IV. "The Influence of Stress and Strain on the Physical Properties of Matter. Part I. Moduli of Elasticity—*continued*. The Viscosity of Metals." By HERBERT TOMLINSON, B.A. Communicated by Professor W. GRYLLS ADAMS, M.A., F.R.S. Received December 9, 1884.

(Abstract.)

After a short account of the researches of Sir William Thomson and Professor G. Wiedemann, on the loss of energy of a wire when vibrating torsionally, the author proceeds to describe his own experiments on the same subject. The wire under examination was clamped at one end into a stout brass block, which was secured to the extremity of a strong iron bracket projecting from a wall. A wooden box nearly 600 cm. in length and 12 cm. square inside, protected the vertically suspended wire from currents of air. The box rested upon another, which measured about 40 cm. each way, provided with a glass window in the front, and a door at the side, which latter was opened only when it was necessary to make a readjustment of a vibrator attached to the lower extremity of the wire, and capable of moving freely inside the box. The bar of the vibrator was either clamped or soldered to the lower extremity of the wire, and on it were suspended two cylinders of equal mass and dimensions; by sliding the cylinders backwards or forwards on the bar the moment of inertia of the whole vibrator could be altered without changing the mass. The torsional vibrations of the wire were observed by the aid of mirror, scale, and lamp, so that, as the length of the wire was upwards of 600 cm., and the diameter rather less than 1 mm., very small molecular displacements were produced when the wire was vibrating.

It was found necessary to allow the wire to rest, after the adjustments had been made, for a length of time varying with the nature of the metal from one day to several weeks; as though great care was taken in arranging the wire, it was impossible in many cases to avoid imparting subpermanent torsion, which gradually came out under the influence of rest and repeated oscillation.

Very great care was taken both in starting the vibrator and in taking the observations, the results of which showed that the diminution of amplitude could in most cases be determined with considerable accuracy.

The various causes of the loss of energy of a vibrating wire are pointed out in the paper, and subsequent experiments proved that all of these are practically neglectable, except the resistance of the air and the internal molecular friction of the metal. A full account is

given of the method of eliminating from the results the effect of the resistance of the air, which with some metals plays a very unimportant part in diminishing the amplitude as compared with the internal friction, but with others, especially under certain conditions, is to be credited with almost the entire loss of energy.

A mathematical investigation of the loss of energy which would be experienced by a wire vibrating under the influence of torsional elasticity, if the internal friction of solids were like that of fluids, proved with reference to the proportionate diminution of amplitude (*a*) that it should be independent of the amplitude; (*b*) that it should vary inversely with the vibration-period. From the experiments it followed that though the condition (*a*) was satisfied, (*b*) was not, so that when the moment of inertia of the vibrator was altered without any change of mass, instead of the proportionate diminution of amplitude being in the inverse ratio of the vibration-period, it seemed to be to a considerable extent independent of the period.

"The fatigue of elasticity" (a term first used by Sir William Thomson), according to which a wire which had been kept vibrating for several hours or days through a certain range, came to rest much more quickly when left to itself than when set in vibration after it had been at rest for several days, was next the subject of investigation, and it was found that this elastic fatigue never showed itself when the wire, whatever might be the metal of which it was composed, was vibrated through amplitudes sufficiently within the limits of elasticity, but, on the contrary, repeated oscillation was in this case always attended with *diminution* of loss of energy, the diminution being subpermanent. No trace of elastic fatigue was discernible in the case of most of the metals examined so long as the vibrations did not exceed in amplitude the limits of the scale, but with nickel it was necessary to confine the amplitude to 100 scale-divisions, in order to avoid "fatigue of elasticity."

Moderate permanent extension was found to diminish the loss of energy of a copper wire and to increase that of an iron wire, provided a rest of one day was allowed; *recent* permanent extension *increased* in both cases the loss of energy. The difference between copper and iron in the above respect is no doubt due to the greater "coercive force" of the latter. Moderate permanent torsion had a much greater effect than permanent extension in increasing the loss of energy of an iron wire.

An examination of the effect of passing an electric current of from ·1 to ·3 ampère through a vibrating iron, nickel, or tin wire ended in proving that such a current, though capable of producing sensible circular magnetisation of the first two metals, had no sensible effect on the loss of energy of any of them.

