"On the Changes produced by Magnetisation in the Length of Rods of Iron, Steel, and Nickel." By Shelford Bidwell, M.A., LL.B. Communicated by Professor Frederick Guthrie, F.R.S. Received April 1. Read April 23, 1885.

The earliest systematic experiments on the effects produced by magnetisation upon the length of iron and steel rods were those of Joule, an account of which is published in the "Phil. Mag." of 1847. The experiments were made with bars 36 inches long, which were placed inside a solenoid 38 inches long; and the variations in the length of the bars when currents of electricity were passed through the solenoid were measured by means of a delicate micrometer, each division of which indicated a change of $\frac{1}{138528}$ inch.

Using bars of iron and soft steel, he found that their length was increased by magnetisation, the elongation varying up to a certain point as the square of the intensity of the magnetisation, temporary or permanent, of the bar, and he remarked that the elongation was, for the same magnetisation, greater in proportion to the softness of the metal.

When the metal was hard steel it appeared that "the bar was slightly *increased* in length every time that contact with the battery was broken." On passing the first current through the magnetising coil the length was unaffected, but when the circuit was broken after the passage of this current there occurred a small elongation equivalent to a fifth of a micrometer division, and each succeeding make and break of the current was accompanied by a further small elongation.

These experiments were made with currents of gradually increasing strength; Joule appears never to have tried what would be the effect of applying the same current twice in succession. Had he done so there is reason to believe, as will appear hereafter, that effects of a somewhat different character would have been observed. He attributed the increase in length when the current was interrupted "to the state of tension in the hardened steel," adding that he "found that soft iron wire presented a similar phenomenon when tightly stretched."

The phenomena were, however, not exactly identical in the two cases. From the account which he proceeds to give of his experiments with stretched wires, it appears that when the tension of the wire exceeded a certain limit, the effects produced by the current were exactly the opposite of those which occurred when the wire was unstretched; magnetisation, instead of causing the wire to lengthen temporarily, caused it to shorten, while it resumed its original length

when the magnetising force ceased to act. Using a soft iron wire a quarter of an inch in diameter, Joule found that when it was loaded with a weight of 408 lbs. the effects were the same in direction (though smaller in degree) as when the wire was unstretched; its length increased when it was magnetised, and diminished to the same extent when it was demagnetised. When, however, the load was increased to 740 lbs. the effects were reversed, and magnetisation produced temporary retraction. After describing this experiment Joule expresses his belief that with a tension of about 600 lbs. (roughly the mean of 408 and 740), " the effect on the dimensions of the wire would cease altogether in the limits of the electric currents employed," *i.e.*, that currents which produced on his tangent galvanometer deflections ranging from 6 to 58 degrees would neither increase nor diminish the length of his quarter inch wire when stretched with a weight of 600 lbs. If he had actually made the experiment he would perhaps have found that the length of the wire was increased by a weak current, that a current of medium strength would have had no effect whatever, and that one of his stronger currents would have caused the wire to retract.

Joule's experiments have many times been repeated, and his results generally confirmed. In particular Professor A. M. Mayer of the United States, carried out a series of very careful experiments with apparatus of elaborate construction and great delicacy.* The conclusions at which he arrived were in accord with those of Joule so far as regards iron; but in the case of steel there is apparently some discrepancy. Mayer found that (after the first magnetisation) the steel rods with which he worked, whether soft or hard, were invariably shortened when the circuit was made and lengthened when it was broken, the same current being used for the first and for the subsequent magnetisations. This result is, however, not necessarily inconsistent with Joule's, because the conditions of the experiment were not the same, the second current which Joule applied being stronger than the first, and the third stronger than the second. Again, while in the case of Joule's " soft steel " the movements were in the same direction as those observed with iron (though smaller in degree), Mayer's " soft steel " behaved in exactly the opposite manner, the movements (after the first magnetisation) being in the same direction as those which occurred when harder steel was employed. This difference may be accounted for, as Mayer himself suggests, by supposing that his so-called " soft steel " was harder than Joule's. Possibly too there was a sufficient difference in the magnetising forces employed in the two cases to affect the results of the experiments. More will be said on this point further on. The effects resulting from the *first* action of the magnetising current are

* "Phil. Mag.," vol. xlvi, p. 177.

altogether distinct. The permanent magnetisation so produced was found by Mayer to impart a small permanent elongation to rods of soft and blue-tempered steel, and a small permanent retraction when the steel was tempered yellow. Mayer's paper also contains some new facts relating to details of minor importance.

In 1882 Professor Barrett published an account in "Nature," vol. xxvi, p. 585, of some experiments which he had made not only on iron but also on bars of nickel and cobalt, with a view of ascertaining the effect of magnetisation on their length; cobalt, he discovered, behaved like iron, but the elongations were smaller; nickel, however, *retracted* under the influence of magnetisation, the amount of its retraction being twice as great as the elongation of iron under similar circumstances.

The knowledge on the subject up to the present time may be summarised as follows:—

1. Magnetisation causes in iron bars an elongation, the amount of which varies up to a certain limit as the square of the magnetising force. When the "saturation point" is approached the elongation is less than this law would require. The effect is greater in proportion to the softness of the metal.

2. When a rod or wire of iron is stretched by a weight, the elongating effect of magnetisation is diminished; and if the ratio of the weight to the section of the wire exceeds a certain limit, magnetisation causes retraction instead of elongation.

