when the magnetising force is so high as to give the wire the maximum magnetisation; thus confirming beyond all doubt what has been pointed out theoretically by Thomson ("Electrostatics and Magnetism," § 667) and indicated experimentally by Rowland.

III. "On Abel's Theorem and Abelian Functions." By A. R. Forsyth, B.A., Fellow of Trinity College, Cambridge, Professor of Mathematics in University College, Liverpool. Communicated by Professor Cayley, F.R.S. Received October 28, 1882.

(Abstract.)

The present paper is divided into two sections. The object of Section I is to obtain an expression for an integral more general than, but intimately connected with, that occurring in Abel's theorem. The latter, as enunciated by Mr. Rowe in his memoir in the Phil. Trans., 1881, is as follows:—If

$$\chi(x, y)=0$$

be a rational algebraical equation between x and y, then an expression can always be found for

$$\Sigma\int\frac{\mathrm{U}dx}{f(x)\dfrac{d\chi}{dy}}$$

where $f(x)$ is a function of x only, U is a rational algebraical integral function of x and y, and the upper limits of the series of integrals are the roots of the eliminant with regard to y of $\chi(x, y)=0$ and a function $\theta(x, y)$.

In the case here considered two equations of the degrees m and n respectively between three variables

$$\mathrm{F}_m(x, y, z)=0$$
$$\mathrm{F}_n(x, y, z)=0$$

are given (these alone being treated, as subsequent generalization to the case of r equations between $r-1$ dependent variables and one independent is obvious); and an expression is obtained for

$$\Sigma\int\frac{\mathrm{U}dx}{f(x)\mathrm{J}\left(\dfrac{\mathrm{F}_m,\ \mathrm{F}_n}{y,\ z}\right)},$$

the upper limits of the integrals being given by the roots of the equation arrived at by the elimination of x and y between F_m, F_n and an arbitrary equation

$$\mathrm{F}_p(x, y, z)=0,$$

or, what is the same thing, by the co-ordinates x of the points of intersection of the three surfaces represented by F_m, F_n, F_p.

Some preliminary considerations (in connexion with §§ 92 sqq. of Salmon's Higher Algebra) are adduced in reference to the eliminants of the three equations in each of the variables; thus if X be the equation in x obtained by eliminating z and y, it is expressed in the form

$$X = B_m F_m + B_n F_n + B_p F_p,$$

which afterwards proves useful. Then the ordinary case (above referred to) of Abel's theorem is treated on the lines laid down in Clebsch and Gordan's treatise on the Abelian functions; and under the guidance of this the more general form is investigated with the result

$$\Sigma \int \frac{U\,dx}{f(x) J\left(\dfrac{F_m, F_n}{y, z}\right)} = \Theta\left[\frac{1}{f(x)}\right] . \Sigma \left\{ \frac{U}{J} \log F_p \right\} + C,$$

Θ being the symbol introduced by Boole.

The remainder of the section is occupied with the discussion of two examples of this theorem. In Example I, by the assumption of suitable forms for F_m, F_n, F_p, it is proved that

$$E(u_1) + E(u_2) + E(u_3) - E(u_1 + u_2 + u_3) = -\frac{8k^2 ABC}{(A^2 + B^2 - k^2 C^2)^2 + 4k^2 A^2 C^2}$$

where E is the second elliptic integral and A, B, C are given by

$$\begin{aligned} As_1 + Bc_1 + Cd_1 &= 1, \\ As_2 + Bc_2 + Cd_2 &= 1, \\ As_3 + Bc_3 + Cd_3 &= 1, \end{aligned}$$

and s, c, d stand for $sn\,u$, $cn\,u$, $dn\,u$ respectively. The corresponding expression for the third elliptic integral is stated.

In Example II an expression is obtained for

$$E(u_1 + u_2 + \ldots . + u_7).$$

In Section II the addition theorem for the functions presented in Weierstrass's memoir in Crelle, t. lii (1856), p. 285, is investigated. It may be pointed out that the fundamental equations in the theory occur as natural examples of the more general form of Abel's theorem proved in Section I; but the equations so obtained are identical with those used by Weierstrass, and this case, therefore, does not belong distinctively to the form of Abel's theorem connected with the curve of double curvature. On this account the simpler form is used on the two occasions (in §§ 14, 19) when required.

The theory is worked out at considerable length, and the necessary

formulæ are obtained in a manner somewhat different from that of Weierstrass.

