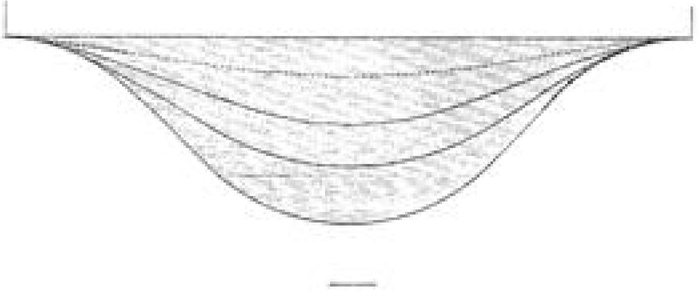


Diam. = ·448 cm.

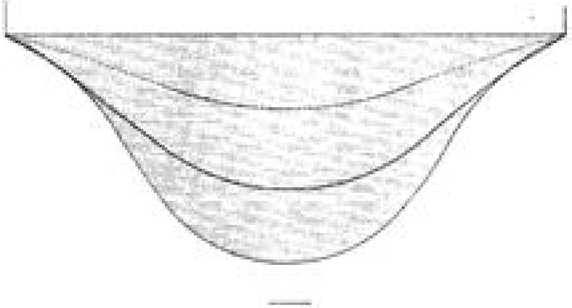


TURPENTINE (*Density* ·8679)

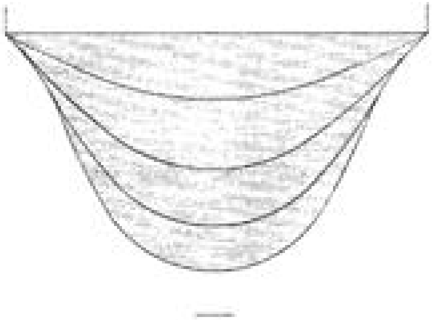
Diam. = 1.153 cm.



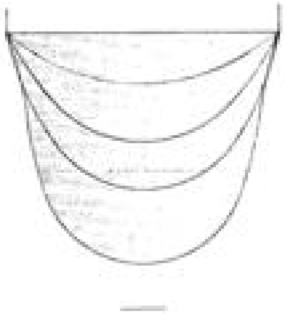
Diam. = ·9525 cm.



Diam. = ·7238 cm.



Diam. = ·448 cm.