3. Soft steel behaves like iron, but the elongation for a given magnetising force is smaller (Joule). Hard steel is slightly elongated both when the magnetising current is made and when it is interrupted, provided that the strength of the successive currents is gradually increased (Joule). The first application of the magnetising force causes elongation of a steel bar if it is tempered blue and retraction if it is tempered yellow; subsequent applications of the *same* external magnetising force cause temporary retraction whether the temper of the steel be blue or yellow (Mayer).

4. The length of a nickel bar is diminished by magnetisation, the maximum retraction being twice as great as the maximum elongation of iron (Barrett).

In order that the results obtained by Joule and Mayer might be comparable with those of my own experiments, I have made an attempt to estimate the magnetising forces with which they worked.

In the first series of Joule's experiments—those in which he observed the elongation of iron and steel rods not under traction, he used a coil of the following dimensions:—

Length of coil........	38 in. =	96·5 cm.
Internal diameter	1·5 " =	3·8 "
Length of wire.......	110 yds. =	10,058 "

Each turn would contain $3{\cdot}8\pi$ cm. of wire (or rather more) = 12 cm. Therefore the number of turns would be 10,058/12=838. If there were more than one layer of wire, the number of turns would be fewer. The magnetising force would be nearly—

$$4\pi\frac{838}{96{\cdot}5}C=109C,$$

C being the current in C.G.S. units.

In Joule's experiments with stretched wires another coil was used.

Length of coil........	11·5 in. =	28·5 cm.
Internal diameter	1 "	
Length of wire.......	33 yds.	=1188 in.
Thickness of wire	0·1 in.	

The number of turns of wire would therefore be about $1188/1{\cdot}1\pi = 344$, and the magnetising force about

$$4\pi\frac{344}{28{\cdot}5}C=152C,$$

C being, as before, the current in C.G.S. units.

Joule also describes his tangent galvanometer, and gives the deflection which the magnetising current produced in every case. The galvanometer "consisted of a circle of thick copper wire 1 foot in diameter, and a needle half an inch long furnished with an index." The radius, therefore, was 6 inches = 15·2 cm., and the constant, G, approximately $2\pi/15{\cdot}2=0{\cdot}41$.* The horizontal component of the earth's magnetic force was, at the date of Joule's paper, about 0·17; thus the factor by which the tangents of the angles of deflection should be multiplied to give the deflecting currents in C.G.S. units is $0{\cdot}17/0{\cdot}41=0{\cdot}41$.

The greatest deflection recorded in Joule's experiments with iron was 62° 48′, the tangent of which is 1·95; the magnetising force was therefore

$$1{\cdot}95\times0{\cdot}41\times109=87.$$

The greatest deflection in his experiments with steel was 70°30′ $=\tan^{-1}2{\cdot}824$, the corresponding magnetising force being 126.

The greatest galvanometer deflection in the experiments with stretched wires was 61° 25′, the tangent of which is 1·835, the corresponding current 0·75 C.G.S. units, and the magnetising force 114.

Mayer used a coil 60·25 inches=153 cm. in length, the number of turns being 1919. The magnetising force at the centre of his coil was therefore about $4\pi\frac{1919}{153}C=157{\cdot}5C$.

* See Clerk Maxwell's "Electricity," vol. ii, p. 325.

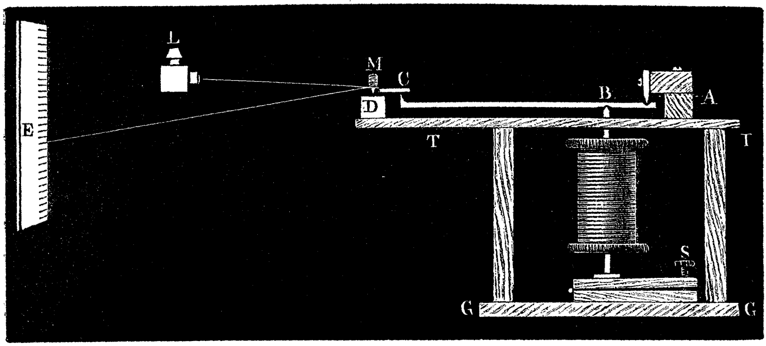
It is less easy to estimate the strength of his current, since he gives no galvanometer readings, nor any details as to the electromotive force and resistance of his battery. The resistance of his coil he states to have been 0·75 ohm, and that of the rest of the circuit (exclusive of the battery) about 0·25 ohm, making 1 ohm in all. He used a battery of twenty-five Bunsen cells, and, in his own words, "the above interpolar resistance showed that the maximum effect of magnetisation would be given by connecting the twenty-five cells five in couple and five in series." This implies that the resistance of the battery as thus arranged would be not far from 1 ohm, which, unless the cells were very small, is surprisingly high. Taking the electromotive force of a Bunsen cell to be 1·9 volts, the electromotive force of Mayer's battery would be 9·5 volts, and the current $9{\cdot}5/2=4{\cdot}75$ ampères$=0{\cdot}475$ C.G.S. unit. The magnetising force would therefore be about $157{\cdot}5\times 0{\cdot}475=75$ nearly. But 1 ohm is probably too high an estimate for the resistance of the battery. Assuming the resistance of the battery, leading wires, and connexions to be 0·5 ohm (which is the lowest reasonably probable estimate), the current would be 7·6 ampères$=0{\cdot}76$ C.G.S. unit, and the magnetising force $157{\cdot}5\times 0{\cdot}76=118$ units. In point of fact the force was probably something more than 75 and less than 118.