During the investigation it was thought advisable to make experi-

ments on the loss of energy of a vibrating magnet, and the following facts were elicited:—

(*a.*) The loss of energy of a vibrating magnet, like that of a vibrating iron wire, is lessened by repeated oscillations, and, after the *first* adjustment, or after a slight jar, is lessened by rest only. Repeated oscillation produces a subpermanent diminution of loss of energy.

(*b.*) The diminution of amplitude due to magnetic causes only is like that due to the resistance of the air, and follows the same law, namely, that for different vibration-periods the diminution is inversely as the period, and therefore is in this respect quite unlike the diminution of amplitude resulting from internal friction.

During the whole of the experiments a most careful watch was kept upon any effect which change of temperature might produce in the loss of energy, and it was discovered that whilst with wires of tin, lead, aluminium, silver, platinum, nickel, *unannealed* piano-steel, zinc, copper, brass, German silver, and platinum-silver the loss of energy became greater when the temperature was raised; with iron, on the contrary, the loss of energy was diminished by the same cause.*

The marked difference between the effects produced on the loss of energy of annealed iron and of the other metals, by rise of temperature, caused an extended investigation to be made with annealed iron wire with the following results:—

(*a.*) The loss of energy of a torsionally vibrating iron wire is *permanently* diminished to a very large extent by repeated heating to 100° C. and cooling combined with long rest.

(*b.*) The loss of energy is very considerably diminished temporarily by rise of temperature, the value of the logarithmic decrement at the temperature 0° C. being about *twice* as great as the value at 100° C. The loss of energy at the temperature of 100° C. of an iron wire which has been repeatedly heated to 100° C. and cooled to the ordinary temperature of the room is so small that it may be *almost entirely* accounted for by the resistance of the air.

An examination was also made of the effect of change of temperature on the torsional rigidity of the metal, and it was found that this change could be represented by the formula—

$$r_t = r_0(1 - \cdot0001443t - \cdot0000015804t^2),$$

where r_t and r_0 represented the torsional rigidity at $t°$ C. and 0° C. respectively. Both from the observed torsional rigidity and from that calculated from the formula, it was found that the decrease of the torsional rigidity caused by rise of temperature from 0° C. to 100° C. is only about *half* of that got by Kohlrausch when making a similar investigation. The great discrepancy above alluded to can only have arisen from the different treatment of the iron previously to the actual

* Probably also annealed piano-steel.

testing, *repeated* heating and cooling, combined with oscillations for a great number of hours and long rest, having had the effect of rendering the iron much less susceptible to alteration of torsional rigidity from change of temperature.

At the present stage of the inquiry it is impossible to arrive at any definite conclusion as to any relationship between the viscosity of metals and their specific electrical resistance. It would seem indeed that in the case of the pure metals, those which have the greatest viscosity, such as lead and tin, are those whose specific electrical resistance is comparatively great, but even with the pure metals so many circumstances influence the loss of energy that a much more extended investigation must be made ere one can write with sufficient certainty on the point. In the case of the alloys, German silver, platinum-silver, and brass, the values of the logarithmic decrements do not seem to be greater than those pertaining to their components, whereas as regards specific electrical resistance we know that this is not so. Again we encounter the curious fact that whereas with iron the *electrical resistance* is more *increased* by rise of temperature than is the case with any other metal, the *logarithmic decrement* is on the contrary *decreased* by the same cause.

A review of the whole experiments shows that the loss of energy due to *internal* friction of a torsionally vibrating wire does not accord with laws of fluid friction, but with those of *external* friction, inasmuch as the loss of energy from internal friction, like that from external friction, is to a great extent *independent of the velocity*. Whether with external as with internal friction the loss of energy would be independent of the pressure *provided the molecules of the two surfaces were brought into very close proximity*, remains yet to be decided.

V. "Professor Malet's Classes of Invariants identified with Sir James Cockle's Criticoids." By the Rev. ROBERT HARLEY, F.R.S. Received December 10, 1884. Read December 18.

1. Professor Malet, in a paper entitled "On a Class of Invariants," printed in the "Philosophical Transactions" for 1882, Part III, pp. 751–776, says he has not seen it noticed by any mathematician that "in the theory of Linear Differential Equations there are two important classes of functions of the coefficients which have remarkable analogies to the invariants of Algebraic Binary Quantics." He then proceeds to determine the forms of such functions, and to give examples of their application. Soon after the publication of the abstract of Professor Malet's paper in the Proceedings of the Society (vol. 33,