Victoria (Queen) Alcune Pagine dal Giornale della Vita di S.M. la Regina Vittoria nell'Alta Scozia (1862–82). Traduzione di V. Brandi. 8vo. *Firenze* 1884. The Translator.

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Copy in Bronze of the Sylvester Medal published by the Johns Hopkins University, Baltimore.

"The Variation of Stability with Draught of Water in Ships." By F. ELGAR, Professor of Naval Architecture in the University of Glasgow. Communicated by Professor Sir WILLIAM THOMSON, F.R.S. Received March 6, 1884. Read March 13.

Of all the properties possessed by a ship none is more vital to her safety and efficiency than that of stability. At the same time none is dependent for its existence and amount upon so many, or such diverse and variable, circumstances as it. The stability of a ship is regulated and determined by her outward size and shape, and also by the separate amounts and positions of all the weights that go to make up her structure, equipment, and loading. No change of any kind can be made in dimensions or form, or in the quantities or distribution of the various items of equipment, stores, or cargo without affecting stability. It is, of course, essential to the safety of every vessel that her stability should not become reduced during all the changing conditions of her employment and career below a certain definite amount. The result of neglect in this respect may be a dangerous inclination or complete capsize when unlooked for, or exceptionally trying, emergencies occur. Deficient stability, whether caused by faulty design or stowage, may admit of a vessel being suddenly capsized by the action of the wind and waves, or of her being forcibly heeled to a dangerous angle of inclination by the shifting of some of her internal weights, such as coals or cargo. Although in every vessel there is a minimum limit below which it is not prudent or safe to diminish the stability, it does not follow that this limit is the same or similar in character in all sizes and types of ships.

It is not only necessary to guard against the stability of a ship becoming reduced below a safe minimum amount, but there is also a maximum limit which it should not be allowed to exceed. Excessive

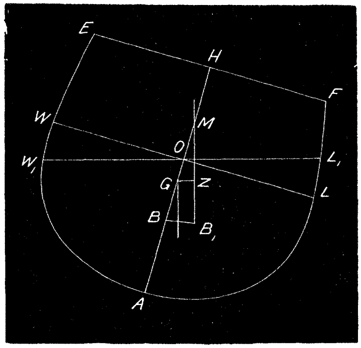
stability has its own peculiar objections and dangers. It causes heavy and uneasy rolling which may not only be uncomfortable and trying to passengers and crew, but at times sufficient to strain and even fracture portions of the structure, and displace or damage some of the fittings. The masts and rigging generally suffer most on account of their distance from the axis of rotation, and the change of motion in them at the end of each roll, being greater than that of any other part of the vessel; and also because their attachment to the hull is less direct and secure. The straining effect of too great stability often shows itself early, and in a marked manner, upon the masts and rigging of a vessel. Seamen frequently say of a ship that is laden with heavy cargo placed low down in the hold, that unless some of the weights are raised, *i.e.*, unless the stability is reduced, she will roll her masts out. The violent and deep rolling caused by excessive stability also tends to move over towards one side of the ship such cargoes as grain or coals which are free, to some extent, to shift as a whole; or such portions of other classes of cargo as admit of being displaced. Speaking generally, it may be said that, whereas large stability helps to prevent inclination to a great angle in the event of cargo shifting, the very possession of such large stability often increases the chances of shifting.

The stability of a ship at a given draught of water, and with a specific description of loading can readily be calculated, and it has become usual to make such calculations at the Admiralty and in a few of our leading mercantile shipyards. The practice is to construct a curve of metacentres, which shows how the height of the metacentre varies with draught of water, and is affected by such changes in the external bulk and form of the immersed portion of the hull as may be caused by increasing or diminishing the draught of water. A curve of stability is also constructed for one or more fixed draughts of water, and for certain intended, or estimated, weights and distributions of load. These curves show what the righting moments for such draughts of water and descriptions of loading amount to at successive angles of inclination from the upright up to 90°, or to the angle at which the stability vanishes.

The curve of metacentres is obtained by calculating the height of the metacentre at several fixed draughts of water, setting up those heights as ordinates of a curve whose abscissæ are proportional to the corresponding draughts of water, and drawing a fair curve through the points thus obtained. The term "metacentre" was originated by Bouguer, who published the first investigations into the subject in his "Traité du Navire," which appeared in 1746. The metacentre of a floating body is the point of intersection of a vertical line through the centre of gravity of the volume of displacement when the body is inclined through an indefinitely small angle from a

position of equilibrium, with the vertical through the centre of gravity of the displaced volume in the original position of equilibrium.*

FIG. 1.



For instance, if AEF in fig. 1 represent the transverse section of a ship upon which WL is the intersection of the water-line plane when the vessel is upright and in a position of equilibrium, and B the position of the centre of gravity of the volume of displacement WAL—or centre of buoyancy as it is commonly called; and if AH be the vertical through B when the vessel is floating in equilibrium at the water-line WL, then if the vessel be inclined through a small angle WOW_1, and the point B_1 represent the centre of buoyancy of the immersed volume W_1AL_1, the vertical B_1M, drawn through B_1, will intersect the original vertical AH in a point M. M is the metacentre when the angle of inclination WOW_1 is indefinitely small. It then represents the ultimate intersection of the new vertical through B_1 with the original vertical BH. In a ship the vertical corresponding to the upright position of equilibrium is, for all practical purposes, the intersection AH of the middle line longitudinal plane with the transverse section.

Bouguer showed that the position of the metacentre limits the height to which the centre of gravity of a floating body may be raised without making it unstable, and that the righting moments at small angles of inclination from a position of stable equilibrium are proportional to the height of the metacentre above the centre of gravity. This is readily seen, because if G in fig. 1 be the position of the centre of gravity of a ship, and GZ the horizontal distance between

* It is a moot question whether the term metacentre should not be made to embrace all the ultimate intersections of consecutive verticals through the centres of gravity of the displaced volume. Bouguer calls the locus of such intersections the *metacentrique*.

G and the vertical through the centre of buoyancy B_1, then $W \times GZ$ is the righting moment; W being the weight of the ship. But $GZ = MG \sin WOW_1$; and therefore when WOW_1 is indefinitely small, the righting moment is proportional to $W \times MG$. While G remains below M the moment is always a righting one, and tends to restore the ship to the upright position, which in this case is one of stable equilibrium; but if it be above M the tendency is to move the ship farther away from the upright till an inclined position of stable equilibrium be reached, or to capsize her. The curve of metacentres for a ship, which gives the height of the metacentre at all draughts of water, indicates therefore the limit above which the centre of gravity cannot be raised by changes in the amounts or positions of any of the weights without causing her to become unstable. Sufficient stability for practical working requirements and for purposes of safety can only be secured by taking care that at the various displacements and draughts of water a vessel may have in different conditions of loading, the centre of gravity is always kept at a proper depth below the corresponding points on the metacentric curve.

Such instability as may be due to deficiency, or absence, of metacentric height is not necessarily dangerous, and may not be sufficient to cause a complete capsize. It will, of course, cause the vessel to incline away from the upright, but a position of stable equilibrium may soon be reached; and the righting moments at greater inclinations, and the range of stability, beyond that point may be so large as to put all danger of upsetting out of the question if there are no openings through which water may find its way inboard, and no large weights free to shift. Many ships are in this condition when light, and some approach it when laden. On the other hand, there are vessels in which small metacentric height involves a serious risk of capsizing.

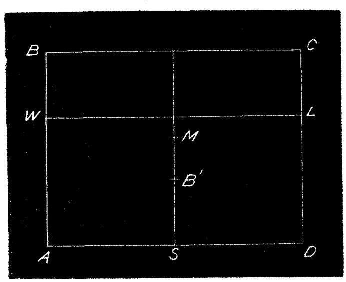
No fixed distance of centre of gravity below metacentre, or metacentric height, as it is commonly termed, can be adopted as a standard for application to all ships, because such a measure of stability is very imperfect and insufficient, and may by itself be misleading. This is chiefly due to the reasons that the form and proportions of the above-water part of the ship are not taken into consideration in the metacentric calculations, and the under-water form not completely so; and that the value, as a general measure of stability, of a given metacentric height is largely affected by the absolute heights in a ship of the centre of gravity and metacentre. The initial stability is, of course, constant for a given metacentric height whatever may be the absolute positions of the points M and G, but the righting moments at moderate and large angles of inclination, and the angle at which such righting moments vanish, or change into upsetting

moments, are largely dependent upon the absolute position of the latter point. Speaking generally, it may be said that, keeping the same distance between the metacentre and centre of gravity, the righting moments at successive angles of inclination, and the range of stability, are increased by lowering these points and reduced by raising them.

Experience proves that some classes of vessels are as safe and seaworthy in respect of stability with 1 foot, or even less, of metacentric height, as others are with 3 or 4 feet; while some of an exceptional character require much greater stability than even the latter figures would give. Examples of this class are to be found among ironclad monitors of very low freeboard and with heavy upper works, including armoured turrets and guns on deck; and also among paddle steamers of extremely light draught, with extensive tiers of houses above them. These are cases in which the metacentre and centre of gravity are both comparatively high in the ship.

The vertical position of the metacentre of a floating body is determined by the consideration that the height of the metacentre above the centre of gravity of the volume of displacement is equal to the moment of inertia of the plane of flotation about a longitudinal axis through its centre of gravity divided by the volume of displacement.

Fig. 2.



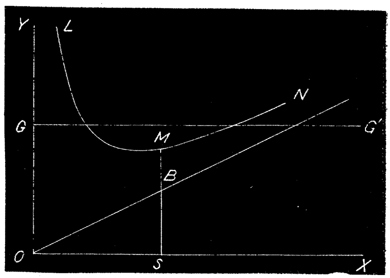
If ABCD (fig. 2) represent the transverse section of a rectangular prismatic body which floats in equilibrium with the side BC horizontal, WL the line of flotation, B′ the position of the centre of gravity of the displaced volume or centre of buoyancy, and M that of the metacentre, then by Bouguer's well known formula

$$B'M = \frac{\text{moment of inertia of plane of flotation WL}}{\text{volume WADL}}.$$

Let AB$=a$, BC$=b$, and WA, the depth of flotation$=d$. The moment of inertia of an unit of length of the water-line plane

$WL=\frac{b^3}{12}$, and the volume of displacement of an unit of length of the prism $=bd$. Therefore $B'M=\frac{b^2}{12d}$. But $B'S=\frac{d}{2}$, and MS is therefore equal to $\frac{b^2}{12d}+\frac{d}{2}$. If a curve LMN (fig. 3) be constructed whose

FIG. 3.



abscissæ represent the various depths of flotation of the body, and ordinates the corresponding heights of the point M, this will be the curve of metacentres. OS is equal to the depth of flotation WA in fig. 2; BS is equal to the corresponding height of the centre of buoyancy B′ above S; and MS is equal to the height of the metacentre above S. It is obvious that the locus of the centre of buoyancy OB is a straight line whose equation is $y=\frac{x}{2}$. The equation to the curve of metacentres is $\frac{b^2}{12x}+\frac{x}{2}=y$, or $6x^2-12xy=-b^2$. This curve is therefore an hyperbola whose asymptotes are the axis OY,—which corresponds with the lower side AD of the section of the floating body in fig. 2,—and the straight line OB, which is the locus of the centres of buoyancy. The curves of metacentres for various geometrical forms of floating bodies possess many interesting properties, but it is foreign to the purpose of this paper to enter upon a full discussion of them. It may, however, be noted, as additional illustrations of these, that the ordinate of the metacentric curve at zero, *i.e.*, the one corresponding with no draught of water, is the radius of curvature of the transverse section of the floating body at its lowest point. Thus, for a body of circular section the height of the metacentre at the point where the draught vanishes is equal to the radius of the circle; for one the lowest point of whose section is angular, it is zero; and for one the bottom of whose section is straight and is parallel to the water-line, it is infinite.