In my own experiments both the magnetising coil and the rods of metal were much shorter than those used by Joule and Mayer. The length of the coil is 11·5 cm.; it contains 876 turns of wire 1·22 mm. in diameter, wound in twelve layers on a brass tube with boxwood ends. The internal diameter of the tube is 1·5 cm., that of the coil is 1·9 cm., and its external diameter is 5·2 cm. The mean length of the diagonals of the cylindrical layers of wire is 12 cm., and the field at the centre due to a current through the coil is approximately $4\pi\frac{876}{12}C=918\,C$ when C is expressed in C.G.S. units, or 91·8C when C denotes the current in ampères. The strongest current which I have hitherto used was 3·27 ampères, and the greatest magnetising force was $3{\cdot}27\times 91{\cdot}8=300$ units, Joule's maximum having been 126, and Mayer's, on the most favourable estimate, not greater than 118.

The apparatus for performing the experiment, which is of a very simple nature, is shown in fig. 1.* A mahogany table, TT, supported by three stout legs, the lower ends of which are let into a base board, G, carries a lever arrangement for causing the elongating or retracting rod under examination to deflect a mirror, M, which turns about its horizontal diameter upon knife edges. The lower end of the rod rests in a conical recess formed in a brass plate which is attached to a hinged board: the height of the plate can be adjusted by turning a screw, S. The rod passes through the coil, and also through a hole

* The diagram is not drawn to scale.

Fig. 1.



in the table, and its upper end, which is chisel-shaped, acts at B upon a brass lever, one end of which abuts upon the knife edge, A, and the other upon a short arm, C, fixed to the back of the mirror. Shallow notches in the form of obtuse angles are cut in the lever (which is of square section) at the points where it bears upon the knife-edge fulcrum and the end of the rod. The end of the lever remote from the fulcrum has the form of a chisel, the edge of which is turned upwards and fits into a shallow groove cut transversely in the arm of the mirror. By means of a magic lantern, L, illuminated by a lime-light, the image of a horizontal wire is, after reflection from the mirror, projected upon a distant vertical scale, E. A slight deflection of the mirror causes a considerable movement of the image of the wire upon the scale. The actual dimensions are as follows:—The distance AB=10 mm., BC=170 mm., DC=7 mm., DE=3200 mm. (10 feet 6 inches). The multiplying power of the arrangement, the beam of light being horizontal, is $3200 \times 17 \times 2/7 = 15{,}543$ times.* The scale is one of Elliott's ordinary galvanometer scales, each division of which is equal to a fortieth of an inch or 0·64 mm. Therefore a movement of the focussed wire through one scale division indicates a difference in the length of the rod of $0{\cdot}64/15{,}543 = 0{\cdot}000041$ mm. The length of magnetic metal in the rods used is 100 mm., so that a movement of one division shows a difference of 0·00000041 in the length, equal to about a two and a half millionth part.

For projecting the image of the wire upon the scale, a half-plate photographic portrait lens of high quality was used. When the best definition was secured, it was possible after a little practice to read the deflections to a quarter of a scale division with tolerable certainty.

* The multiplier 2 is used because the angle through which the reflected beam of light is deflected is twice the angle of deflection of the mirror.

In working with this apparatus, three possible sources of error were soon revealed. The first was due to the expansion of the rod in consequence of the heating of the coil by the current. This effect could be distinguished from the elongation resulting from magnetisation, by the fact that the latter took place quite instantaneously, while the expansion due to heat was gradual; but it was likely to lead to uncertainty in estimating the amount of permanent elongation accompanying the permanent or residual magnetism of the rod. This uncertainty could be reduced to a minimum by taking care to close the circuit only for a second or two when making an observation. The second possibility of error resulted from the gradual yielding of the magnetic rod, or, more probably, of some part of the base or frame of the apparatus, under the pressure, small though it was, of the brass lever. This may to a great extent be obviated by adjusting the apparatus and leaving it for half an hour before making an observation; but I am not quite sure that it ever entirely disappeared, for even though the image of the wire remained perfectly steady upon the scale so long as the apparatus was quite undisturbed, it is possible that the shocks produced by magnetising and demagnetising the rod might cause a sudden slight upward movement of the image, thus making the permanent elongation of the rod appear to be somewhat less than it in fact was. I think, however, that the error, after the apparatus has been at rest for a sufficient time, must be very small. In observing the purely temporary elongation resulting from so much of the magnetisation as is purely temporary, no uncertainty whatever need arise from this cause, for the experiment can easily be repeated as often as may be necessary to obtain uniform results; but an observation of the permanent extension cannot be repeated without dismantling the apparatus and demagnetising the rod, after which its condition will probably not be exactly the same as before.

Lastly, errors may arise from the electromagnetic attraction existing between the coil and the rod, which tends to draw a uniform rod into such a position that the middle point of its axis shall coincide with the centre of the coil. As at first constructed, the coil in my apparatus was attached by means of screws to the under side of the table, and the rod under examination passed freely through it, touching nothing whatever except the brass plate at the bottom and the lever at the top. The length of the *magnetic* portion of the rod—that which was the subject of the experiment—was in every case, as already stated, exactly 10 cm., or 1·5 cm. less than the length of the coil. But the distance from the brass base plate to the lever was 21 cm., and in order to increase the experimental rods to this length, pieces of thick brass wire were screwed or soldered to their two ends; thus the rods when prepared for the experiment were of a compound form, consisting of iron, steel, or nickel in the middle, and brass at each end