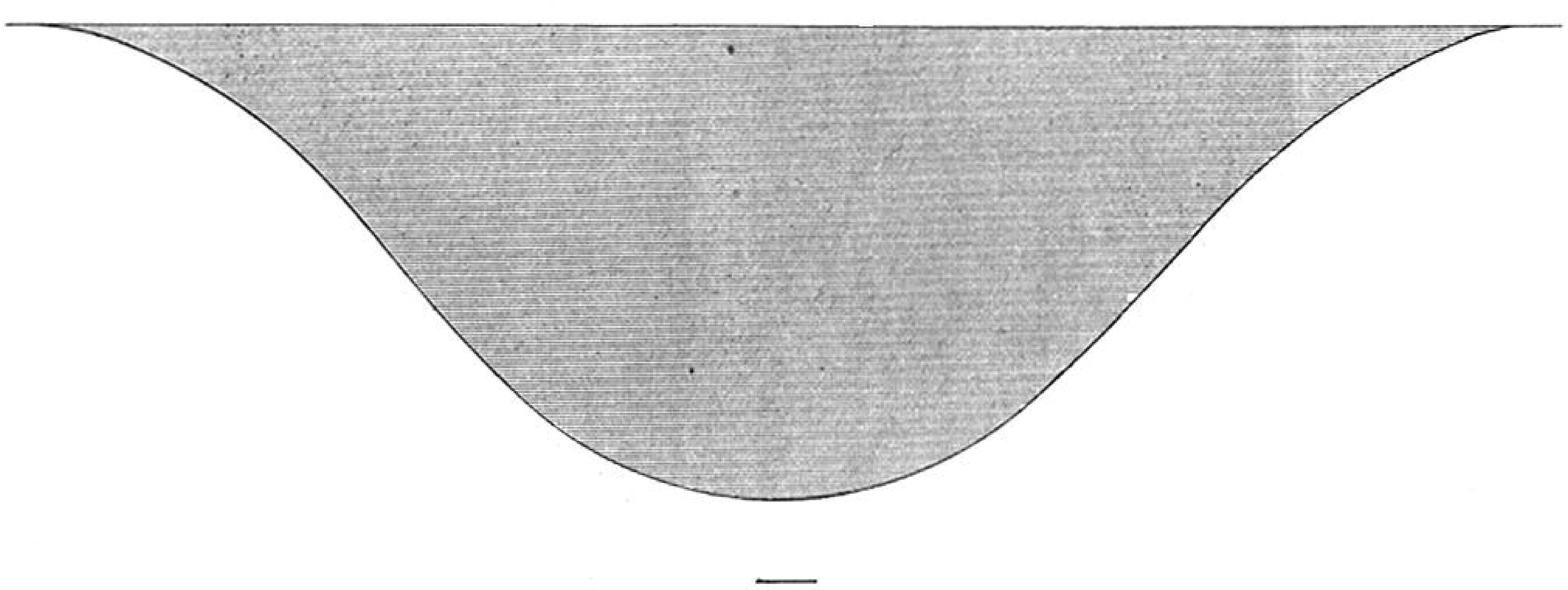


West Newman & Co. lith.

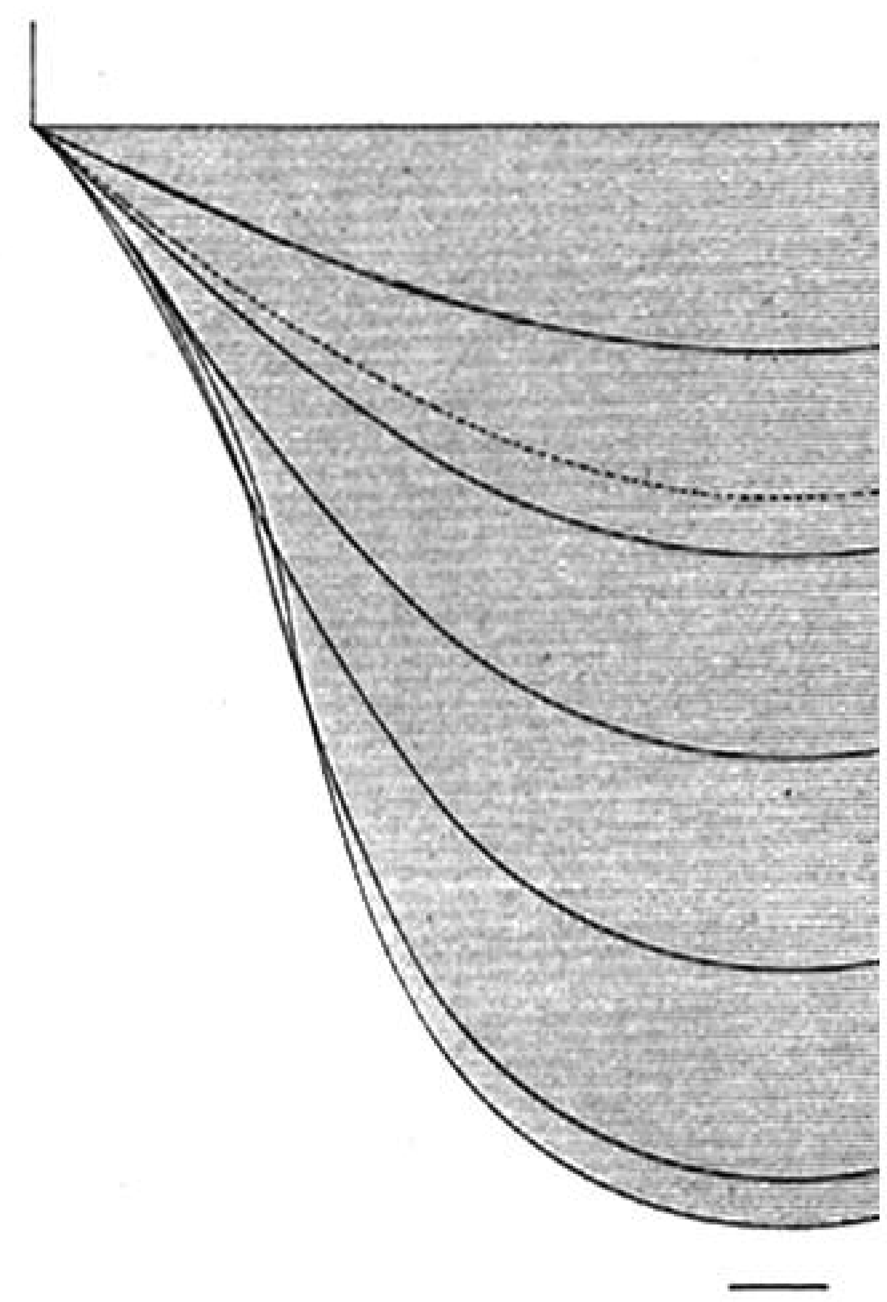
Worthington.