When the body is completely immersed the metacentre is identical

with the centre of buoyancy, but if the upper surface is bounded by a plane which is parallel to, and coincident with, the water-surface, the curve of metacentres does not, when produced, end in the centre of buoyancy, as may be seen by fig. 3. The immersion of the plane of the upper surface BC causes a point of discontinuity in the curve of metacentres, which drops at once to the curve of centres of buoyancy. Curves of metacentres are given in fig. 7 for prismatic bodies of triangular and elliptical sections, and for a similar body the lower half of whose section is elliptical and the upper half rectangular (figs. 4, 5, and 6). The section in fig. 4 is an isosceles triangle with the base upwards and horizontal. In figs. 5 and 6 the major axis of the ellipse is horizontal. A comparison of the metacentric curves in fig. 7 will show how they are affected by changes in the form of the floating body. In the case of the triangular section the curve of metacentres is a straight line which passes through the immersed angle of the triangle.

FIG. 4.

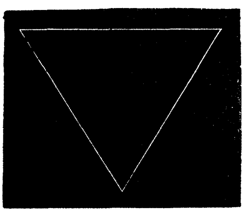


FIG. 5.

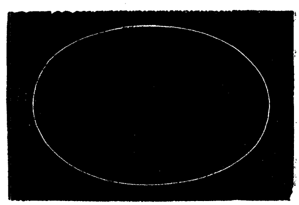
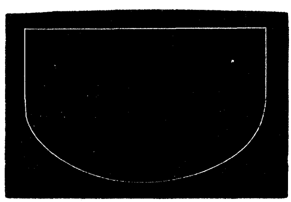


FIG. 6.



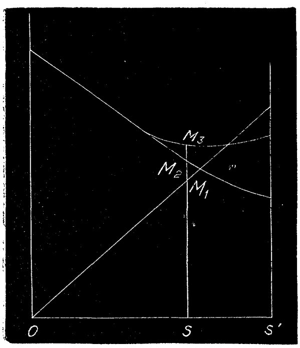
If OS represent any draught in fig. 7, then M_1, M_2, and M_3 are the positions of the metacentres at that draught for the three bodies under consideration; OS′ being the draught at which they become completely immersed.

If the rectangular floating body shown in fig. 2 be homogeneous, and the changes in its depth of flotation be caused by merely altering the density throughout, or by otherwise altering its weight so that the position of the centre of gravity remain the same, the latter will always be in the centre of the body. The locus of the centre of

P 2

gravity will therefore appear in fig. 3 as a straight line parallel to the axis OX. Let GG′ be this line. The equilibrium of the floating body will best able in the upright position for those depths of flotation at which the ordinates to the curve of metacentres LMN are greater than OG, and unstable when they are less. At the points where the curve LMN intersects GG′ the equilibrium will be neutral.

FIG. 7.



Triangular section.

Section whose upper half is rectangular and lower half elliptical, as in fig. 6.

Elliptical section.

In actual ships the locus of the centre of gravity is not a straight line such as GG′, any more than the curve of metacentres is a hyperbola like LMN; and the fundamental difference exists between them in practice, that whereas the curve of metacentres is constant for a ship, the locus of the centre of gravity is very variable in its character. The diagram in fig. 3 may serve, nevertheless, to illustrate the nature of the problem that has to be dealt with in investigating the stability of ships. The curve of metacentres for a ship can be readily constructed by applying Bouguer's theorem, viz., height of metacentre above centre of gravity of displaced volume$=\frac{\frac{2}{3}\int y^3dx}{\text{V}}$, where y is the half-ordinate of the plane of flotation and V the volume of displacement. The integration of y^3dx is effected by one of Simpson's rules; and the volume of displacement and corresponding height of centre of gravity of displaced volume are computed by the same means. The curve of metacentres when once constructed is the same for all conditions of the ship, as it can only be altered by changes in her dimensions or form. In this important respect it differs entirely from the locus of the centre of gravity.

The locus of the centre of gravity of a ship is usually very irregular, and is neither fixed in character nor position. It varies with different weights and descriptions of loading; and, unlike the curve of metacentres, its ordinates cannot always be expressed in terms of the

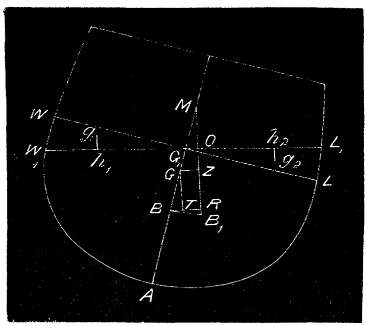
depth of immersion. What is usually done in practice, is to ascertain the position of the centre of gravity for such conditions of loading as may often be expected to occur; or to determine the limits between which it will lie in certain extreme circumstances. This variable and uncertain factor, of height of centre of gravity, is one of the greatest practical difficulties that have to be overcome in fully or accurately determining and regulating a ship's stability. The difficulty is not the same in all cases. In war-ships, for instance, and many which merely carry passengers and light baggage, the centre of gravity may either be regarded as fixed, or to vary with the draught of water in a specific manner which may be readily determined. On the other hand, many mercantile vessels employed in the carrying trade are laden with cargoes which, together with the coals that are required for consumption upon a voyage, weigh twice their own total weight. In such cases, the position of the centre of gravity of the laden ship is largely dependent upon stowage, and the stability may be entirely dependent upon it. The consumption of coal at sea introduces a cause of variation in position of centre of gravity, and metacentric height, which operates during a voyage; so that the stability may be materially altered after a steamer has left port, by reason of the consumption of coal. Such alteration frequently has the effect of diminishing the stability: and there are cases of steamers whose gross weight when fully laden is reduced at sea from this cause by over 25 per cent., and some in which the metacentric height is reduced by $1\frac{1}{2}$ feet. These large changes in the amount and distribution of a ship's weights—some of which take place at sea—sometimes make the problem of regulating the stability of a ship, so as to prevent its ever becoming deficient or excessive during her voyages, a very difficult and extremely delicate one.

The curve of metacentres and the positions of the centre of gravity for all possible draughts of water and conditions of loading are not sufficient, when obtained, for enabling the condition of a ship in respect of stability to be completely determined. Atwood showed, in two papers communicated to the Royal Society in 1796 and 1798, that much more than this is required. In his second paper, read on the 8th March, 1798, he says, "M. Bouguer, in his treatise entitled 'Traité du Navire,' has investigated a theorem for estimating the exact measure of the stability of floating bodies. This theorem, in one sense, is general, not being confined to bodies of any particular form; but in respect to the angles of inclination, it is restrained to the condition that the inclinations from the upright shall be evanescent, or, in a practical sense, very small angles. In consequence of this restriction, the rule in question cannot be generally applied to ascertain the stability of ships at sea, because the angles to which they are inclined, both by rolling and pitching, being of considerable magnitude, the

stability will depend, not only on the conditions which enter into M. Bouguer's solution, but also on the shape given to the sides of the vessel above and beneath the water-line or section, of which M. Bouguer's theorem takes no account." It may be added that Bouguer's theorem also neglects to take into account the volume of the above-water part of a ship, and to some extent the form of the below-water part; as well as the absolute height of the centre of gravity, which has been already referred to.

Atwood lays down a general theorem for determining the righting moments, at any required angles of inclination, possessed by a ship having a given draught of water and a fixed height of centre of

Fig. 8.



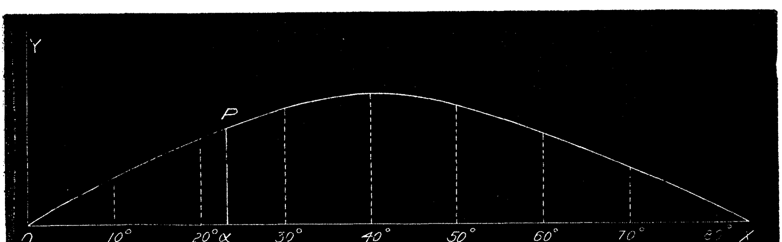
gravity. It is the following:—Let fig. 8 represent the transverse section of a ship which is inclined to an angle $WOW_1 = \theta$ from the upright water-line WL. Let G be the position of the centre of gravity; B the centre of gravity of the volume of the displaced fluid—or centre of buoyancy—when upright; and B_1 the position to which it has been moved by the inclination of the vessel through the angle θ. The original form of the under-water volume WAL has now been changed by the addition of the wedge-shaped piece LOL_1 and the deduction of the wedge-shaped piece WOW_1. The volumes of these wedges must be equal, because the displacement has not been changed by the mere act of inclination. Let g_1 be the centre of gravity of the wedge WOW_1, g_2 that of LOL_1, and v the volume of each wedge. Then the horizontal shift, BR, of the centre of buoyancy $\times$ the volume of displacement, or $V \times BR, = v \times h_1 h_2$. But $BR = GZ + BG \sin\theta$, and therefore $GZ = \frac{v}{V} \times h_1 h_2 - BG \sin\theta$.

This is the formula by which the stability of a ship at various angles of inclination is ordinarily computed; GZ being the arm of the couple at the ends of which the weight of the ship and the

upward pressure of the water act, either to restore the vessel to the upright position or to produce further inclination. The factors v, V, and $h_1 h_2$ are readily calculated from the external dimensions and form of the ship by means of Simpson's rules; the position of the point B, the centre of buoyancy in the upright position, is similarly obtained; and G is either determined by experiment, or by calculating in detail the weights and statical moments of the component parts of the structure and loading. Mr. F. K. Barnes, one of the present Chief Constructors of the Navy, read a paper before the Institution of Naval Architects, in 1860, in which he showed how the requisite calculations could be made concisely and with facility.

Notwithstanding Atwood's demonstration of the imperfect and unreliable standard of stability furnished by mere metacentric height, and his theorem for enabling the righting moments at large angles of inclination to be determined, the step which it may now appear would naturally follow was not actually taken till 1867. In that year a question arose at the Admiralty respecting the stability of some low freeboard monitors at very large angles of inclination; and Sir E. J. Reed, then Chief Constructor of the Navy, directed the matter to be investigated. It was placed in the hands of Mr. William John, who made the calculations, and embodied them in the graphic form now known as the curve of stability.

Fig. 9.



Thus in fig. 9, if OX be an abscissa line, upon which the various equal divisions represent angles of inclination of a ship, and any ordinate, such as Pα, be the length of GZ (see fig. 8) at the angle of inclination α, OPX will be the curve of stability for the particular draught of water and position of centre of gravity under consideration. The results of Mr. John's calculations were described in a paper read by Sir E. J. Reed before the Institution of Naval Architects in 1868; and a further paper containing an improved method of applying Atwood's theorem to the calculation of stability upon this extended scale, was read before the same Institution by Messrs. John and White, in 1871.