The relative lengths of the pieces of brass were so adjusted that when the compound rod was fixed in position, the centre of the magnetic portion of it coincided as nearly as possible with the centre of the coil. This arrangement was, however, found not to be entirely satisfactory. It was difficult to secure the required coincidence with perfect accuracy, and it was necessarily somewhat disturbed during the adjustment of the image of the wire upon the scale; while, even supposing that the geometrical coincidence was perfect, it might well happen that owing to inequalities in the magnetic properties or physical condition of the rod, the source of error might still exist. A simple experiment showed conclusively the immense importance of guarding against any trace of this interaction between the rod and the coil. A compound rod of iron and brass was prepared, such that when it was placed with one end uppermost the centre of the iron was somewhat below that of the coil, and when the other end was uppermost the centre of the iron was about 5 mm. above that of the coil. A current was passed through the coil when the iron was in the first position and a certain elongation was indicated. The position of the rod was then reversed and the same current passed. It was expected that the apparent elongation would be diminished, but in point of fact an actual *retraction* equivalent to two or three scale divisions was indicated. The sucking action of the coil caused the lower end of the rod to press upon the base with increased force; the base yielded a few hundred thousandths of a millimetre, and this was sufficient, in spite of the real elongation of the rod, to cause the image of the wire upon the scale to move in the direction of retraction.

It appeared that the only method of avoiding with certainty the misleading effects of this attraction between the coil and the rod would be to attach the two together. The pressure upon the base would then depend simply upon the joint weights of the coil and the rod, and would not be varied by any interaction between them. The coil was therefore detached from the table, and its ends were fitted with corks through which the experimental rod was passed, care being taken that it fitted tightly at both ends. The arrangement was then exactly as shown in fig. 1, the coil being supported solely by the rod; and it was so used in all the experiments described in this paper.

Before giving an account of the new effects which I have obtained, it may be well to state how far the maximum elongations and retractions of iron and nickel bars, as indicated by my apparatus, accord with those published by previous experimenters. This is done in the subjoined table.

Table I.

Metal.	Diameter in millimetres.	Observer.	Magnetising force.	Total elongation in fractions of the length.
Soft iron	6·35	Joule	64	·00000562
"	12·7	Mayer	Between 75 and 118	·00000457
Iron	25·4	Barrett	"Maximum magnetisation"	·00000385
"	2·65	Bidwell	45	·00000450
"	3·65	"	73	·00000389
Nickel	25·4	Barrett	Unknown	·00000769
"	9 × 0·75	Bidwell	300	·00001000

These figures include the total elongation of the rods, *i.e.*, that due to the permanent as well as to temporary magnetisation.

The magnitude of the effect undoubtedly varies considerably with the quality of the iron employed; that used in my own experiments was ordinary commercial iron wire annealed by being heated red hot and slowly cooled. The permanent elongation was, in both cases, rather more than one-third of the whole. For reasons already given, it was much easier to measure with certainty the temporary than the permanent effect.

By using thinner rods and greater magnetising forces than those previously employed, I have arrived at the curious and interesting fact which it is the main purpose of this paper to describe. If the magnetisation be carried beyond a certain critical point, the consequent elongation, instead of remaining stationary at a maximum, becomes diminished, the diminution increasing approximately with the magnetising force. If the force be sufficiently increased a point is arrived at, varying according to the dimensions and quality of the iron, where the original length of the rod is totally unaffected by magnetisation. And if the magnetisation be carried beyond this point, the original length of the rod will be reduced. To take a concrete example: the maximum temporary elongation of my thinnest iron rod occurs when the external magnetising force is about 45; an external force of about 212 has no effect whatever upon the length, which remains exactly the same as when the rod is unmagnetised; a force of about 300 causes the length of the rod to diminish, the amount of the retraction thus produced being about one-half that of the maximum elongation. Here I had exhausted the capability of my battery power—seven Grove cells—but so far I could detect no indication that a limit of retraction was being approached.

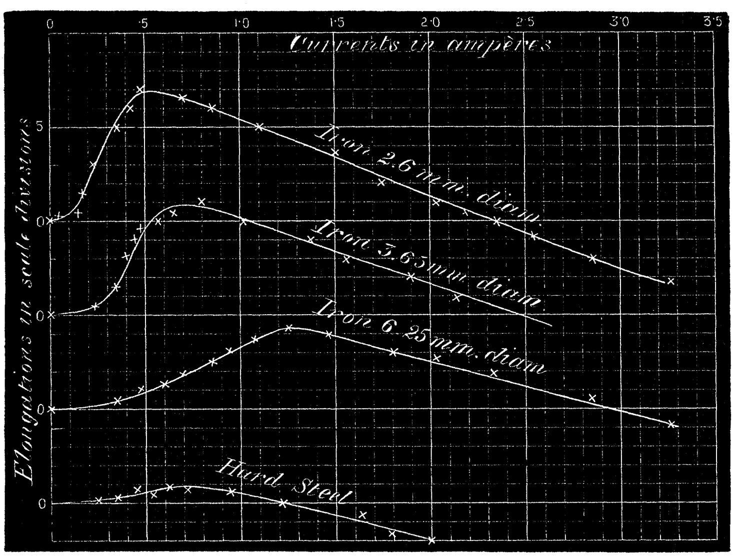
Table II.