WATER.

Indefinite plane (glass)

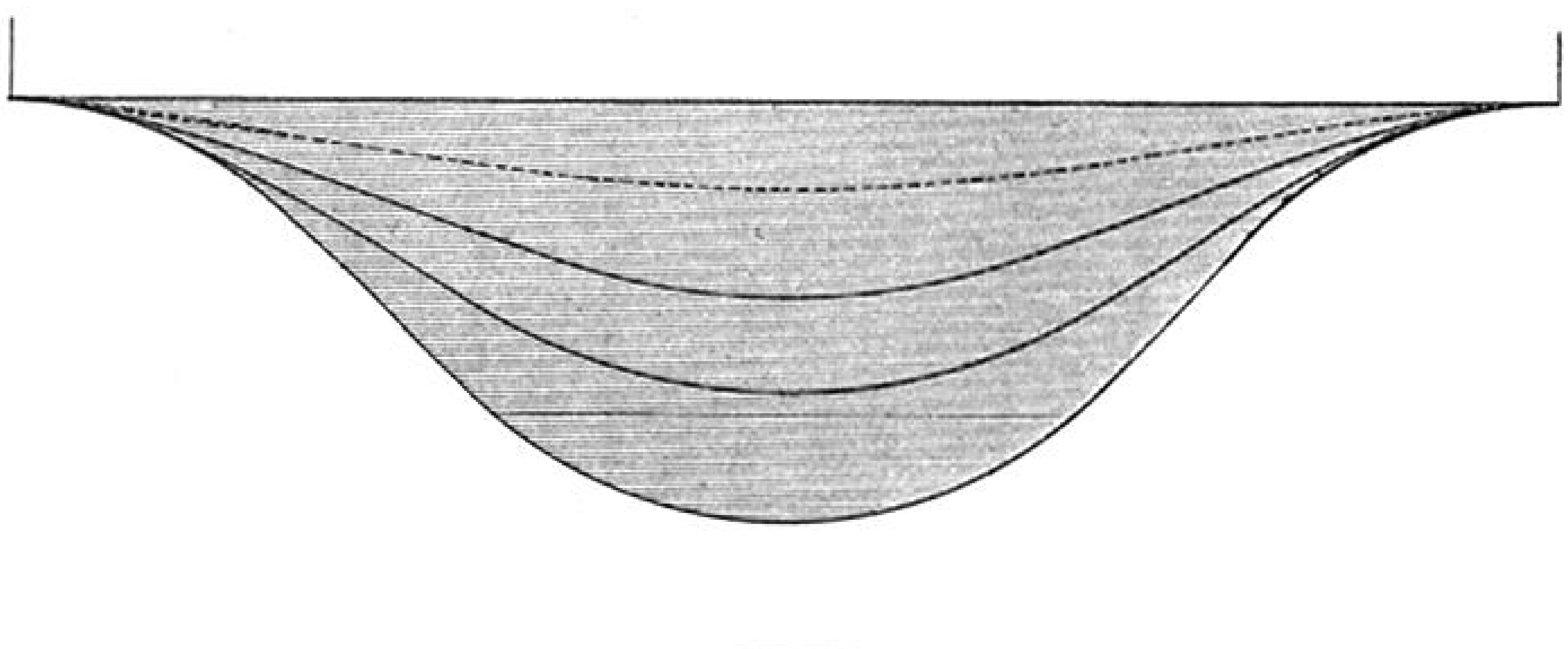


Diam. = ·

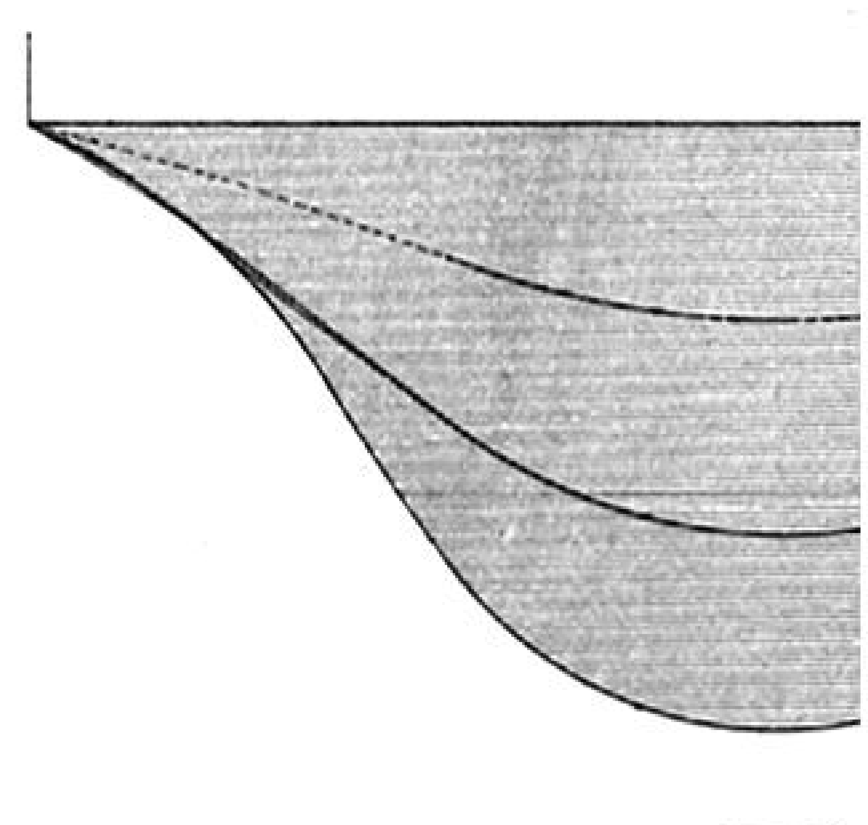


TURPENTINE (*Densit*

Diam. = 1.153. cm.