The curve of stability, as thus constructed, has been in common use at the Admiralty, and in a few of our leading mercantile ship-yards for some years. The loss of H.M.S. "Captain," by capsizing at sea, furnished the impetus which led to the practice of producing stability information in this complete and instructive form, becoming established at the Admiralty. Many losses have occurred of late years in the mercantile marine from a similar cause, and forcibly directed the attention of mercantile naval architects to the same point. Curves of stability have been constructed for large numbers of vessels of various classes, many of which have been published; and the general character of a ship's stability can now be judged of with much greater accuracy than was possible a few years ago. It appears that prior to 1867 no calculations had been made which showed how the stability of a ship became affected by inclining her till the water-line came up over the deck; or at what angle the stability vanished. Messrs. John and White say in the paper before referred to: "The metacentric stability, as it was termed, was by general consent taken as a sufficiently good standard of comparison, and no approximation was made, nor any great importance attached to the angle of inclination when a ship ceased to be stable. It was very generally known that up to very considerable angles of heel, the stability of high-sided ships continued to grow, even more rapidly than it appeared to do from the metacentric method, and the vague impression that the angle would be very large at which the ship became unstable, was considered sufficient to render investigation needless."

The investigations conducted at the Admiralty into the stability of war-ships of low freeboard, and those made by naval architects outside of the Admiralty into the condition of deeply laden merchant steamers of low freeboard and with high centres of gravity of cargo, prove that the mere application of the metacentric method may often lead to a false sense of security being established respecting the stability of certain types of vessels.

H.M.S. "Captain" had a metacentric height of about $2\frac{1}{2}$ feet when laden, which—in the absence of definite information, at that time most unusual and not considered absolutely necessary, respecting the righting moments at large inclinations, and the angle at which the stability vanished—was not supposed to be insufficient. Some of our low freeboard monitors and deeply laden merchant vessels with flush decks and low freeboards, have metacentric heights which, by themselves, furnish no clue to the rapidity with which the stability diminishes after a moderate angle of inclination has been passed; or the smallness of the angle at which it vanishes. The introduction of curves of stability, and the extent to which they have been applied in practice, have led to a due and precise appreciation being formed, by many, of the dangers attendant upon low freeboard. The

opinions previously existing upon the point, which were usually based upon mere surmise or vague impression, and often influenced by prejudice, can now be corrected by means of exact and conclusive data. The effect of low freeboard upon stability has latterly been largely made known and discussed.

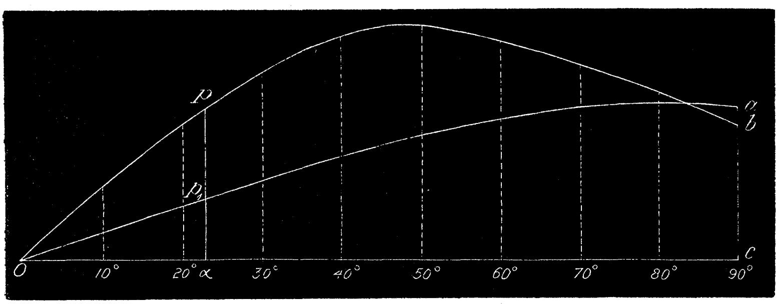
It has been seen that the mere act of setting off the righting moments obtained by Atwood's formula, at various angles of inclination, as ordinates of a curve, and thus obtaining a complete graphic representation of the variation of righting moment with angle of inclination, threw a flood of light upon the general problem of a ship's stability, and enabled it to be far more comprehensively and accurately treated than before. It also enabled definite and instructive generalisations to be made which were previously impossible. It will be observed, however, that the curves of stability referred to only deal with the matter so far as certain fixed draughts of water and positions of centre of gravity are concerned. A curve of stability for a given draught of water and position of centre of gravity ceases to be applicable if changes are made in the weight and consequent draught of water of a ship, or the position of the centre of gravity, or both.

In the case of war-ships, curves of stability are usually constructed for three conditions. 1st, for the draught of water and position of centre of gravity of the vessel when fully laden; 2nd, the same, but with all coal consumed; and, 3rd, when quite light, and without any coal, ammunition, or consumable stores. In certain cases there may be special conditions for which additional curves are required, but usually the above are all that are considered necessary. The stability at intermediate points is not often supposed, or found in practice, to vary sufficiently to call for further attention. In mercantile steamers, however, which are often launched in a very light condition and are constructed for carrying heavy cargoes, and large supplies of coal, the question of stability requires a more exhaustive, and somewhat different, mode of treatment. It becomes necessary, in fact, to make another step in the same direction as was taken when the righting moments for various angles of inclination, described by Atwood, were represented as a whole for a fixed draught of water and loading by means of a curve of stability. What is still further required to complete the representation of a ship's stability is to show how the curves of stability themselves vary with draught of water and position of centre of gravity, and to record this variation in a form that will render it easy to construct curves of stability for any specific draughts of water and positions of centre of gravity.

The curves of stability of merchant steamers are required for so many different draughts of water and positions of centre of gravity, that I have found it convenient in practice, after obtaining curves

for the extreme light and loaded conditions, and one or two intermediate ones, to construct cross-curves, from which the ordinates of a curve of stability at any draught between the two extreme ones can be readily measured off. In order to do this, the curve of stability already described has to be analysed, and dealt with under a different form. The ordinary curve of stability, as illustrated in fig. 9, is a curve showing the variation in the length of GZ in fig. 8—or the arm of the inclining couple—with the angle of inclination. Pα in fig. 9 is the length of GZ in fig. 8, at the angle of inclination α. But $GZ=BR-BT=BR-BG\sin\theta$. Therefore Pα in fig. 9 is the difference of these two quantities at the angle of inclination α. The curve of stability, or curve of GZ, is a curve whose ordinates are equal to the differences between the corresponding ordinates of two curves, one of which represents the variation in the length of BR with change of inclination, and the other is a curve of sines with

Fig. 10.



radius BG. Thus if Opb, fig. 10, be a curve, of which the ordinates, such as $p\alpha$, are equal to the length of BR at the angle of inclination α; and Op_1a a curve of sines from 0° to 90° with radius ac=BG; then the differences between the ordinates of these curves will give the corresponding values of GZ, or the ordinates of the curve of stability. Thus pp_1 is the length of GZ at the angle of inclination α, and is equal to the ordinate Pα of the curve given in fig. 9.

It will be obvious that after calculating three or four curves of BR for a vessel, including those for the two extremes of draught, no further calculation is necessary for obtaining a similar curve at any intermediate draught, since it is only requisite to construct cross-curves at given angles of inclination—say at intervals of 10°—and from these cross-curves to measure off the ordinates of the curve BR for the draught required. Having thus obtained the curve of BR for the draught in question, the ordinary curve of stability,

or lengths of GZ, can be obtained by setting off a curve of sines as in fig. 10 to radius BG, and deducting the ordinates of this curve from the corresponding ones of the curve of BR. This mode of constructing curves of stability has been rendered necessary by the great variations in draught of water and position of centre of gravity that have to be dealt with in many mercantile steamers. The demand that has thus arisen for constructing cross-curves of stability, *i.e.*, curves showing the variation of righting moment with draught of water at constant angles of inclination, has recently led to greatly improved and more rapid methods of calculation being devised. Amsler's mechanical integrator has been invaluable in bringing about this desirable result, and it is now possible to make a complete set of cross-curves of stability for a ship in a few days.*

In considering the stability of a ship from the point of view of variation of righting moment with draught of water, the angle of inclination being constant, instead of from that of variation of righting moment with angle of inclination, the draught being constant, as was formerly done; or, rather, in considering the subject from both points of view instead of almost exclusively from the latter, several interesting and important results are obtained, which do not appear to have received the attention they deserve.

It has already been stated that one of the first lessons taught by the introduction of curves of stability was the decrease in the righting moments at large angles of inclination, and the rapidity with which stability frequently vanishes altogether, in vessels of low freeboard. These are matters of great practical importance, and have attracted a great deal of attention, on account of the numerous vessels of various types which are more or less affected by them. The connexion between low freeboard and range of stability, and the manner in which the latter is affected by metacentric height and the position of the centre of gravity, has latterly been well discussed and explained;

* The method thus described of calculating a series of ordinary curves of stability and afterwards constructing cross-curves from them is tedious and complicated. It is simpler and very much more expeditious to calculate the cross-curves directly by applying the integrator to the under-water part of the ship, instead of to the wedges of immersion and emersion; and thus determining at once the positions of the vertical lines through the centres of buoyancy at the required angles of inclination. By this means the necessity is avoided of calculating separately the volumes of the wedges of immersion and emersion, and of correcting the positions first assumed for the inclined water-lines in order to make these wedges approximately equal. Mr. William Denny, of Dumbarton, was the first to call my attention to this very important and useful simplification; and it was described by him in a paper read before the Institution of Naval Architects in April last. Other investigators have also been working in the same direction, and several papers dealing with the extension and simplification of stability calculations will be found in the "Transactions of the Institution of Naval Architects" for 1884, including one of great interest and value by M. Daymard.—25th July, 1884.

and the dangers which may be incurred by low-sided ships are well understood by many. In former times, it was not so necessary as it has recently become, to carry theoretical investigations to the point of ascertaining the angle at which a ship would capsize. It is only during the last twenty years that small height of side out of water has been thought a desirable quality for sea-going war-ships to possess, or that certain classes of merchant vessels have been evolved in which range of stability has been unduly limited by lowness of freeboard. When Atwood wrote his papers, and for very many years after, war-ships were built with such lofty sides, and merchant vessels were so comparatively uniform in type, and so deep in proportion to their other dimensions, that no demand arose for complete curves of stability. Besides, all the vessels of Atwood's day, and for long after, were sailing ships; and such few investigations as were made respecting their stability, were for the purpose of determining their sail-carrying power. The constructors of the old types of vessels judged of and regulated stability by reference to the practical test of sail-carrying power at sea, and this was usually sufficient for the purpose. Modern variations of type, which began after the introduction of steam, have for some time, however, rendered scientific calculations necessary, which before were considered merely interesting or curious. The many departures from the comparative uniformity of proportions and form that once prevailed, and particularly the extent to which some of these departures have gone in the direction of reducing freeboard, have created the necessity referred to.

Curves of stability, having been first produced for the purpose of ascertaining the effect of low freeboard, and having disclosed the dangers which lowness of freeboard may cause, have been largely used for that purpose. If we consider the cross-curves of stability—or curves of righting moments for various draughts of water, the angle of inclination being constant—it will be seen that while the upper and middle portions of those curves have been often dealt with, the lower portions have been almost, if not quite, neglected. The stability of a floating body at light draughts is, however, similar in character to that at deep draughts, and the same peculiar features and dangers that have been found to exist with low freeboard are frequently connected also with lightness of draught.