Iron.						Hard steel.		Nickel.	
No. 1 diameter=2·65 mm.		No. 2 diameter=3·65 mm.		No. 3 diameter=6·25 mm.					
Magnetising current.	Elongation in scale divisions.	Magnetising current.	Elongation in scale divisions.	Magnetising current.	Elongation in scale divisions.	Magnetising current.	Elongation in scale divisions.	Magnetising current.	Elongation in scale divisions.
0·047	0·25	0·23	0·5	0·34	0·50	0·26	0·12	0·11	− 1·0
0·14	0·5	0·34	1·5	0·49	1·00	0·37	0·25	0·16	− 2·0
0·18	1·5	0·41	3·0	0·60	1·25	0·46	0·50	0·26	− 3·0
0·23	3·0	0·45	4·0	0·70	1·75	0·53	0·50	0·34	− 4·0
0·34	5·0	0·49	4·5	0·86	2·50	0·62	0·87	0·49	− 5·5
0·42	6·0	0·56	5·0	0·95	3·12	0·72	0·87	0·60	− 6·0
0·49	7·0	0·66	5·5	1·08	3·75	0·95	0·62	0·82	− 8·5
0·70	6·5	0·80	6·0	1·24	4·25	1·20	0·00	1·03	−11·0
0·84	6·0	1·01	5·0	1·48	4·00	1·64	−0·75	1·32	−12·0
1·10	5·0	1·38	4·0	1·80	3·00	1·80	−1·50	1·73	−16·0
1·51	3·5	1·68	3·0	2·03	2·50	2·00	−2·00	2·09	−18·0
1·75	2·0	1·89	2·0	2·31	1·75			2·54	−21·0
2·04	1·0	2·16	1·0	2·85	0·50		Mean of two experiments.	2·85	−22·0
2·19	0·5			3·27	−1·00			3·27	−24·0
2·35	0·0								
2·54	−0·75				Mean of two experiments.				
2·85	−2·0								
3·27	−3·25								

I made a systematic course of observations with three iron rods of different thicknesses prepared in the manner described above. All were of ordinary commercial iron annealed in the usual manner. Their respective diameters were 2·65, 3·65, and 6·25 mm., and the length of each was 100 mm. The strength of the successive magnetising currents in ampères and the corresponding temporary elongations in scale divisions are given in Table II. The currents can be approximately expressed as magnetising forces by multiplying by 91·8, and the scale divisions can be reduced to ten-millionths of the length of the rod by multiplying them by 4.

The results are also shown graphically in the first three curves of fig. 2, in which the abscissæ represent the magnetising currents and the ordinates the elongations.

Fig. 2.



The elongations are those due to *temporary* magnetisation only. In order to avoid the uncertainty attached to the elongating effects of permanent magnetism, I always at the beginning of an experiment, and before making an observation, passed through the coil the strongest current at my disposal, thus permanently magnetising the rod to saturation. There appear to be two kinds of residual magnetism. When a strong current has been passed through the coil, the magnetism which remains after the current has ceased to flow is for the most part of a sub-permanent nature. If the rod is undis-

turbed, the sub-permanent magnetism will remain without material diminution for perhaps half an hour, but in a short time it rapidly falls off. If, however, the rod is shaken, or even if it is removed from the coil with the greatest care, the magnetism which I have called sub-permanent instantly disappears. But, after the destruction of the sub-permanent magnetism, there still remains a small quantity of magnetism of a nature which may properly be called permanent, since it persists for days, or perhaps indefinitely, unless violent measures are resorted to for its removal. I find, by experiments which will be referred to hereafter, that when an iron rod has once been sub-permanently magnetised by a strong current, the intensity of the sub-permanent magnetisation is absolutely unaffected by the action of currents weaker, or not stronger, than the first. For a limited time (say half an hour) currents of varying strength may be passed through the coil, and the *additional* magnetism produced by their action is of a *purely* temporary nature, disappearing completely when the current ceases to flow, and leaving the sub-permanent magnetism exactly where it was before. The elongations referred to in my tables and curves are due to purely temporary magnetisation.

The strength of the magnetising current was varied by means of a box of resistance coils, and was measured by a Helmholtz tangent galvanometer with four separate coils, of Elliott's manufacture. After the first two or three preliminary experiments, no attempt was made to read the galvanometer at the time when the observations of the elongations were made: for, in order to do so, it was found necessary to keep the circuit closed for a period which was sufficiently long to cause the coil to become heated, and confusion was introduced owing to the heat expansion of the rod. A note was made of the resistances successively inserted, and the currents corresponding to the several resistances were afterwards leisurely and carefully determined. It was soon found that the action of the battery was so constant that several elongation experiments might be made on the assumption that the same currents accompanied the same resistances without any sensible error, except perhaps a slight one in the case of the strongest currents; but the estimated currents were from time to time checked by reference to the galvanometer, and when any material variation was observed, a fresh series of galvanometer readings was made.

An examination of the three iron curves discloses the following facts:—In every case the form of the curve for the first part of the ascent is sufficiently nearly parabolic in form to afford confirmation of Joule's law, that the elongation varies up to a certain point as the square of the magnetisation. After passing the maximum, the curve assumes a form which is apparently intended to be a straight line; at all events, no single observation deviates from the straight line by an amount equivalent to more than half a scale division. If this is so,

the retraction after the maximum elongation increases with the external magnetising force.

No certain indication of an approach to a limit of retraction is observable in the curves. Stronger magnetising forces would of course show one, and I hope to be able to repeat the experiments with greater battery power.

The maximum elongation is reached by the three rods with magnetising currents which are the same in order of magnitude as the diameters of the rods.