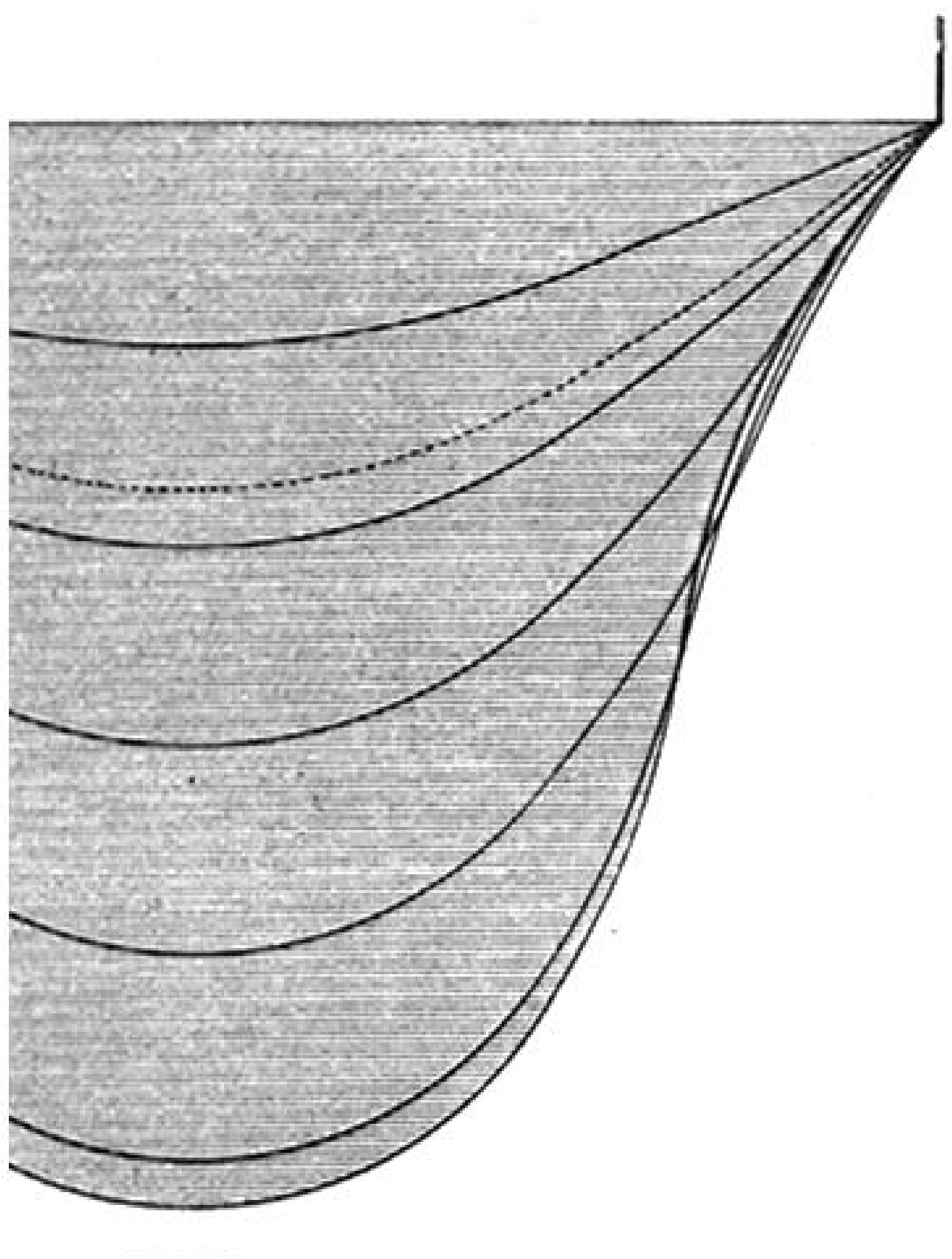
Diam. = ·



