other in the same manner as the corresponding curves for different values of stress, a fact to be anticipated from the earlier discoveries of Baur.

The experiments, which have been of a very extended character, were made, during 1881–1883, in the Laboratory of the University of Tokio, Japan, with the help of Japanese students, Messrs. Fujisawa, Tanakadate, Tanaka, and Sakai, to whom the author is indebted for much valuable assistance. The results have been, almost without exception, reduced to absolute measure, and are for the most part presented graphically in curves which accompany the paper.

II. "On certain Definite Integrals. No. 12." By W. H. L. Russell, F.R.S. Received December 8, 1884.

The following theorem must be implicitly known, although I have never seen it in print:—

$$\begin{aligned} \text{Let } \xi &= \lambda x + \mu y + \nu z, & \Delta x &= \Lambda\xi + \mathrm{M}\eta + \mathrm{N}\zeta. \\ \eta &= \lambda' x + \mu' y + \nu' z, & \Delta y &= \Lambda'\xi + \mathrm{M}'\eta + \mathrm{N}'\zeta. \\ \zeta &= \lambda'' x + \mu'' y + \nu'' z, & \Delta z &= \Lambda''\xi + \mathrm{M}''\eta + \mathrm{N}''\zeta. \end{aligned}$$

Then $\iiint dxdydz\phi(\lambda x+\mu y+\nu z,\ \lambda' x+\mu' y+\nu' z,\ \lambda'' x+\mu'' y+\nu'' z)$

$$=\iiint \Theta\phi(\xi,\ \eta,\ \zeta)d\xi d\eta d\zeta,$$

where the limits of the first integral, and consequently those of the second, are given by an equation of limits, and Θ is a well-known constant. Now let

$$\mathrm{P}=ax^3+by^3+cz^3+a_1x^2y+a'_1x^2z+b_1y^2x+b'_1y^2z+c_1z^2x+c'_1z^2y+mxyz,$$

and let the expression break up into three linear factors, so that

$$\mathrm{P}=(\lambda x+\mu y+\nu z)(\lambda' x+\mu' y+\nu' z)(\lambda'' x+\mu'' y+\nu'' z),$$

which will subject the constants a, b, a_1, &c., to three conditions.

Then we have $$\iiint\frac{dxdydzx^{l-1}y^{m-1}z^{n-1}}{\sqrt[r]{\mathrm{P}}}=$$

$$\Sigma\frac{\rho\Delta^3}{(\Lambda+\Lambda'+\Lambda'')(\mathrm{M}+\mathrm{M}'+\mathrm{M}'')(\mathrm{N}+\mathrm{N}'+\mathrm{N}'')}\,\frac{\Gamma\left(\alpha-\frac{1}{r}\right)\Gamma\left(\beta-\frac{1}{r}\right)\Gamma\left(\gamma-\frac{1}{r}\right)}{\Gamma\left(1+\alpha+\beta+\gamma-\frac{3}{r}\right)},$$

where l, m, n are positive integers, ρ a function of λ, μ, ν, λ', &c., which is different for each term, and $\alpha+\beta+\gamma=l+m+n$. The limits of the integral are given by the equation $x+y+z=1$.

Let P be the sum of the three cubes

$$(\lambda x+\mu y+\nu z)^3+(\lambda' x+\mu' y+\nu' z)^3+(\lambda'' x+\mu'' y+\nu'' z)^3,$$

which implies one condition between the constants a, b, and c. Then we have

$$\iiint dx dy dz x^{l-1} y^{m-1} z^{n-1} \phi(\mathrm{P}) = \Sigma\rho \frac{\Gamma\frac{\alpha}{3}\Gamma\frac{\beta}{3}\Gamma\frac{\gamma}{3}}{\Gamma\left(\frac{\alpha}{3}+\frac{\beta}{3}+\frac{\gamma}{3}\right)}\int_0^1 du\phi(u) u^{\frac{\alpha}{3}+\frac{\beta}{3}+\frac{\gamma}{3}-1}.$$

where other things being the same, the limits are determined by the equation

$$\mathrm{P}=1.$$

It is scarcely necessary to point out that these investigations may be much extended when the radical sign is different, when the algebraical function under it is of a higher degree, and when there are more than three variables.

I enter now on a different subject.

In a partial differential equation

$$\int \mathrm{F}\left(x\frac{d}{dx},\ y\frac{d}{dy}\right)u=0,$$

substitute for (u) a series of which the general term is $\mathrm{A}x^m y^n$. Then if this series satisfies the equation, term by term, we have an algebraical equation $f(m,\ n)=0$, whence $m=\phi(n)$, and the equation is satisfied by an infinite series of the form

$$\mathrm{A}x^{\phi(n)}y^n+\mathrm{B}x^{\phi(n_1)}y^{n_1}+\mathrm{C}x^{\phi(n_2)}y^{n_2}+\ \ldots$$

Now suppose $y^{\phi(n)}$ to be expressed in the form $\int \mathrm{PQ}^n d\phi$, then the series is transformed into

$$\mathrm{A}\int \mathrm{P}(x\mathrm{Q})^n d\phi+\mathrm{B}\int \mathrm{P}(x\mathrm{Q})^{n_1} d\phi+\ \ldots$$

where A, B, &c., are arbitrary constants.