It fell to my lot to make some investigations respecting the stability possessed by the "Daphne" at the time of the disaster which befell her, and to give evidence respecting the same. I afterwards, by way of explanation of a portion of the evidence, wrote a letter to the "Times," which appeared on the 1st September last, calling attention to the relation which exists between the righting moments at deep and light draughts in certain elementary forms of floating bodies. The proposition I then enunciated, which illustrates

the point under consideration, is the following: If any homogeneous body, which is symmetrical about the three principal axes at its centre of gravity, be of such density as to float with its lowest point at a depth x below the water; then, if the density be altered so as to make it float with its highest point at a height x above the water, the righting moments will be the same in both cases at equal angles of inclination, and consequently the range of stability and complete curve of righting moments will be the same. This proposition can be made still more general, as was shown by Mr. William John, in a letter to the "Times" of the 5th September; as it applies to all homogeneous floating bodies of irregular form revolving about a horizontal axis fixed only in direction. In this general form, the condition of turning the body through an angle of 180°, or upside down, must, however, be included, because the immersed volume in the one case must be of the same form as the emersed volume in the other, and this can only be obtained with irregularly-shaped figures by turning them through an angle of 180°. For this reason I chose symmetrical figures for the purpose of giving a popular illustration of the analogous effects of low freeboard and light draught upon the stability of ships, and avoided introducing the condition of turning through 180° in order to get similar volumes above and below water in each case. The general proof of the proposition laid down is that the line joining the centre of gravity of the immersed volume with that of the volume above water must pass through the centre of gravity of the whole body, and the distances of the centres of gravity of the two sections from that of the whole body are inversely proportional to their volumes; so that the moment of stability, which is proportional to the immersed volume multiplied by the distance of its centre of gravity from that of the whole body, will be the same in both cases.

As the righting moments at equal angles of inclination at the deep and light draughts described are the same in a homogeneous floating body of symmetrical form, it follows that at the same draughts the lengths of GZ, or the arms of the righting lever at equal angles of inclination, and also the metacentric heights, are in the inverse ratio of the displaced volumes.

The moment of stability $V \times GZ$ is by Atwood's formula, see fig. 8, equal to $v \times h_1h_2 - V \times BG \sin \theta$. Now $v \times h_1h_2$, which is the moment of the wedges of immersion and emersion, is the same whether WAL be the below-water or above-water volume; the immersed wedge in the one case being the emersed in the other, and *vice versâ*. $V \times BR$ is therefore the same in floating bodies of any form that revolve about a horizontal axis fixed in direction, whether WAL be the above-water or below-water volume. For such bodies as are homogeneous $V \times BG \sin \theta$ is also equal in the two cases, because

BG is inversely proportional to V, or the immersed volume; from whence we again derive the result that the moments of stability at equal angles of inclination are the same. In the case of a body which is not homogeneous, and in which the centre of gravity is at a distance GG_1 from G, the centre of gravity of a similar homogeneous body, the moments of stability at equal angles of inclination when WAL represents above-water and below-water volumes respectively, differ from each other by an amount equal to $(V_1+V_2)GG_1 \sin\theta$, where V_1 and V_2 are the two volumes into which the whole body is divided by the water-line plane WL. When the change of immersed volume is produced by merely increasing the depth of immersion, as in a ship—and not by rotating the body so as to make WAL represent above-water and below-water volumes alternately—the difference between the moments of stability is (V_1-V_2) $GG_1 \sin\theta$. In the first case it is $GG_1 \sin\theta \times$ the whole volume of the body: and in the second it is $GG_1 \sin\theta \times$ the difference between the volumes into which the body is divided by the water-line plane WL.

Some of the results which follow from the above considerations have been previously noticed. Atwood, in his paper read before the Royal Society in 1796, discusses at great length the positions of equilibrium of homogeneous rectangular bodies, and prisms of square sections, with varying specific gravities. He shows that whether the specific gravity of a square parallelopiped be a or $1-a$, it will float in equilibrium with its faces at the same angles to the water-surface, and will pass through the same number of positions of equilibrium in turning through 360°. These cases are included in the general proposition that the righting moments at equal angles of inclination are the same for both densities of the body, because the righting moments will vanish, *i.e.*, positions of equilibrium will occur, at equal angles of inclination. Atwood also shows, in a paper read before the Royal Society in 1798, that the stability of a vessel whose sides are inclined at a given angle below the water-surface, is equal to that of a vessel whose sides are inclined to similar angles above the water-surface; the breadth of the water-line and other conditions being the same in both cases. He goes on to say that this proposition is not confined to the case which he demonstrates, but is equally true whatever figure be given to the sides of the ship, and whether they are plain or curved, provided that the sides under the water in one vessel are similar and equal, and similarly disposed in respect of the water-line, to the sides of the other vessel above the water-line. The unnecessary condition, so far as homogeneous bodies are concerned, introduced by Atwood, that the displacements shall be equal, prevented him from pushing his conclusions to the extent to which they have been carried in this paper.

It should here be remarked that in dealing with cross-curves of

stability, and thus considering the variation of stability with draught of water, the curves of righting moments require to be constructed, and not merely curves of GZ, or lengths of righting arm. The ordinary curve of stability usually has for its ordinates the lengths of GZ at the various angles of inclination. This is right enough for the condition under which such curves are constructed; because the displacement is then constant, and the curves represent either lengths of righting arm or righting moments, according to the scale upon which the ordinates are measured. In the cross-curves of stability, however, draught is one of the variable elements, and the displacement changes with it. A cross-curve whose ordinates represent the lengths of righting arm at various draughts of water is therefore quite different in character from a cross-curve of righting moments, whose ordinates are lengths of righting arm × displacement. It is necessary, in order to judge accurately of the variation of stability with draught of water, to use curves of righting moments, and not merely curves of GZ, such as are usually considered sufficient when the draught of water is fixed.

Fig. 11.

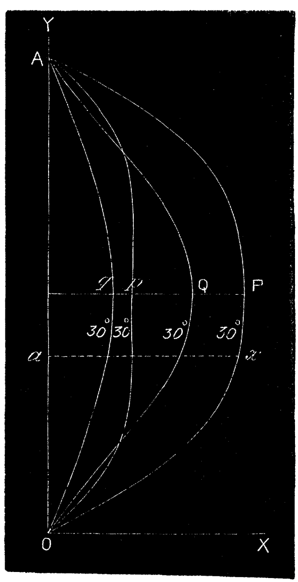


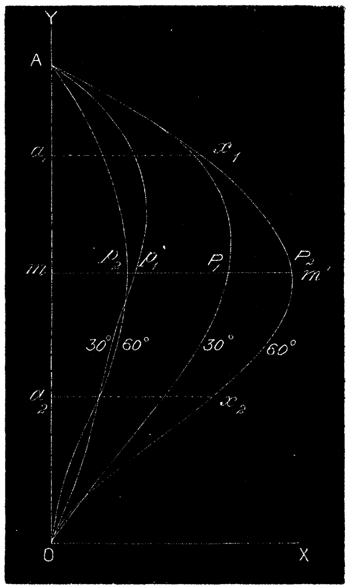
Fig. 11 shows cross-curves of stability, at angles of inclination of 30°, for two homogeneous floating bodies of prismatic form, and of the same breadth and depth; one being rectangular and the other ellip-

tical in cross section. The shorter axis is vertical when the bodies are upright, and is two-thirds of the longer axis, or extreme breadth. The measurements in the direction of OY give the depths of immersion, and those in the direction of OX represent moments. The curve APO gives the values, for the rectangular body when inclined to an angle of 30° from the upright, of the horizontal shift of the centre of buoyancy multiplied by the immersed volume, or $BR \times V$, see fig. 8. Thus if O*a* be any draught of water, the ordinate *ax* gives the value of $BR \times V$ at that draught. The curve AQO is the corresponding curve of moments for a prismatic body of elliptical section, and of equal length to the above, when inclined also to an angle of 30°. The axes of the ellipse are of the same length as the sides of the rectangular section taken in the former case; the minor axis being two-thirds of the major, and the minor axis being vertical when the body is upright. A*po* and A*qo* are the corresponding curves of $GZ \times V$ at the various draughts of water, and are obtained by deducting $V \times BG \sin 30°$ from the ordinates of the curves APO and AQO. The bodies being homogeneous, G is taken at the middle of the depth. These curves therefore represent the actual righting moments of the two bodies under consideration, for an inclination of 30°, at all draughts of water. The ordinates measured to the right of AO giving righting moments, and those to the left, if there were any, would be upsetting moments. It will be seen that the whole of the curves in fig. 11 are symmetrical with respect to a line drawn parallel to OX at one half the depth of total immersion. The elliptical prism tends to return to the upright, when at the inclination of 30°, at all draughts of water; and exerts the maximum effort to do so when immersed to the middle of its depth. The rectangular prism, when inclined to the same angle, also tends to return to the upright at all depths of immersion; the maximum righting moment is not, however, obtained when floating at the middle of its depth, but at draughts which are at equal distances above and below it.

Fig. 12 represents similar curves for a prismatic body, the upper half of whose section is rectangular, and the lower half elliptical as shown in fig. 6; the extreme dimensions of the section being the same as in the previous cases. This form of section is an example of the kind of departure from symmetry of form which exists in ships. It has been seen that if homogeneous bodies of symmetrical form be altered in density so as to float alternately at water-lines which are at equal distances above and below the centres of such bodies, the righting moments at equal angles of inclination will be the same at each draught. In the body for which the curves in fig. 12 have been constructed, the departure from similarity of the immersed and out of water volumes causes a difference between the righting moments at the draughts described. AP_1O and AP_2O represent the curves of

$BR \times V$ at angles of inclination of 30° and 60° respectively; and Ap_1o, Ap_2o are the corresponding curves of $GZ \times V$, or curves of righting moments, when G is taken in the position it would occupy if the body were homogeneous. The lines a_1x_1 and a_2x_2 indicate draughts at

FIG. 12.



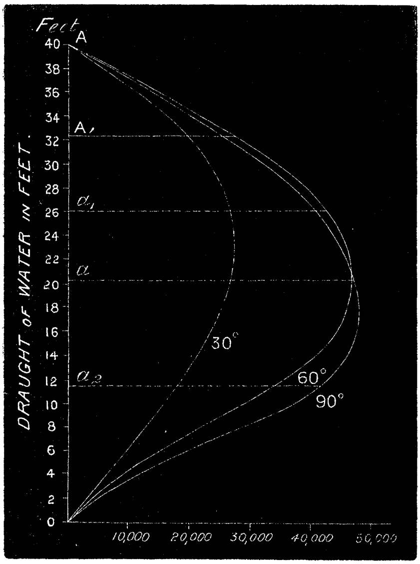
which equal volumes are cut off above and below water, and *mm′* shows the depth at which the immersed portion is one-half of the total volume of the body.

It will be seen that the righting moments are greater at 30° and 60° of inclination when the body is deeply immersed than when it is floating at light draughts with equal volumes below water to those which are above in the other case. The relation between the righting moments at the two extremes of draught is, however, largely determined in a ship by the position of the centre of gravity, which in this case has been taken as for a homogeneous body. This will be seen by the next example.

Fig. 13 gives curves of $BR \times V$ for an actual ship, at 30°, 60°, and 90° of inclination respectively. The vessel for which these have been constructed is 400 feet in length, 44 feet in breadth, and 32 feet 6 inches in moulded depth. The extreme depth from the top of keel to the highest point of the sheer of the upper deck is 40 feet. The point O is the top of the keel, A is the highest point of the sheer of the deck, and A_1 the lowest point of the upper deck at side, from

which freeboard is measured. The horizontal ordinates of these curves represent the moments BR × V, at the draughts to which they correspond; the scale of moments in foot-tons being shown upon the base line. The displacement of the vessel when wholly immersed

FIG. 13.