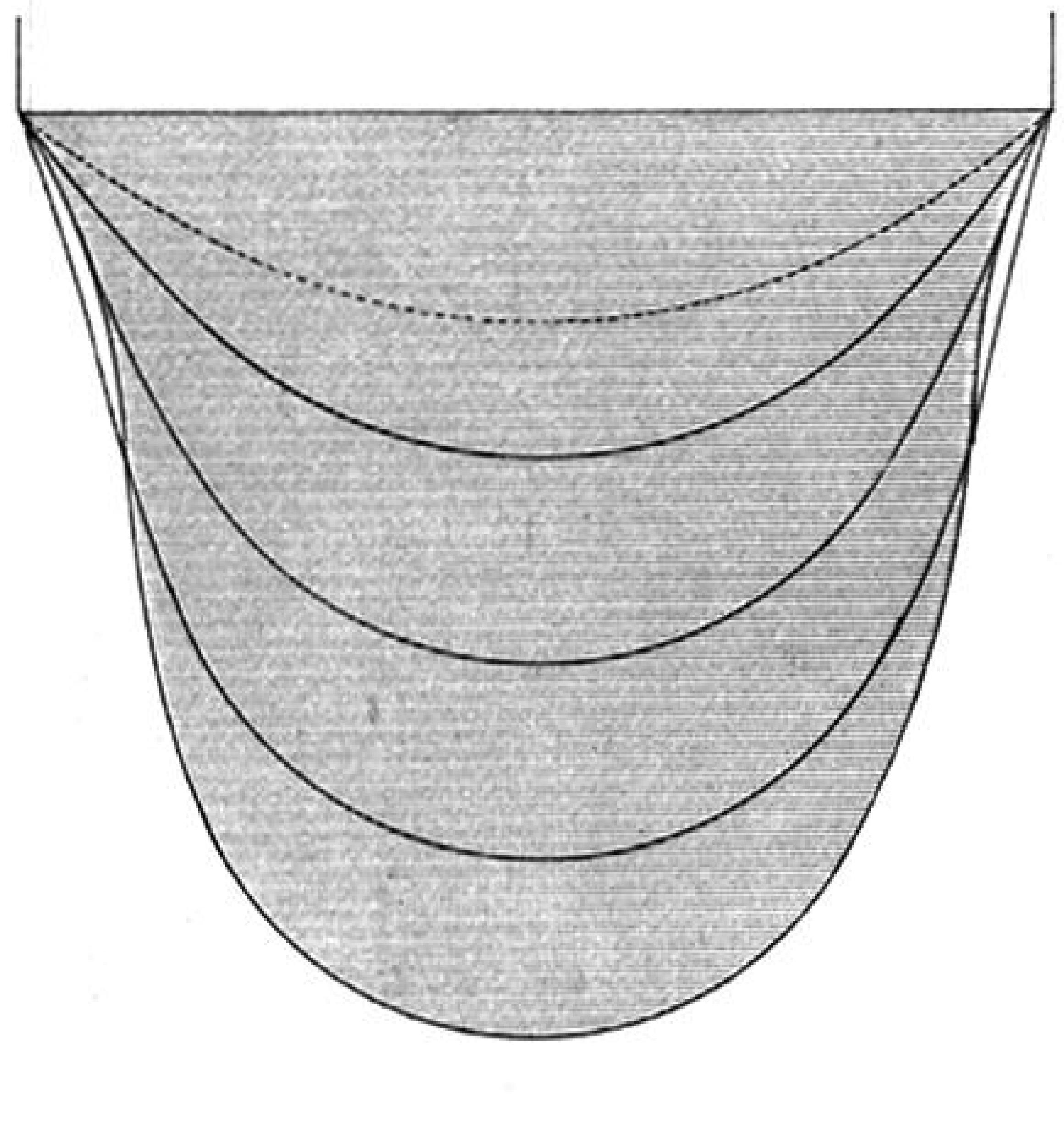
West, Newman & C^{o} lith.