The fundamental equations being

$$y^2-\mathrm{P}(x)=y^2-(x-a_1)(x-a_2)\ \ldots\ldots\ (x-a_\rho)\ =0,$$
$$z^2-\mathrm{Q}(x)=z^2-(x-a_{\rho+1})(x-a_{\rho+2})\ \ldots\ (x-a_{2\rho+1})=0,$$
$$0=\mathrm{M}y+\mathrm{N}z$$

where

$$\mathrm{M}=x^\rho+\mathrm{M}_1x^{\rho-1}+\ \ldots\ +\mathrm{M}_{\rho-1}x+\mathrm{M}_\rho,$$
$$\mathrm{N}=\qquad \mathrm{N}_1x^{\rho-1}+\ \ldots\ +\mathrm{N}_{\rho-1}x+\mathrm{N}_\rho,$$

the equation giving the roots x is

$$\mathrm{M}^2y^2-\mathrm{N}^2z^2=0.$$

The 3ρ roots are denoted by $x_1, x_2, \ldots, x_\rho$; $\xi_1, \xi_2, \ldots, \xi_\rho$; $p_1, p_2, \ldots, p_\rho$; and there are obviously ρ relations between them. Writing

$$\mathrm{R}(x)=\mathrm{P}(x)\mathrm{Q}(x),$$

and

$$u_\mu=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\int_{a_\lambda}^{x_\lambda}\frac{\mathrm{P}(x)\,dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}\quad(\mu=1, 2, \ldots, \rho),$$

and v, w corresponding functions of ξ, p, it is shown that

$$u_\mu+v_\mu+w_\mu=0.$$

Writing, with Weierstrass,

$$\phi(x)=(x-x_1)(x-x_2)\ \ldots\ (x-x_\rho),$$
$$-\mathrm{Q}(a_r)=l_r\qquad(r=1, 2, \ldots, \rho),$$
$$\mathrm{P}(a_{\rho+s})=l_{\rho+s}\quad(s=1, 2, \ldots, \rho+1),$$

then $2\rho+1$ of the functions of the theory are given by

$$l_r al_r^2=\phi(a_r)$$

for values $1, 2, \ldots, 2\rho+1$ of r. Then if

$$\mathrm{U}=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\int_{a\lambda}^{x_\lambda}\frac{\mathrm{P}(x)\,dx}{\sqrt{\mathrm{R}(x)}},$$

it is proved that

$$\frac{l_\mu al_\mu^2}{\mathrm{P}'(a_\mu)}=-\frac{d\mathrm{U}}{du_\mu}.$$

If V, W are respectively the same functions of the ξ's and p's as U is of the x's, then the theorem

$$\mathrm{U}+\mathrm{V}+\mathrm{W}=\sum_{m=1}^{m=\rho}\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)al_m(v)al_m(u+v)$$

is obtained in § 21, a verification being furnished by expansion in

terms of the u's and v's. From this equation is deduced the addition-theorem for the functions.

In §§ 25 and 26 is given the discussion of a particular case of the above, viz., that in which the functions are of the order 2, the fifteen functions being the quotients of all but one of the double theta-functions by that one. The addition-theorem in these functions has already formed the subject of a paper by Cayley in Crelle, t. lxxxviii (1878), p. 74.

IV. "Note on the Recent and Coming Total Solar Eclipses." By J. NORMAN LOCKYER, F.R.S. Received November 17, 1882.

The following note has been drawn up in anticipation of the detailed accounts of the work done by me in Egypt on the eclipsed sun of 1882, May 17, which I am preparing to lay before the Royal Society, because as the next total eclipse occurs next May, there is no time to be lost if any attempt is to be made to secure observations, and I am of opinion that such observations are most important.

I have prefaced the statement of the work done by a reference to the considerations which led me to undertake it, and I have added a scheme of observations which, in the present state of our knowledge is, I think, most likely to produce results of value.

1. In order to understand the recent change of front in solar research which has followed the introduction of the view of the possible dissociation of elementary bodies at solar temperatures, and suggested the later laboratory, and especially the later eclipse observations with which we are now chiefly concerned, we must first consider what facts we may expect on the two hypotheses. In this way we can see which hypothesis fits the facts best, and whether there are any inquiries possible during eclipses of a nature to throw light on the question.

2. On the old hypothesis the construction of the solar atmosphere was imaged as follows :—

(1.) We have terrestrial elements in the sun's atmosphere.

(2.) They thin out in the order of vapour density, all being represented in the lower strata, since the solar atmosphere at the lower levels is incompetent to dissociate them.

(3.) In the lower strata we have especially those of higher atomic weight, all together forming a so-called "reversing layer" by which chiefly the Fraunhofer spectrum is produced.

3. The new hypothesis necessitates a radical change in the above views. According to it the three main statements made in paragraph 2 require to be changed as follows :—