

he considers this only an approximation to the true normal gradient, and that the readings of the Coal-mines and Artesian-well experiments are, owing to the causes he enumerates, still too high. He also discusses the question whether or not the gradient changes with the depth. His own reduction of the observations gave no result, but he points out that in all probability the circulation of water arising from the extreme tension of its vapour is stayed at a certain depth; while as it is known experimentally that the conductivity of iron diminishes rapidly as the temperature increases, this may possibly in a different degree apply to rocks. If, therefore, there is any change, these indications would be in favour of a more rapid gradient.

Taking all these conditions into consideration, the author inquires whether a gradient of 45 feet per degree may not be nearer the true normal than even the one of 48 feet obtained by the observations.

III. "On the Connexion between Electric Current and the Electric and Magnetic Inductions in the surrounding Field." By J. H. POYNTING, M.A., late Fellow of Trinity College, Cambridge, Professor of Physics, Mason College, Birmingham. Communicated by Lord RAYLEIGH, M.A., D.C.L., F.R.S. Received January 31, 1885.

(Abstract.)

This paper describes a hypothesis as to the connexion between current in conductors and the transfer of electric and magnetic inductions in the surrounding field. The hypothesis is suggested by the mode of transfer of energy in the electromagnetic field, resulting from Maxwell's equations investigated in a former paper ("Phil. Trans.," vol. 175, pp. 343—361, 1884). It was there shown that according to Maxwell's electromagnetic theory the energy which is dissipated in the circuit is transferred through the medium, always moving perpendicularly to the plane containing the lines of electric and magnetic intensity, and that it comes into the conductor from the surrounding insulator, not flowing along the wire.

Symbolising the nature of the induction by unit tubes drawn in the direction of the induction in the usual way, *i.e.*, so that there is unit quantity of induction over every section of a tube, the electric induction is equal to  $K \times$  electric intensity  $\div 4\pi$ , and the magnetic induction is equal to  $\mu \times$  magnetic intensity. The electric induction is the same quantity as Maxwell's "displacement." The hypothesis now made

consists in the supposition that the electric and magnetic inductions are transferred through the medium in a similar manner to the energy, the induction being regarded as propagated sideways rather than along the lines or tubes of induction. The tubes of electric induction move in upon the wire containing a current, and are there broken up or dissolved, while the magnetic tubes, which are ring-shaped, contract upon the wire and finally disappear, the places of the tubes thus lost being supplied by fresh tubes sent out from the seat of electromotive force.

Examining the basis of Maxwell's electromagnetic theory, it is seen to rest on three principles, viz.:—

I. The electric and magnetic energies are distributed through the field in a manner which can be assigned. Dividing the unit tubes into unit cells by level surfaces drawn at unit differences of potential, each electrical cell, according to Maxwell, contains half a unit of energy, and each magnetic cell  $\frac{1}{8\pi}$  unit.

II. The line integral of the electric intensity round any closed curve is equal to the rate of decrease of the total magnetic induction through the curve.

III. The line integral of the magnetic intensity round any closed curve, is equal to  $4\pi \times$  the current through the curve.

The second and third principles are modified in a manner suggested by the supposed movement of the tubes of induction.

II modified. Whenever electromotive force is produced by a change in the magnetic field or by motion of matter through the field, the E.M.F. per unit length, or the electric intensity, is equal to the number of tubes of magnetic induction, cutting or cut by the unit length per second, the E.M.F. tending to produce induction in the direction in which a right-handed screw would move if turned round from the direction of motion, relatively to the tubes, towards the direction of the magnetic induction.

This will give the result of II above, if the quantity of magnetic induction through a curve only alters by the movement of tubes in or out across the boundary. Reasons for the supposition are given.

III modified. Using the term magnetomotive force, a term suggested by Mr. Bosanquet, to denote the line integral of the magnetic intensity round the axis of a tube of induction, the third principle is as follows:—

Whenever magnetomotive force is produced by change in the electric field, or by motion of matter through the field, the magnetomotive force per unit length is equal to  $4\pi \times$  the number of tubes of electric induction cutting or cut by unit length per second, the magnetomotive force tending to produce induction in the direction in which a right-handed screw would move if turned round from the direction of the

electric induction towards the direction of motion of the unit length, relatively to the tubes of induction.

These principles are applied to special cases.

*A Straight Wire carrying a Steady Current.*

When a current  $C$  is in a wire,  $C$  induction tubes are supposed to close in upon the wire per second, these being, as it were, broken up or dissolved, their energy appearing finally as heat.

This accounts for the constancy of current at all parts of the wire in the steady state, in so far as it reduces this constancy to a particular case of the law, according to which there is the same total induction over all sections of a tube.

Also, since  $C$  tubes are broken up in the wire per second, and the field is steady,  $C$  tubes must move inwards through any curve encircling the wire, and this will, by the third principle as modified, give a line integral of magnetic intensity round the curve equal to  $4\pi C$ .