R.

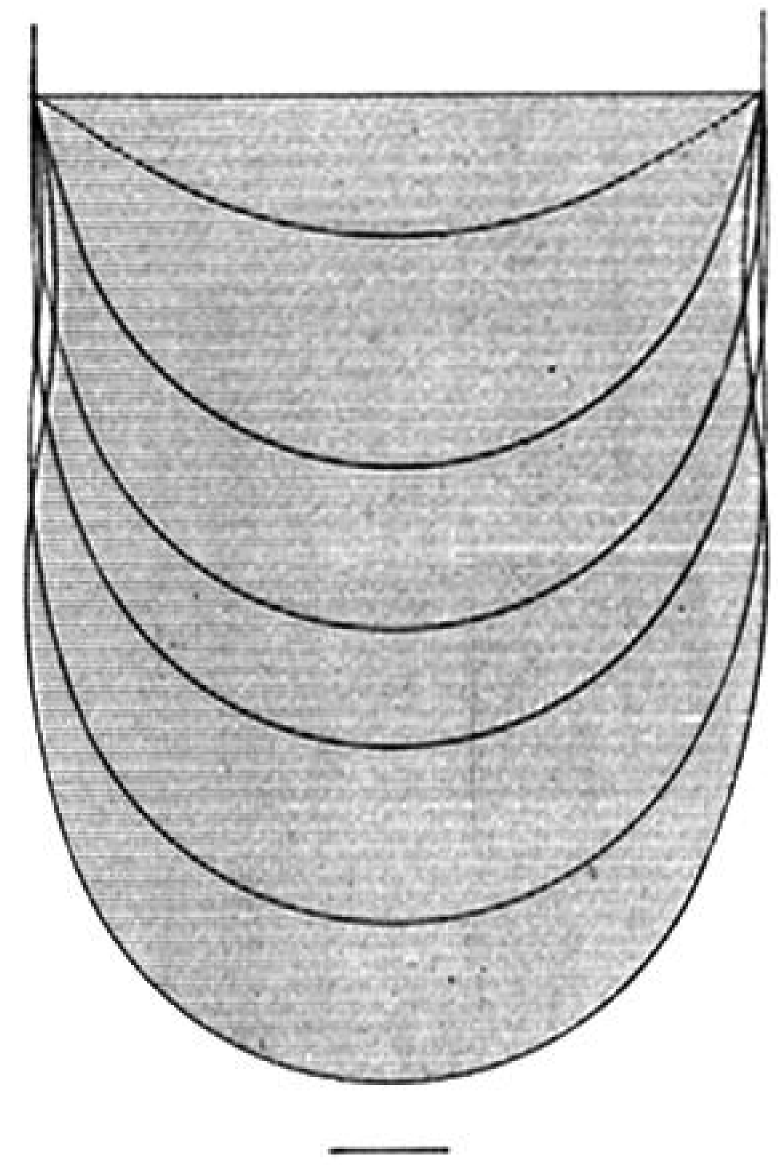
Diam. = ·9525 cm.



Diam. = ·724 cm.