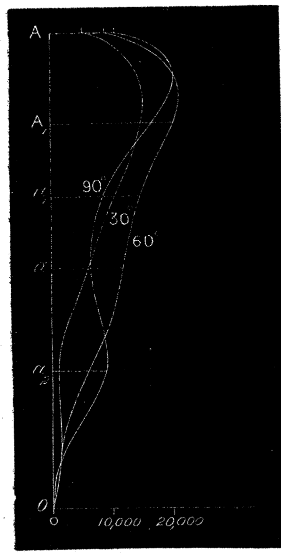


is 11,800 tons, and when displacing half this amount—or 5,900 tons—she draws 20 feet 6 inches of water; and this depth of flotation, with the corresponding values of BR × V, are shown by the ordinate drawn at the point a. a_2 represents the draught of water at which the vessel was launched, and a_1 the draught at which there is an equal volume out of water to that below water at the draught a_2. The draught of water at the point a_2 is 11 feet, and the freeboard above the point a_1 is 7 feet.

If the centre of gravity be taken at 19 feet above the top of keel for all draughts of water—it always varies, and in some cases considerably, with the draught, as has been stated, but 19 feet is found to be a fair mean height for the ship in question—and the moment $V \times BG \sin \theta$ be deducted from the ordinates of the curves in fig. 13, we obtain new ordinates, which represent the curves of righting moments, V × GZ; and these are shown in fig. 14. It will be seen that the righting moments which correspond with the ordinates of the usual curves of stability are much larger at deep draughts than at light draughts. For instance, the ordinates of the usual

curve of stability for the launching draught of 11 feet at a_2 give very much smaller moments than the curve for the deep draught at a_1, where there is an equal volume above water to what there is below in the other case, and the freeboard is only 7 feet.

FIG. 14.

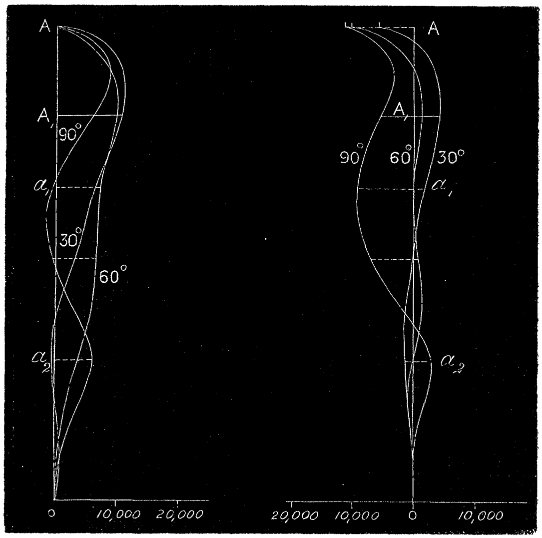


The centre of gravity taken in fig. 14 is 1·1 feet below where it would be if the external surface of the ship inclosed a homogeneous volume. Its position with reference to that of the centre of gravity of a homogeneous body of the same form largely determines the stability at all except small angles of inclination, and also the relation between the stability at light and deep draughts. Figs. 15 and 16 show what fig. 14 becomes changed into, if the centre of gravity is first raised 1·1 feet, so as to be in the same position as if the ship were a homogeneous body, and, second, if it is raised a further 1·1 feet, so as to be as much above this point in fig. 16 as it is below it in fig. 14. A comparison of these figures will show that, except in the case of a very high centre of gravity, the stability at light draughts with various positions of centre of gravity is less than at deep draughts.

It appears, therefore, that in the case of this ship, and she is a type of many mercantile passenger steamers, the proposition respecting the equality of the stability at light and deep draughts which has been shown to apply to homogeneous symmetrical bodies, requires

modification in a direction which is disadvantageous to light draughts. When there are equal volumes above and below water in this vessel, the righting moments at the light draughts are generally much less than at the deep draughts, except when the centre of gravity is raised excessively, and, for this class of ship, unusually high.

FIG. 15. Fig. 16.



The analogy that exists between light draught and deep draught stability in floating bodies of approximately symmetrical forms, and particularly the point of resemblance afforded by the fact that what is a wedge of immersion in one case is that of emersion in the other, and *vice versâ*, cannot fail to have struck some who have had to calculate the stability of bodies floating at light draughts, but attention has never been prominently called to it. It is desirable, however, that the connexion between the two cases should be fully realised, and the dangers peculiar to very light draught of water appreciated as thoroughly as are those which attach to low freeboard. Just as was said in 1871, in a passage already quoted, that metacentric height was formerly, by general consent, taken as a sufficiently good standard of comparison in judging of the stability a ship would have, on account of "the vague impression that the angle would be very large at which the ship became unstable," so, since the introduction of curves of stability, the dangers attaching to light draught have been frequently lost sight of, because of the equally vague impression that, so long as a vessel has a high side out of water, and any

metacentric height, she will have large righting moments at great inclinations and a large range of stability. The investigations made at the time of the "Daphne" disaster, and the discussions which these and the elaborate report of the Government Commissioner, Sir E. J. Reed, have caused, now place this question upon a somewhat different footing; but at the time referred to the general belief appears to have been as stated.

The considerations set forth in this paper are mainly intended to draw attention to the necessity for taking a more comprehensive view of the problem of stability than has formerly prevailed, by investigating the cross-curves of stability of ships, and thus ascertaining how the righting moments at fixed angles of inclination vary with draught of water. They also aim at showing how the stability at fixed inclinations does vary in some ships with draught of water, and becomes comparatively small at light draughts. In designing ships and other structures which are required to float safely at very light draughts of water, such calculations are necessary if accidents are to be prevented. In some cases the necessity is as great, or even greater, than for vessels of low freeboard.

The subject is so extensive that I feel quite unable to attempt any exhaustive treatment of it at the present time. All that has now been possible is to call attention to some of its main features and to show why it requires to be followed up and thoroughly investigated.

"Notes on the Varieties and Morphology of the Human Lachrymal Bone and its Accessory Ossicles." By A. MACALISTER, F.R.S., Professor of Anatomy in the University of Cambridge. Received March 14, 1884. Read March 27.

[PLATES 1—3.]

Having examined 1000 lachrymal bones and the soft parts of the lachrymal region of over 300 orbits, I have compiled therefrom the following notes on the anatomy of these parts. The lachrymal is one of the most variable bones in the human skull, and one of the most perishable, being frequently destroyed by careless or ignorant methods of handling crania.

I. I have notes of two* instances of deficiency of the lachrymal bone. In one of these cases the whole wall of the groove is formed by the nasal process of the maxilla in front, and by the ethmoid and

* I have since seen eight additional instances of absence, in the Hunterian and other collections, five being in Hindoo crania, two in Negroes, in none of which was there any trace of synostosis.

FIG. 1.

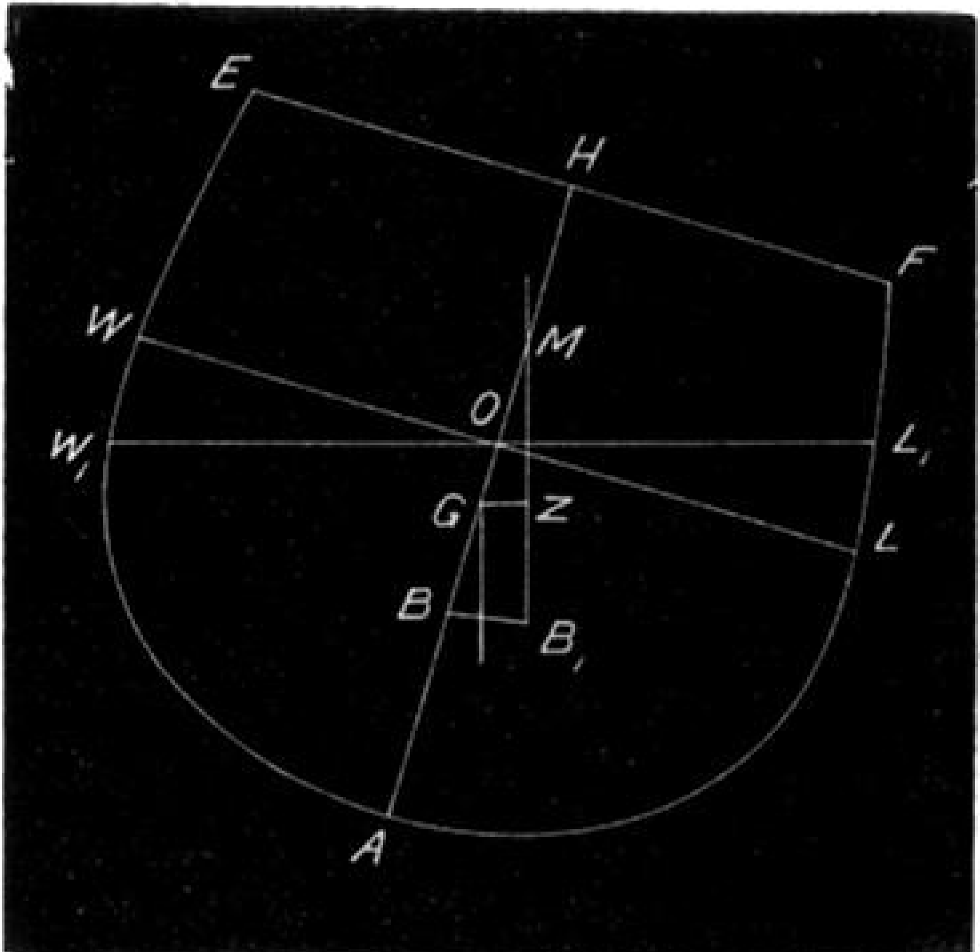


FIG. 2.

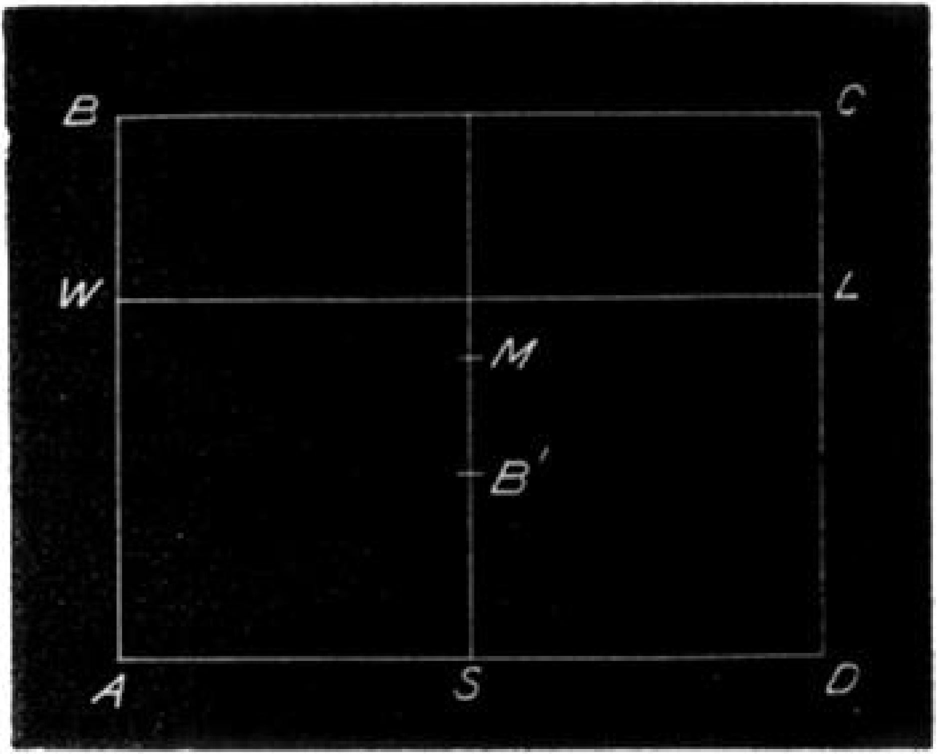


FIG. 3.