Lastly, it appears from the curves that the amount of maximum elongation is smaller when the diameter of the wire is greater. The successive maxima are 7, 6, and 4·25, and if an error of a quarter of a scale division be allowed, these maxima will be found to be inversely proportional to the square roots of the diameters of the respective wires.

$$7 \times \sqrt{265} = 112$$
$$6 \times \sqrt{365} = 114$$
$$4 \cdot 25 \times \sqrt{625} = 106^*$$

It seems to me difficult to account satisfactorily for this variation of the maximum elongation. It is of course easy to understand why a greater external magnetising force should be required to produce a given intensity of magnetisation in a thick rod than in a thin one. But it is not at first sight at all evident why, when the same magnetisation is produced, the elongation should not be the same in both cases. Possibly my results may be due to a mere accident, such as a difference in the qualities of the three specimens of iron; but their apparent regularity renders such an explanation somewhat improbable.

It seemed extremely desirable that, if possible, a connexion should be established between the point of maximum elongation and some definite phase of the magnetisation of the rod. Much time and labour were spent in endeavours to investigate the magnetisation by a method of induction; but probably owing to the fact that the galvanometer used—one of Elliott's Thomson galvanometers, with the usual astatic system of magnets and an aluminium damping vane—was unsuited for the purpose, no results of any value were obtained. I was more successful in an attempt to measure by a deflection method the relative values of the temporary moments which various magnetising currents produced in the three rods. The coil was placed in a horizontal position, and one of the rods inserted in the tube, where it was supported axially by means of corks at the two ends. A reflecting galvanometer was placed at a suitable distance from it, as determined by preliminary trials, and the height and disposition of the galvanometer were so adjusted that its magnet was on a level with

* If the last elongation had been 4·5 the product would have been 113.

the axis of the coil, while the direction of the magnet bisected the axis of the coil at right angles. The galvanometer and the coil were connected in circuit with the box of resistance coils, the tangent galvanometer, and the battery, the connexions being so arranged that the electromagnetic actions of the magnetising coil and the galvanometer coil urged the galvanometer needle in opposite directions. The iron rod being temporarily removed from the coil, the galvanometer, which had a resistance of 1400 ohms, was shunted with a few centimetres of German silver wire, and the length of the shunt was adjusted by trial until, when a strong current was passing, the action of the galvanometer coil upon the needle exactly balanced that of the magnetising coil and the connecting wires of the circuit, so that, on depressing the contact key, no movement of the needle occurred except (with the strongest currents) a slight momentary kick due to induction.

The iron rod being replaced inside the coil, a strong current was caused to circulate round it for two or three seconds. The rod was thus sub-permanently magnetised as in the former experiments. The line of light was then brought to zero of the galvanometer scale by means of the controlling magnet. Great care was taken in setting up the scale to place it accurately at right angles to the magnetising coil, and in such a position that the perpendicular upon it from the middle of the galvanometer needle met it exactly at the zero point, a specially made T-square being used for the purpose. The magnetic field in which the galvanometer needle hung was the resultant of those due to the controlling magnet, the horizontal component of the earth's force, and the sub-permanent magnetism of the experimental rod. No attempt was made to determine its strength.

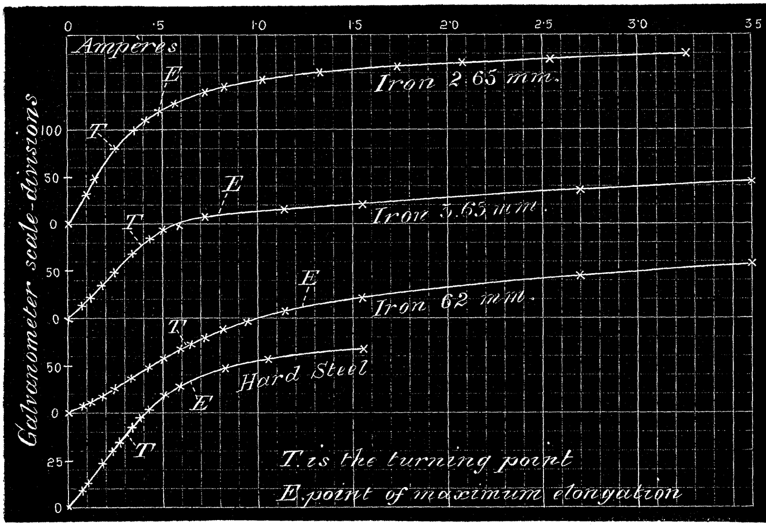
Currents of gradually increasing strength were successively passed through the coil, and the deflections corresponding to the temporary moments of the rod were noted. When the circuit was opened, the spot of light returned in most cases accurately to zero, and the permanent deviations from zero never exceeded two or three divisions, equal to one-fortieth of an inch. The results are given in Table III, and shown graphically in the curves in fig. 3. In each separate experiment the galvanometer deflections are proportional to the temporary moments; but these deflections and the ordinates of the curves are purely arbitrary, and as regards the absolute values of the moments, they give no information, nor are the ordinates of one curve comparable with those of another.

The relative changes in the values of the temporary moments with increasing magnetising forces are, however, clearly shown, and to ascertain the nature of these was the sole object of the experiment. The distance between the galvanometer magnet and the three rods was respectively 25 cm., 36·5 cm., and 53 cm.

Table III.