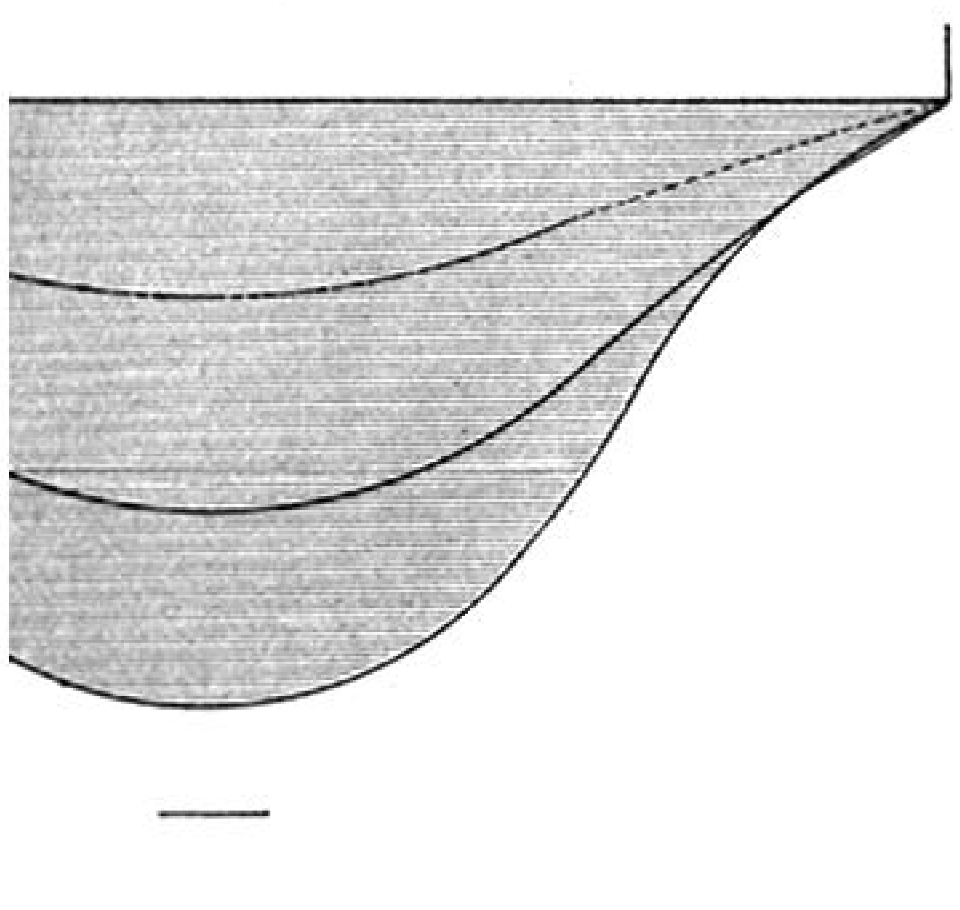


Diam. = ·448 cm.