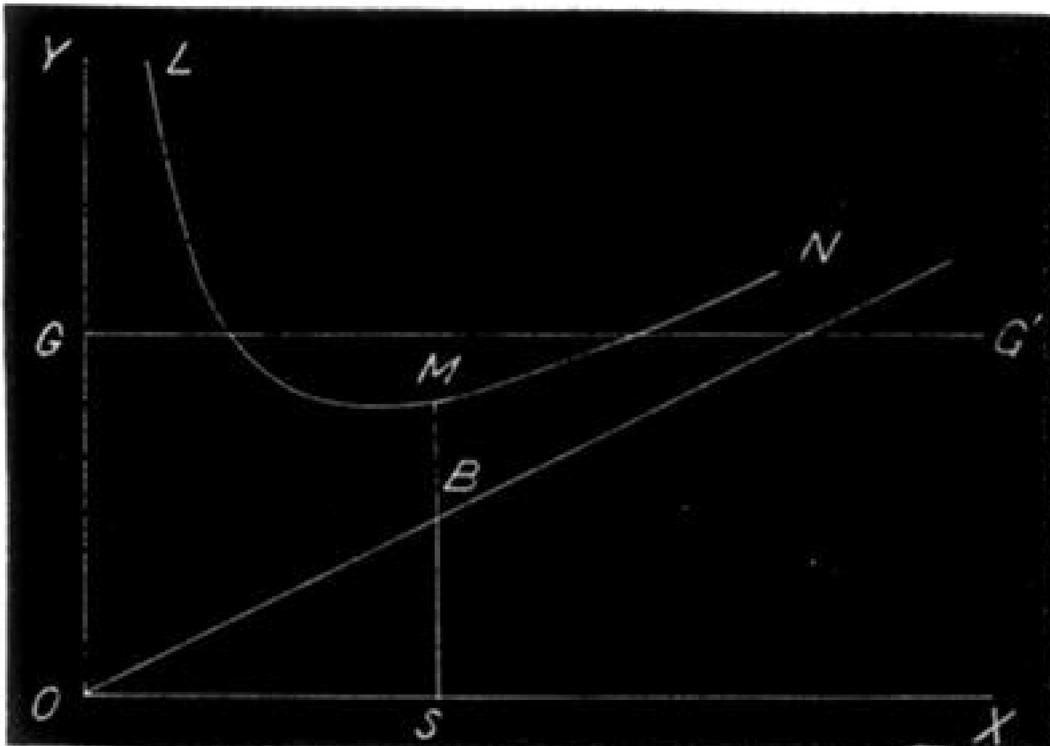
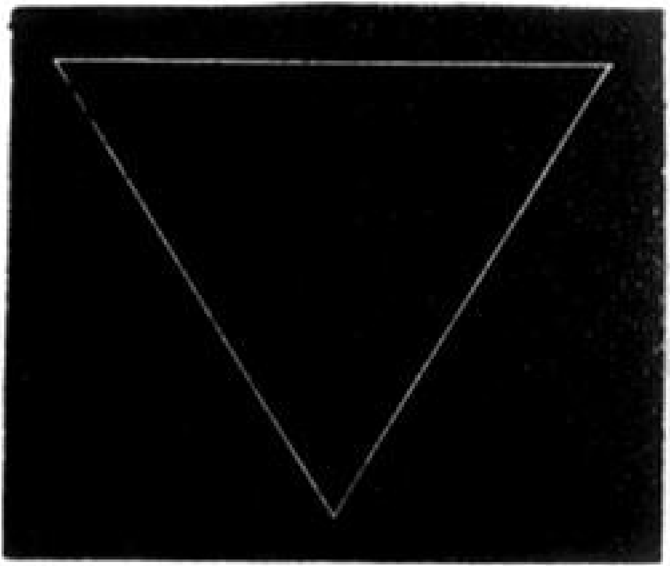


FIG. 4.



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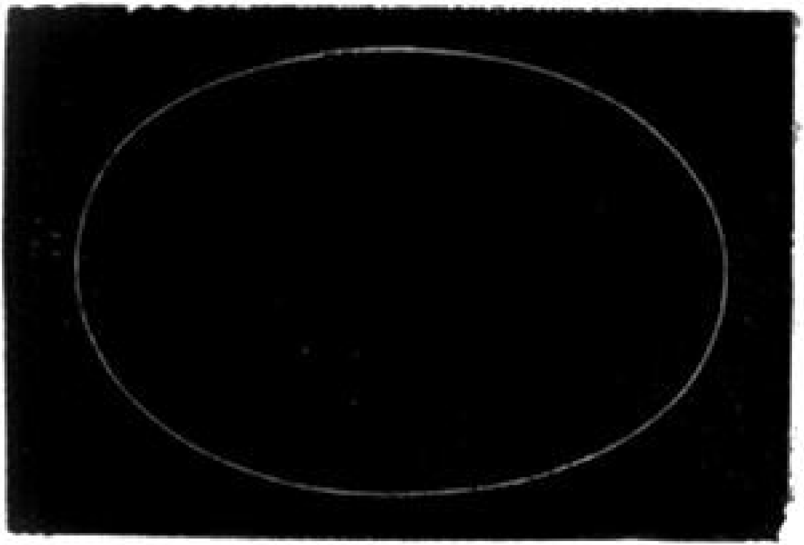

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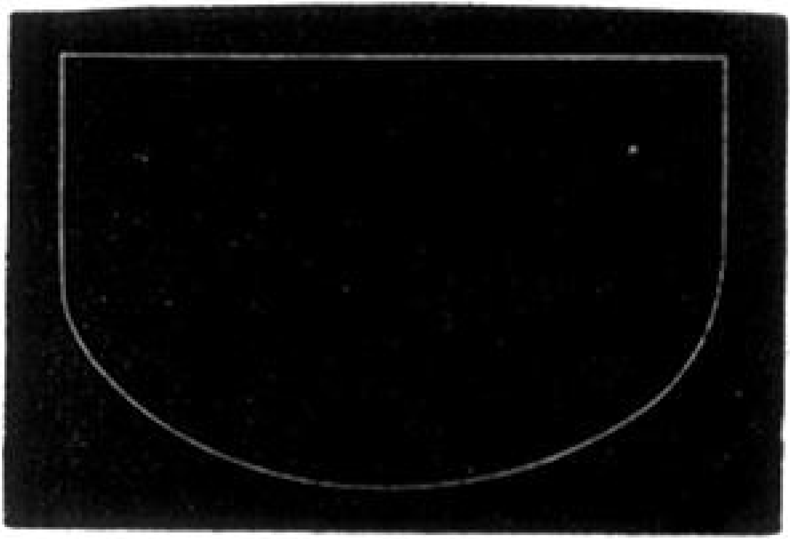
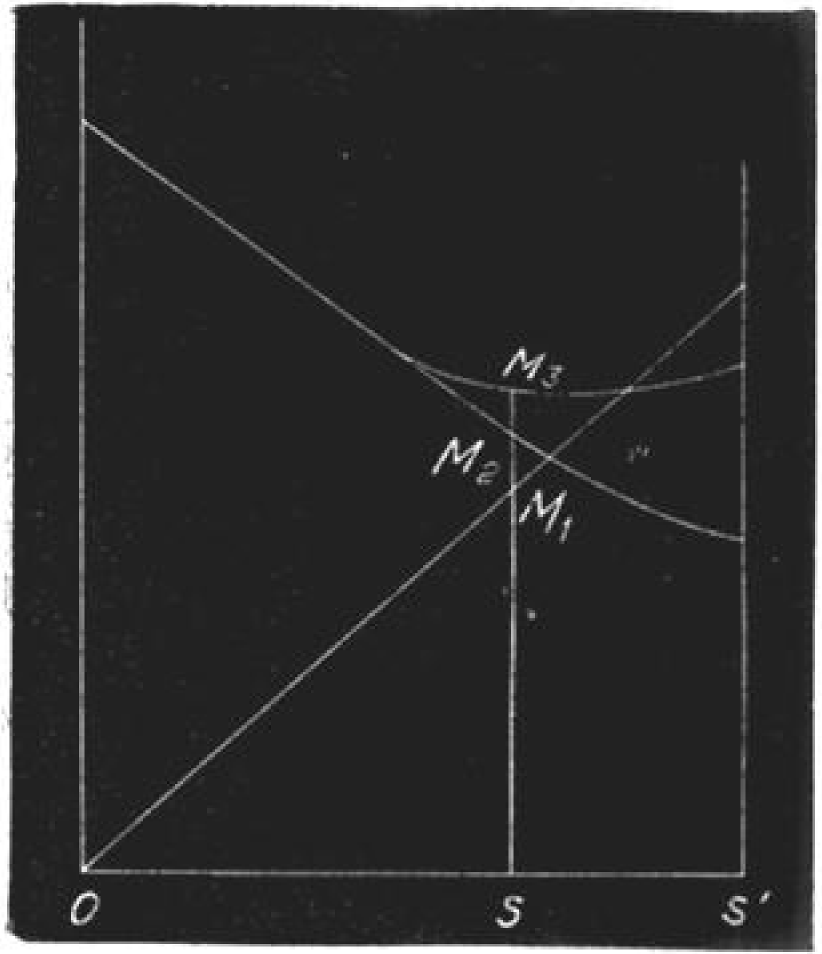


FIG. 7.



Triangular section.

Section whose upper half is rectangular and lower half elliptical, as in fig. 6.

Elliptical section.

Fig. 8.

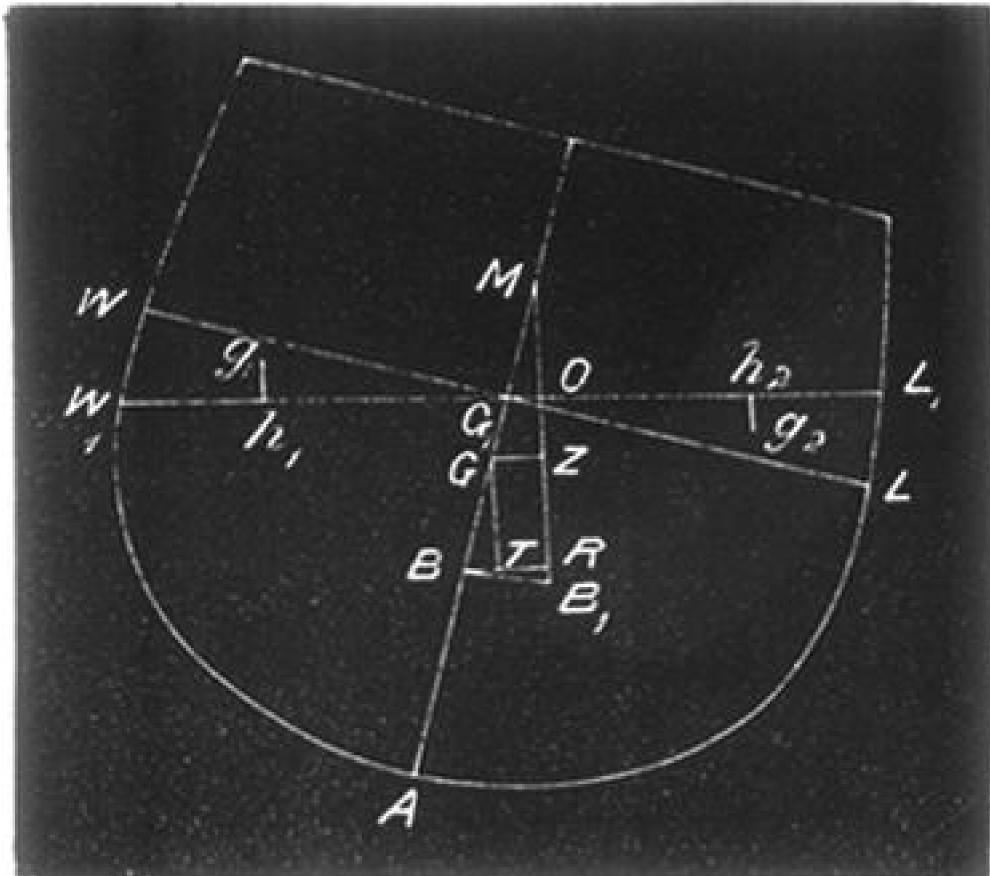


FIG. 9.