Iron.						Hard steel.		Nickel.	
No. 1 diameter=3·65 mm.		No. 2 diameter=3·65 mm.		No. 3 diameter=6·25 mm.					
Current.	Deflection.	Current.	Deflection.	Current.	Deflection.	Current.	Deflection.	Current.	Deflection.
0·11	32	0·012	2	0·012	1	0·012	2	0·012	1
0·16	48	0·073	15	0·073	8	0·073	9	0·073	8
0·26	81	0·108	22	0·108	11	0·108	13	0·108	11
0·34	98	0·185	38·5	0·185	19	0·185	23	0·185	19
0·41	110	0·237	50	0·237	26	0·237	30	0·237	27
0·49	121	0·336	69	0·336	37	0·274	35	0·274	31
0·57	129	0·426	85	0·426	48	0·336	42	0·336	40
0·60	133	0·501	94	0·501	58	0·389	47	0·426	52
0·72	140	0·597	99	0·597	66	0·426	51	0·501	60
0·82	145	0·720	108	0·642	73	0·501	59	0·597	67
1·03	153	1·137	116	0·720	79	0·597	64	0·819	80
1·32	160	1·563	124	0·819	87	0·819	74	1·137	90
1·73	167	2·700	137	0·960	96	1·137	79	1·563	98
2·09	171	3·606	146	1·137	106	1·563	84		
2·54	176			1·563	122				
3·27	180			2·700	144				
				3·606	158				

Fig. 3.



The points corresponding to the maximum elongation in the three curves are marked with the letter E. At first sight they seem to possess no particular distinguishing characteristic in common. Indeed the only points in the curves which appear to be marked by any special property are those which are called by Chrystal ("Enc. Brit.") after Wiedemann, the "turning points." Up to these points the temporary moments increase with the magnetising force, or even more rapidly; after the points are passed, the rate of increase in the temporary moments is less than that of the magnetising force. When the curve does not begin to ascend in a straight line, the turning points are found by drawing tangents from the origin: they are indicated in the curves by the letter T,* but it is not easy to determine their positions with perfect accuracy.

From a careful examination of these curves it appears probable that a simple relation does exist between the turning points and the points of maximum elongation, the abscissæ of the points of maximum elongation being almost exactly equal to twice those of the turning points.

* According to Rowland ("Phil. Mag.," 1873, vol. ii, p. 155), "the temporary magnetism increases continually with the current." This may be strictly true (up to the turning point) for rods or rings having the diameter of those used by Rowland. Thus the curve for my thickest iron rod ascends in a perfectly straight line; but a slight convexity towards the axis of x may be suspected in the medium one, and in the thinnest such convexity is quite evident. It is even more marked in the curve for a wire, 0·77 mm. in diameter, given in fig. 5.

I have also made a great number of experiments with steel. The results obtained appeared at first to be of the most inconsistent character, and it was with difficulty that I finally succeeded in evolving order out of them. The fact clearly is that the point of maximum elongation (when there is one) depends in a very remarkable manner upon the hardness or temper of the steel. Like Joule, I found that a soft steel rod which had been neither annealed nor tempered behaved in very much the same manner as iron, though the effects were smaller. There was a point of maximum elongation which was well defined, but I was not able by any current at my command to produce actual retraction. A rod which was made exceedingly hard by being dropped into cold water when at a bright red heat, had no point of maximum elongation within the limits of my magnetising currents, the temporary elongation continually increasing with increasing magnetisation, and giving no evidence of an approach to a limit. But when the same rod was let down to a yellow temper its behaviour was altogether different. With a very small magnetising force it showed signs of retraction, and the retraction increased with stronger magnetising currents, ultimately becoming very considerable. A rod tempered blue also retracted when magnetised, but the effect did not begin to appear until the magnetising force was much greater than was necessary when the temper was yellow, and after the rod had been still further let down by heating, a measurable *elongation* occurred before the magnetising force was sufficient to cause retraction.

Again, another piece of steel was hardened by raising it to a *dull* red heat and dropping it into cold water. It could easily be marked with a file, and was certainly softer than the last-mentioned rod before it was tempered, though it appeared to be harder than the same rod in the yellow condition. The new hard rod was slightly elongated by feeble magnetisation, and after passing a maximum retracted at about the same rate as iron.

All these various results, which at first sight appeared to be disconnected and inharmonious, point to the following conclusion:—The critical value of the magnetising force for a steel rod diminishes as the hardness becomes greater up to a certain point corresponding to a yellow temper, after which it increases, and, with very hard steel, becomes very high. There is therefore a critical degree of hardness for which the value of the critical magnetising force is a minimum; in steel of a yellow temper the value of the critical magnetising force is lower than in steel which is either softer or harder.

Some careful experiments were made with the hard steel rod last referred to. The results are contained in Table II, and the corresponding curve in fig. 2. As in the case of iron, the rod was first permanently magnetised by a current equal to the strongest subse-

quently used. The relations of the magnetising force and temporary moment appear in Table III and in the last curve of fig. 3. In this experiment the distance between the steel rod and the galvanometer magnet was 15 cm.

In the light of these experiments I have endeavoured to find an explanation of the anomalous results obtained by Joule and by Mayer with hard steel. It will be remembered that Joule, using gradually increasing currents, found that (after the first current, which produced no effect whatever while it circulated, but was followed by a small elongation when it had ceased) his hard steel bar was slightly elongated both when the current was made and when it was broken, the length of the bar being thus continuously increased. Though I have made many attempts, using steel rods in different conditions, to obtain Joule's results, I have never succeeded in finding a rod which behaved in the manner described by him. Below I give Joule's table, and also a diagram in which I have plotted his results, the abscissæ representing the magnetic intensity of the bar, and the ordinates the corresponding elongations. Both are on an arbitrary scale:—

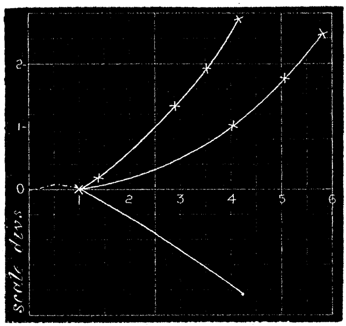
Table IV (Joule's).