Since the electric intensity is not produced by static charges, we must suppose it due to the motion of magnetic tubes. If  $E$  is the electric intensity,  $E$  magnetic tubes must cut any unit length parallel to the axis of the wire per second moving inwards.

On the supposition that the tubes bring their energy in with them, it is shown that the electric and magnetic tubes each account for half the energy, so that we may suppose that the energy crossing any surface is equally divided between the two kinds.

On the supposition that the tubes can be identified throughout their motion in the insulating medium, their velocities are calculated.

*Discharge of a Condenser through a Fine Wire.*

When the terminals of a charged condenser consisting of two parallel plates are connected by a wire, the energy which was before the discharge chiefly between the two plates, now appears as heat in the wire. The electric induction tubes are supposed to move outwards from the space between the plates, keeping their ends upon the plates or wires. They finally converge upon the wire and are there broken up. The hypothesis is in accordance with the doctrine of closed currents. For the total result is equivalent to the addition of so many closed induction tubes to the circuit, the induction running the same way relatively to the circuit throughout. When the condenser is discharged by imperfect insulation of the dielectric, we may represent the process still as a closed current, the two parts of which are the loss of induction and its dissipation, but this is artificial, and it is more natural to look upon the process as a decay of induction without movement of fresh induction tubes inwards, and therefore without the formation of magnetic induction. This case is discussed at some length, as we can here realize what goes on at the source of

energy, and the results suggest that a similar action occurs at the source of energy or seat of E.M.F. in other cases.

*A Circuit containing a Voltaic Cell.*

The chemical theory of the cell is adopted. It is first shown that the difference of potential of the two terminals in open circuit is equal to the E.M.F. immediately after closure, because chemical action will go on charging the terminals, *i.e.*, putting out energy into the medium, until any further charge would require more energy in the medium than is supplied by the chemical action necessary, according to Faraday's law of electrolysis.

The level surfaces are discussed, and it is supposed that the potentials being in ascending order, zinc, copper, acid, all of the surfaces pass between the zinc and the acid, some of them bending round and passing between the copper and acid, the rest going between the terminals. When the circuit is closed, tubes of electric induction, running from acid to zinc, are supposed to diverge outwards and close in on the rest of the circuit, there running in the opposite direction, *i.e.*, from copper to zinc. A divergence of negative tubes being magnetically equivalent to a convergence of positive tubes, the magnetic relations of the circuit or the direction of the current will be the same throughout. The tendency to a steady state is discussed. The existence of charges in the circuit is taken to show that the tubes of electric induction do not enter the wire at the same time throughout the whole of their length.

*Current produced by Motion of a Conductor in a Magnetic Field.*

In this case it is shown that according to the hypothesis we must suppose a divergence of negative tubes from the seat of E.M.F.

*The General Equations of the Electromagnetic Field.*

The assumption that if we take any closed curve, the number of tubes of magnetic induction passing through it is equal to the excess of the number which have moved in over the number which have moved out through the boundary since the beginning of the formation of the field, suggests a historical mode of describing the state of the field at any moment.

Taking  $L, M, N$  as the numbers of magnetic tubes which have cut unit lengths of the axes through a point since the beginning of the system, those being considered positive which tend to produce positive electric intensity, the equations  $a = \frac{dM}{dz} - \frac{dN}{dy}$ , and two others are obtained.

From these and the ordinary current equations  $4\pi\mu u = \frac{dc}{dy} - \frac{db}{dz}$ , and

two others, we obtain  $4\pi\mu u = -\nabla^2 L - \frac{d}{dx}\left(\frac{dL}{dx} + \frac{dM}{dy} + \frac{dN}{dz}\right)$ , and two others, which can be solved in the same manner as Maxwell's equations, from which they only differ in form, in signs. The quantity  $\frac{dL}{dx} + \frac{dM}{dy} + \frac{dN}{dz}$  is, however, not zero, and on forming the equations for electric intensity, it is found to give rise to a triple integral having value at surfaces of contact of dissimilar substances and at charged surfaces.

In the case when there is no material motion the components of electric intensity are—

$$P = -\mu \iiint \frac{du}{dt} \cdot \frac{1}{r} dx dy dz - \frac{1}{4\pi} \frac{d}{dx} \iiint \left( \frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz} \right) \frac{1}{r} dx dy dz,$$

and two other equations.

If the system is steady,  $\frac{du}{dt} = 0$ .

On putting

$$\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz} = 4\pi\rho, \text{ and } V = \iiint \frac{\rho}{r} dx dy dz,$$

we obtain  $P = -\frac{dV}{dx}$ , and two other equations.

It is shown that these equations may be obtained without the special hypothesis as to the mode of motion of the magnetic induction by assuming  $L = \int P dt$ ,  $M = \int Q dt$ ,  $N = \int R dt$ .

Equations are also obtained by considering the growth of the electric induction.