Since the number of these constants is infinite, we may write this (after Poisson)

$$\int \mathrm{PF}(x\mathrm{Q})d\phi$$

As an example, take the equation

$$\alpha.\frac{d^{\mu+1}u}{d\eta^\mu d\xi}+\beta\frac{d^\mu u}{d\eta^{\mu-1}d\xi}+\gamma\frac{d^{\mu-1}u}{d\eta^{\mu-2}ud\xi}+\ \ldots=a\frac{d^\nu u}{d\eta^\nu}+b\frac{d^{\nu-1}u}{d\eta^{\nu-1}}+c\frac{d^{\nu-2}u}{d\eta^{\nu-2}}+\ \ldots$$

This may be written

$$\alpha\left(y\frac{d}{dy}\right)^{\mu}\left(x\frac{d}{dx}\right)u+\beta\left(y\frac{d}{dy}\right)^{\mu-1}\left(x\frac{d}{dx}\right)u+\gamma\left(y\frac{d}{dy}\right)^{\mu-2}\left(x\frac{d}{dx}\right)u+\ \dots$$

$$=a\left(y\frac{d}{dy}\right)^{\nu}u+b\left(y\frac{d}{dy}\right)^{\nu-1}u+c\left(y\frac{d}{dy}\right)^{\nu-2}u+\ \dots$$

Hence if Ax^my^n be a specimen term, we have, substituting for u,

$$(\alpha n^{\mu}+\beta n^{\mu-1}+\gamma n^{\mu-2}+\ \dots\ )m=an^{\nu}+bn^{\nu-1}+cn^{\nu-2}+\ \dots,$$

whence we have

$$m=\frac{an^{\nu}+bn^{\nu-1}+cn^{\nu-2}+\ \dots}{\alpha n^{\mu}+\beta n^{\mu-1}+\gamma n^{\mu-2}+\ \dots}.$$

Hence we have to reduce

$$\epsilon^{\log_{\epsilon}x}\cdot\frac{an^{\nu}+bn^{\nu-1}+cn^{\nu-2}+\ \dots}{\alpha n^{\mu}+\beta n^{\mu-1}+cn^{\mu-2}+\ \dots}$$

to the form $\int PQ^n d\theta$. This is easily done, for the function is equivalent to

$$\epsilon^{\frac{\log_{\epsilon}x}{r_1+s_1n}+\frac{\log_{\epsilon}x}{r_2+s_2n}+\frac{\log_{\epsilon}x}{r_3+s_3n}+\ \dots}$$

Now we have

$$\epsilon^{\frac{\log_{\epsilon}x}{r_1+s_1n}}=\frac{\sqrt{r_1+s_1n}}{2\sqrt{\pi\log_{\epsilon}x}}\int_{-\infty}^{\infty}\epsilon^{v-\frac{(r_1+s_1n)v^2}{4\log_{\epsilon}x}}dv,$$

and
$$\sqrt{r_1+s_1n}=\frac{2}{\sqrt{\pi}}(r_1+s_1n)\int_0^{\infty}\epsilon^{-(r_1+s_1n)v^2}dv.$$

Hence we see that the function can be reduced to the form

$$(A+Bn+Cn^2+\ \dots\ +En^{\mu})\iint PQ^n d\theta d\phi\,.\iint P_1Q_1{}^n d\theta_1 d\phi_1$$

Now if
$$Fx=Mx^{\alpha}+Nx^{\beta}+Px^{\gamma}+\ \dots$$

$$\left(x\frac{d}{dx}\right)^r Fx=M\alpha^r x^{\alpha}+N\beta^r x^{\beta}+\ \dots$$

and consequently we have, if we put

$$\left(x\frac{d}{dx}\right)^r F(u)=F_r(x),$$

$$u=A\iint\ \dots\ PP_1\ \dots\ F(QQ_1\ \dots)\,d\theta d\phi\ \dots$$
$$+B\iint\ \dots\ PP_1\ \dots\ F_1(QQ_1\ \dots)\,d\theta d\phi\ \dots$$

It will be seen at once that this is an extension of Poisson's solution of the equation $\frac{du}{dt}=a^2\frac{d^2u}{dx^2}$. There is only one arbitrary function in my solution, and only one in Poisson's, as thus treated. But he has given one with two arbitrary functions, and I believe a similar investigation would apply to my general equations if the equation,

$$\frac{an^{\nu}+bn^{\nu-1}+\ .\ .\ .}{\alpha n^{\mu}+\beta n^{\mu-1}+\ .\ .\ .}=m,$$

were solved with regard to (n), and thus n found in terms of (m).

III. "The Force Function in Crystals." By ALFRED EINHORN, Ph.D. Communicated by G. MATTHEY, F.R.S. Received November 27, 1884.

(Abstract.)

The first part of the paper which appears at present restricts itself to the consideration of the Tesseral, Tetragonal, and Rhombic systems. By means of a well founded assumption in regard to the stress-distribution in crystals of the above systems, the equilibrium conditions are deduced which further involve that the boundary of the configuration must either be plane or spherical.

It also appears that the statical conditions of the agency which causes crystallisation are the same as those so well investigated for gravitation and electricity.

The paper is divided into three chapters. The first chapter treats of the "Foundation of the Assumption." The assumption is that the stress upon any particle can only be transmitted in six direction-lines respectively at right angles in pairs to the three crystallographic axes—it is a consequence of the internal structure which is shown to be analogous to that of an ordinary cannon-ball pile by means of the cleavage properties, the external form and inertia relations of crystals.

The second chapter—"Derivation of the Force Function"—applies the three general differential equilibrium equations of an elastic solid subject to internal forces to the stated stress-distribution. In order to effect this it was necessary to deduce some peculiarities of the force function in a system of uniform density in equilibrium, and subject to internal forces when referred to the three principal axes of inertia through the mass centre. The character of the attracting agency here becomes evident.

The third heading, "Determination of the Boundary." Under this