Hence $$\int_{-\infty}^{\infty} \epsilon^{2au-u^2} du = \epsilon^{a^2} \sqrt{\pi}$$

and therefore $$\epsilon^{(m+n)^2} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \epsilon^{2(m+n)u-u^2} d\theta$$

also $$\epsilon^{-m^2} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \epsilon^{-\rho^2} \cos 2m\rho d\rho, \text{ and so for } \epsilon^{-n^2}.$$

These transformations give the required form.

If we have two partial differential equations—

$$F_1\left(x\frac{d}{dx}, \ y\frac{dx}{dy}, \ z\frac{d}{dz}\right)u = 0,$$

$$F_2\left(x\frac{d}{dx}, \ y\frac{d}{dy}, \ z\frac{d}{dz}\right)u = 0,$$

then substitute as before $Ax^m y^n z^r$ for u; then we have the equations

$$F_1(m, n, r) = 0, \ F_2(m, n, r) = 0,$$

whence $m = \phi(r)$, $n = \chi(r)$, and we fall back on the first case.

"On Certain Definite Integrals." No. 14. By W. H. L. Russell, A.B., F.R.S. Received June 18, 1885.

It follows from the expansion of $\cos^n\theta$ in terms of the cosines of the multiples of θ, that

$$n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ . \ . \ . \ \frac{n-r+1}{r} = \frac{2^n}{\pi} \int_0^{\pi} \cos n\theta \cos (n-2r)\theta d\theta,$$

and consequently this theorem can be used in the summation of series involving binomial coefficients. I propose to give a few examples of this.

From the binomial theorem, when the index is even, we have

$$\int_0^{\pi} d\theta \frac{\cos^{2n}\theta \sin (n-1)\theta \cos n\theta}{\sin \theta} = \frac{\pi}{2^{2n}} \left\{ 2^{2n-1} - 1 - 1 - \frac{(2n-1) \ . \ . \ (n+1)}{1 \ . \ 2 \ . \ . \ . \ (n-1)} \right\}$$

and when the index is odd,

$$\int_0^{\pi} d\theta \frac{\cos^{2n+1}\theta \sin n\theta \cos n\theta}{\sin \theta} = \pi \left\{ \tfrac{1}{2} - \frac{1}{2^{2n+1}} \right\}$$

Since $(1+x)^{n-1} = (1+x)^n(1-x+x^2-x^3+ \ . \ . \ . \ . \ )$, therefore equating the coefficients of x^r, we have

$$1-n+n\,.\,\frac{n-1}{2}-n\,.\,\frac{n-1}{2}\,.\,\frac{n-2}{3}+\,.\,.(r+1)\text{ terms}$$

$$=(-1)^r.\frac{(n-1)(n-2)(n-3)\;\ldots\ldots\;(n-r)}{1\,.\,2\,.\,3\,\ldots\ldots\,r}$$

Hence $$\int_0^\pi \cos^{n-1}\theta d\theta\left\{\cos(n+1)\theta+(-1^r\cos(n-2r+1)\theta)\right\}$$

$$=\frac{\pi}{2^n}(-1)^r\frac{(n-1)(n-2)\;\ldots\ldots\;(n-r)}{1\,.\,2\,.\,3\,\ldots\ldots\,r}$$

"The Vortex Ring Theory of Gases. On the Law of the Distribution of Energy among the Molecules." By J. J. THOMSON, M.A., F.R.S., Fellow of Trinity College, Cavendish Professor of Experimental Physics in the University of Cambridge. Received June 4, 1885.

In any kinetic theory of gases the statistical method of investigation must be used, and since the separate molecules of the gas are supposed to possess some properties to very different extents, it is necessary to know how many molecules there are which have the measure of any given property between certain limits. Thus the question of the distribution of configuration and velocity amongst the molecules is one of the most important problems in any theory of gases.

This problem has been solved for the ordinary solid particle theory by Maxwell and Boltzmann, and their researches are the more valuable as the results do not depend on any assumption about the law of force between the molecules.

In this paper I shall attempt to solve the same problem for the vortex atom theory of gases. In this case the question is a little more complicated, as the radii of the vortex rings can vary as well as their velocities. This is one of the most striking differences between the two theories; according to the ordinary theory all the molecules of a gas are of the same size, according to the vortex atom theory the molecules of the same gas vary in size. If this be true, a porous plate of the requisite degree of fineness might play in this theory the part which Maxwell's demons play in the ordinary theory. For let us suppose that we have two chambers, A and B, separated by a porous plate, and that A is filled with gas initially while B is empty, then if the pores in the porous plate are so fine that only the smaller molecules can get through from A to B, then, though some of the molecules will recross the plate, some gas will remain in B, and the