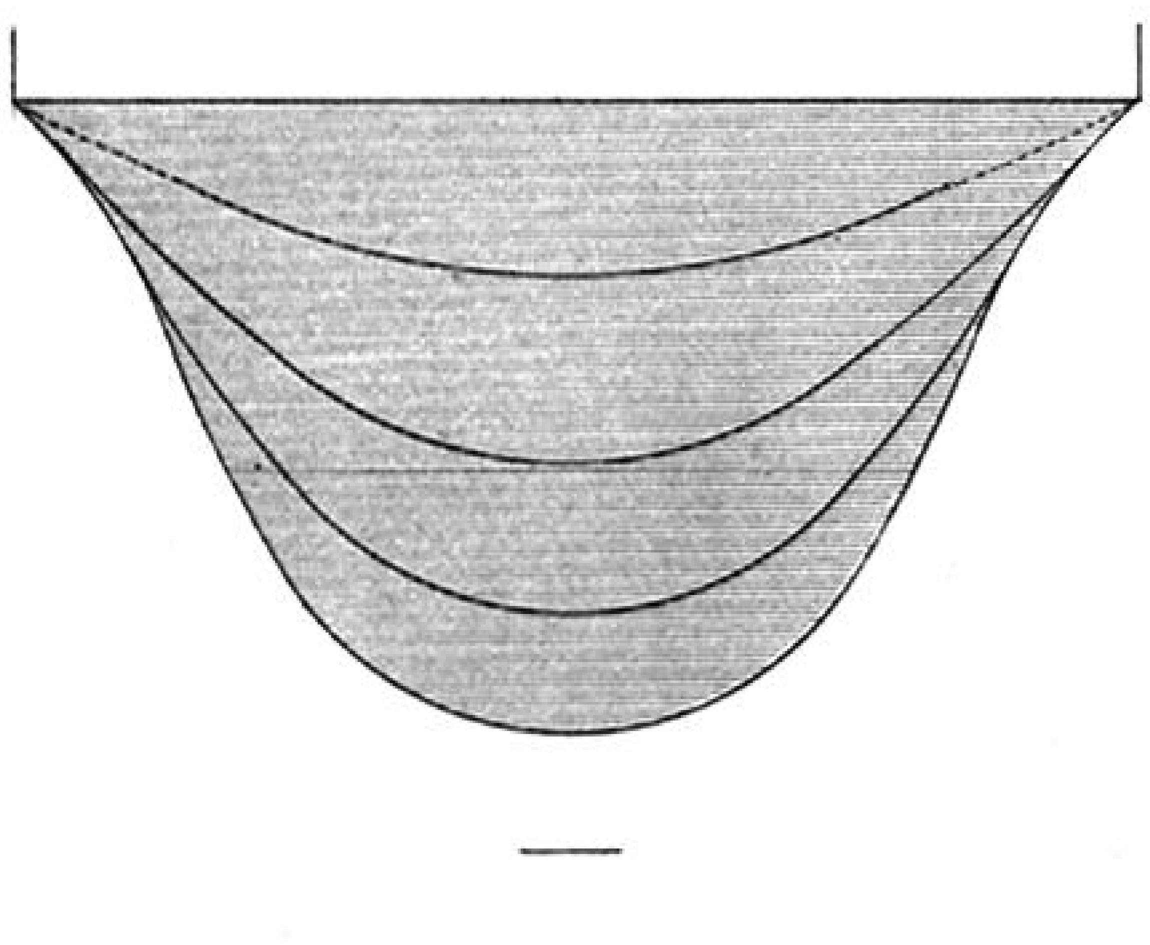


(Density. ·8679.)

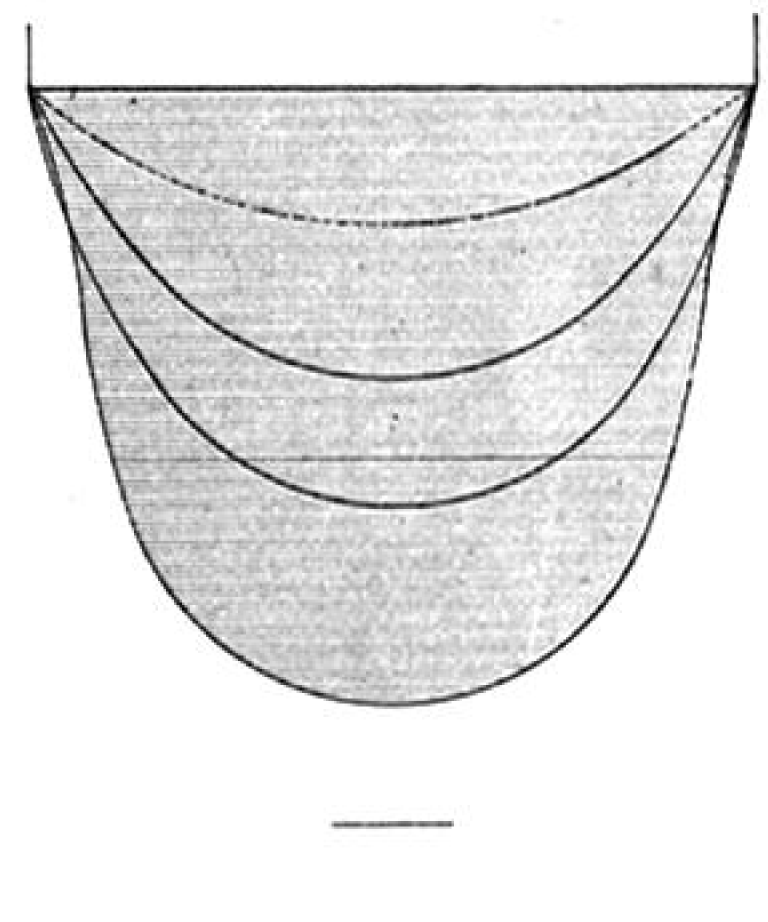
Diam. = ·9525 cm.



Diam. = ·7238 cm.



Diam. = ·448 cm.



ith.