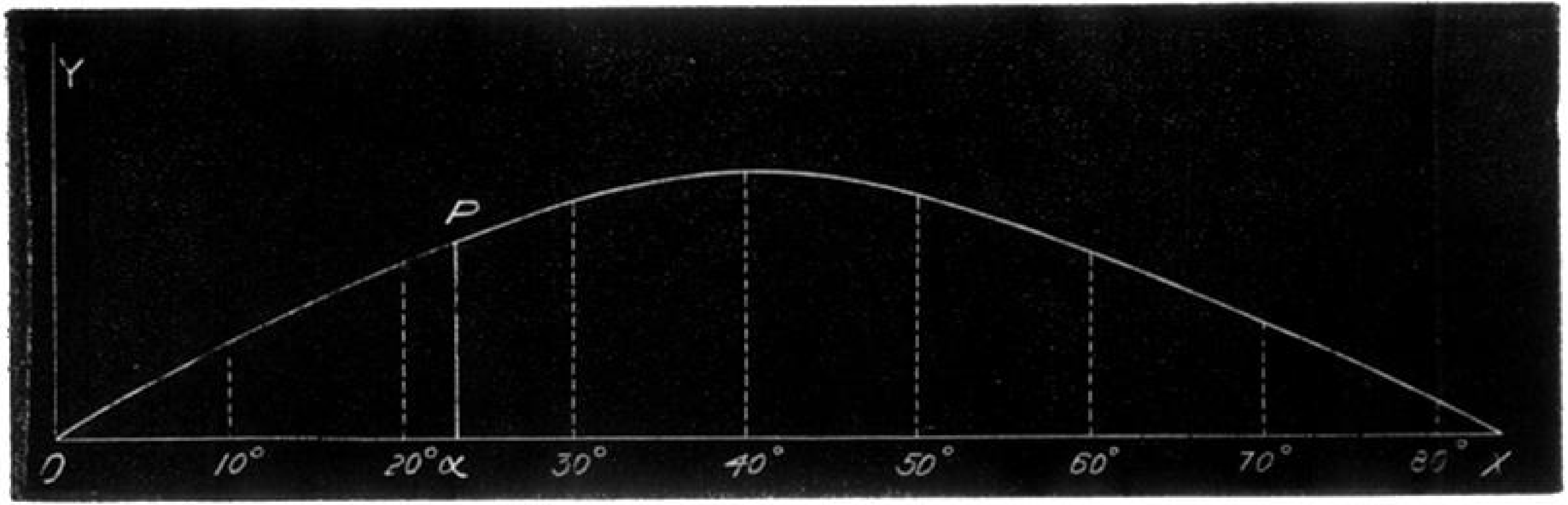


FIG. 10.

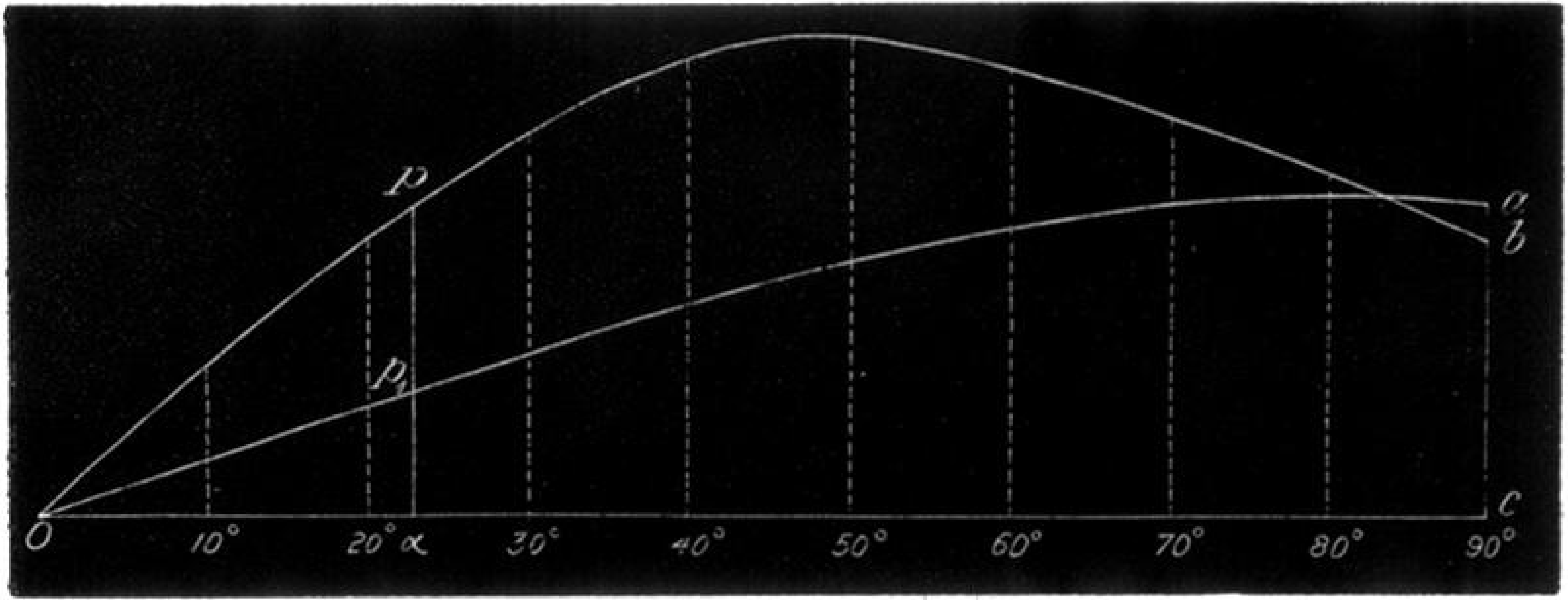


FIG. 11.

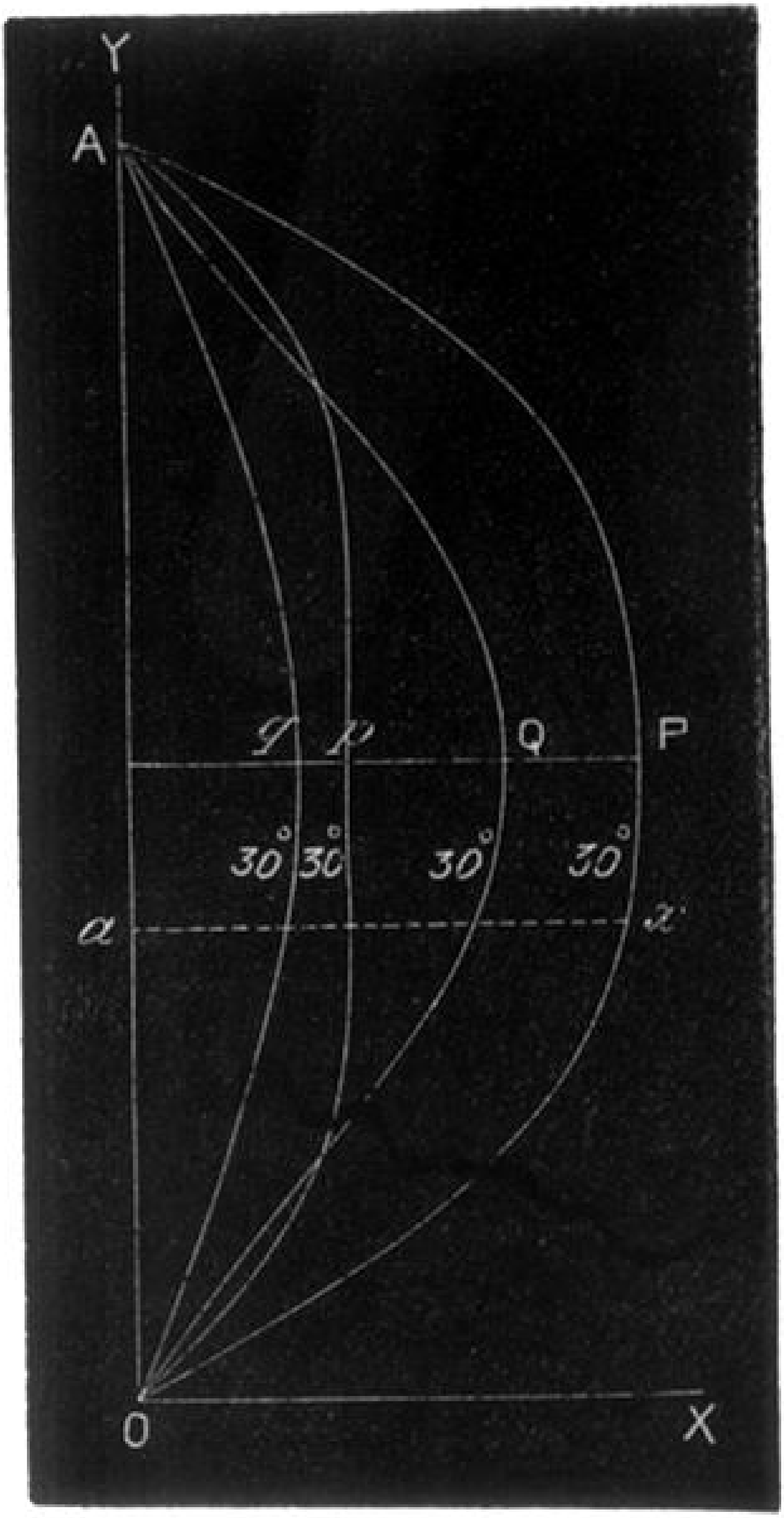


FIG. 12.

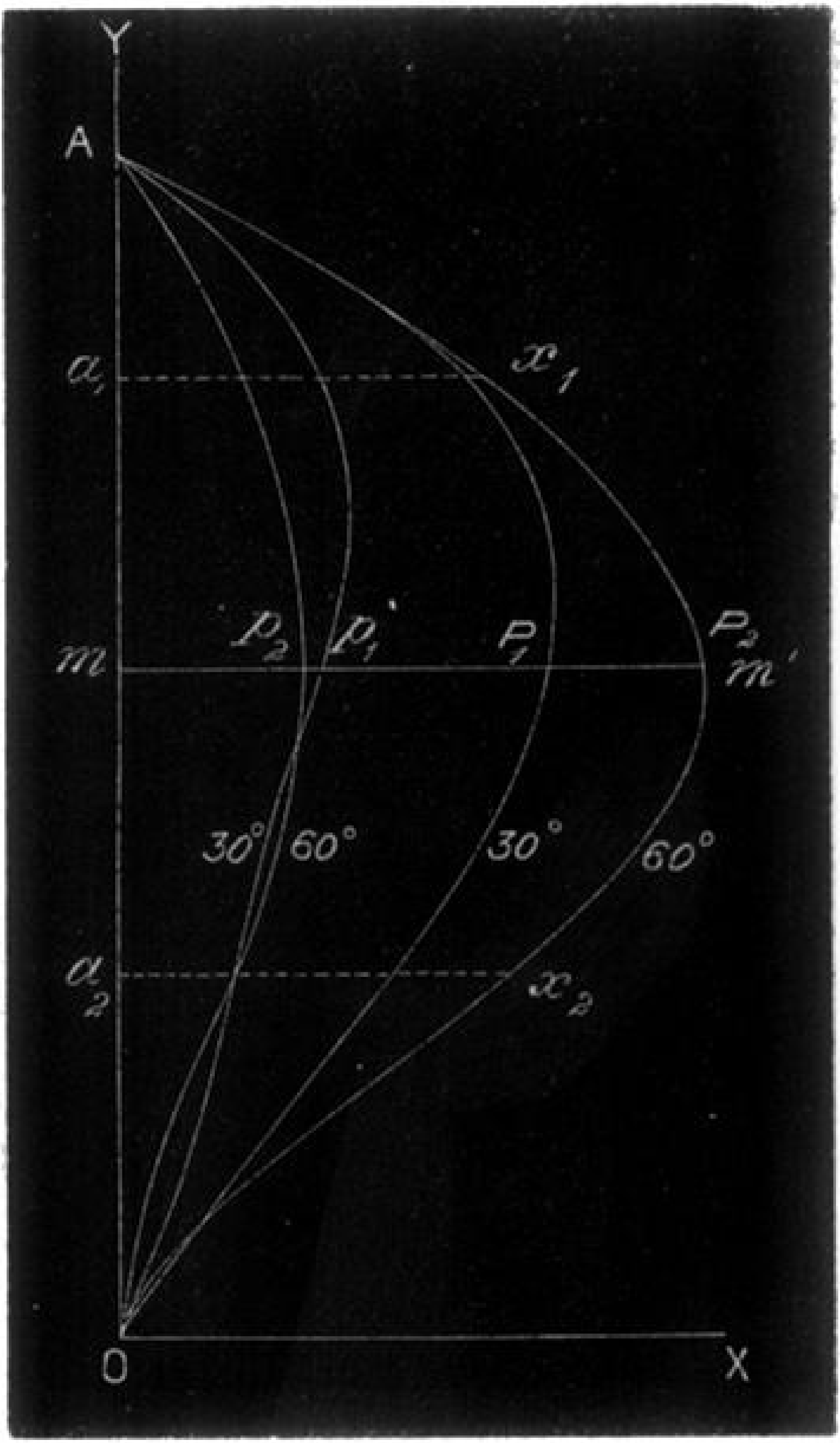
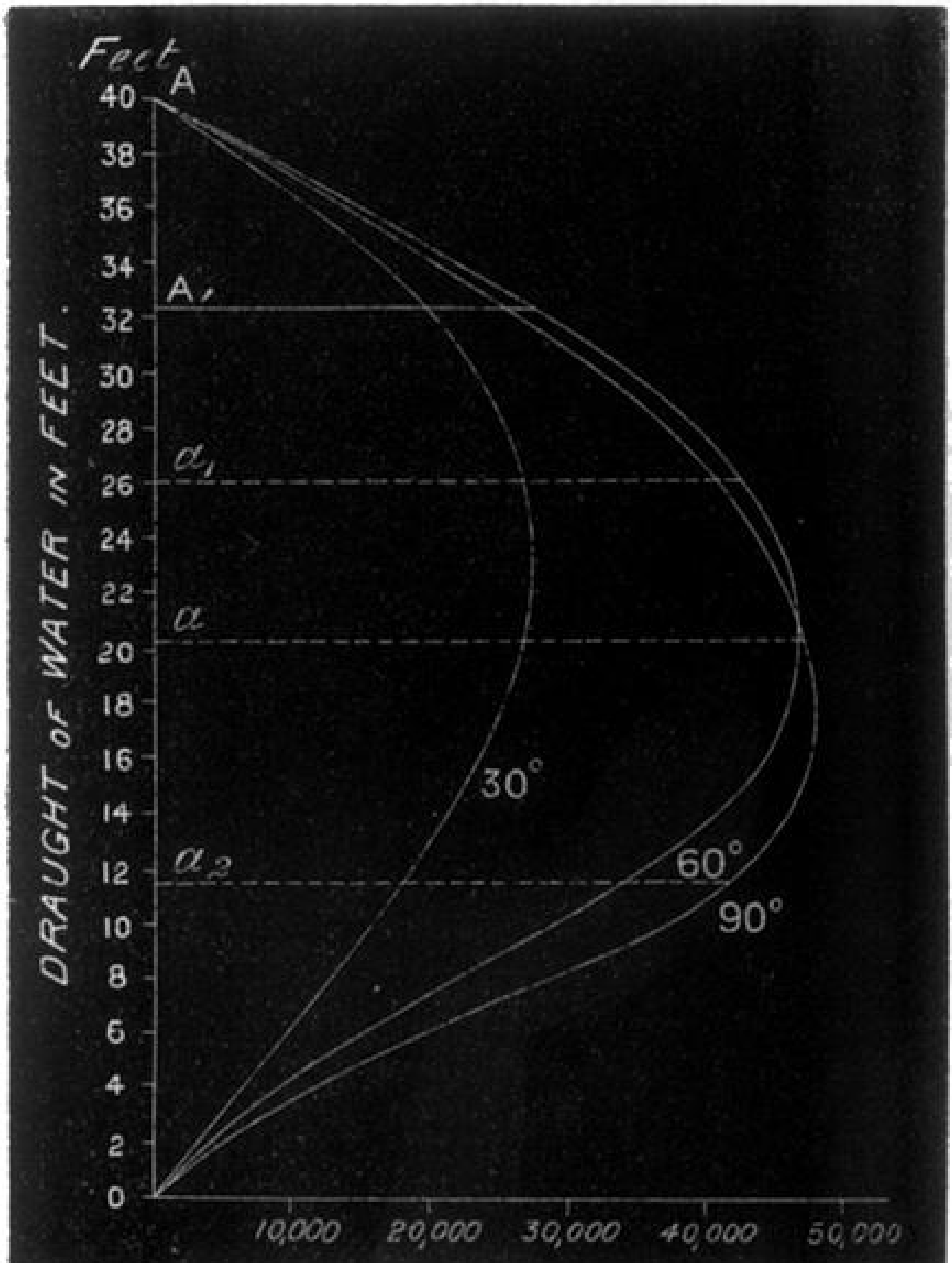


FIG. 13.



Feet
A
A,
a_1
a
a_2
DRAUGHT OF WATER IN FEET.
40
38
36
34
32
30
28
26
24
22
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18
16
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12
10
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FIG. 14.

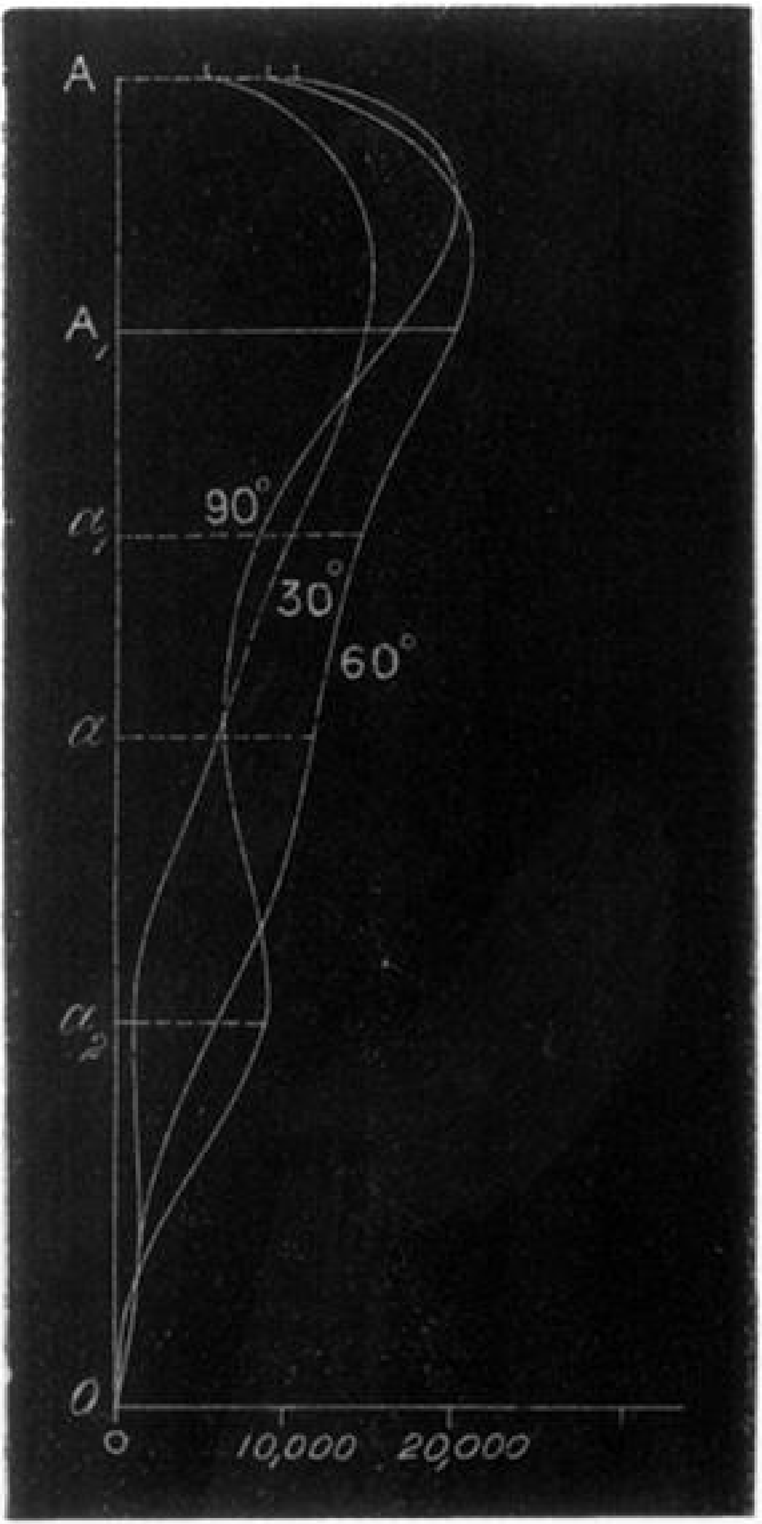

FIG. 15.

Fig. 16.