Deflection of galvanometer.	Magnetic intensity.	Elongation of bar.
39° 0′	1·11	0
0	1·36*	0·2
52 35	4·09	1·0
0	2·85	1·3
60 15	5·10	1·8
0	3·52	1·9
69 45	5·91	2·5
0	4·20	2·7

It clearly appears that the elongations due to permanent magnetisation and to permanent + temporary (*i.e.*, total) magnetisation lie upon separate curves. And, since the total curve is below the permanent curve, it follows that the temporary magnetisation has a negative or retracting effect. Taking ordinates equal to the differences between those of the permanent and total curves, I have plotted the curve for temporary magnetisation, which, of course, lies on the negative side of the horizontal axis, and starts from the point representing a magnetic intensity of 1·11. Following the analogy of other experiments, I have continued the curve above the horizontal axis representing this part of it by a dotted line ; it is probable that

* It is very extraordinary that the "magnetic intensity" of the bar should be greater after the current had been cut off than it was when the current was flowing. Joule makes no reference to the fact.

FIG. 4.



any elongation represented by the dotted portion would be too small to be measurable.

The results of this experiment of Joule's are thus shown to be reconcilable with others, if we may make the assumption that in this particular specimen of steel the elongations due to temporary and permanent magnetisation followed different laws, and that while the critical point of the former occurred at an unusually early stage, that of the latter was not reached within the limits of the experiment. It may be indeed that this is always the case, though under ordinary circumstances the difference is too small to lead to the anomalous result under discussion. Having confined my attention almost entirely to the investigation of temporary effects, I have little experimental evidence to bring forward bearing upon the point.*

Mayer's results may be much more easily accounted for. The fact that his steel rods were invariably shortened by magnetisation (after the first magnetisation, the effect of which varied in different specimens) clearly indicates that his magnetising force exceeded the critical value, which was smaller for the steel bars than for the iron which he had previously used. He apparently never magnetised his

* Were it not for the proverbial accuracy of Joule's work, a simpler explanation of the anomaly would have suggested itself. The lower of the two curves above the horizontal axis represents the state of things *while a current is passing*, and the fact that this curve does not coincide with the upper one might, perhaps, be accounted for by the "solenoidal suction" which would occur if the rod were not quite symmetrically placed with respect to the coil, or even if it were not perfectly homogeneous throughout. Thus, the apparent elongations when the circuit was broken would be really due to the cessation of the suction, while the elongations indicated when the circuit was closed would be less than those which actually occurred. Each of the vertical divisions in the diagram represents only one thirteen-millionth part of the length of the rod: a very small variation in the pressure of the end of the rod upon its support would, therefore, have a sensible effect.—(February 23, 1886.)

steel with currents of less than the maximum strength, and a smaller magnetising force would perhaps have produced elongation, unless indeed the permanent magnetisation induced by the first current equalled or exceeded the critical value. This was almost certainly the case with his yellow-tempered steel, which was permanently *shortened* by the first magnetisation, while all the other specimens were permanently elongated. These considerations are consistent with all the phenomena exhibited by Mayer's steel bars.

In working with a rod of steel which had been neither annealed nor hardened, I obtained some very curious effects of which I am not at present prepared to offer a complete explanation. I therefore describe the experiments exactly as they were performed, without attempting to account for the results.

Experiment 1.—A current of 2 ampères was passed through the coil, whereupon the rod elongated 3 scale divisions. Without breaking the circuit, the current was reduced by inserting resistance to 0·6 ampère. The rod underwent a further elongation of 3 divisions, making the total elongation equal to 6 divisions. On breaking the circuit the rod retracted 6 divisions, returning to its original length; but when the circuit was again closed, the resistance still being inserted and the current consequently 0·6 ampère, the resulting elongation was only 3 divisions.

It appears therefore that a strong magnetising force subsequently diminished causes a greater temporary elongation than the diminished force is capable of producing if applied in the first place.

Experiment 2.—A current of 2 ampères being passed through the coil, an elongation of 3 scale divisions was produced. The current was reduced to 0·26 ampère, when a further elongation of 1 division occurred. On breaking the current the rod returned to its original length. Once more a current of 0·26 ampère was passed through the coil, but no movement whatever occurred.

From this it appears that the temporary elongation of a steel rod when once produced may be maintained by a magnetising force which is itself too small to cause any perceptible elongation whatever.

Something of the same kind, though in a smaller degree, was observed by Mayer in rods of iron.

Both these experiments were repeated many times, the results being invariably of the same character, and there is no doubt whatever as to the reality of the effects described.

On a small scale, I have repeated some of Joule's experiments with stretched wires, and found, as he did, that when a wire was loaded with a certain weight, the effect of magnetisation was not elongation but retraction. No measurements, however, were attempted, my apparatus not being well adapted for the purpose.

It appeared, upon consideration, that the results of this class

of experiments would be brought into perfect harmony with those already described, if it could be shown that the critical value of the magnetising force was lowered when the rod was stretched. There were reasons arising from the nature of my apparatus why I could not attempt to prove this directly; but an indirect method affords strong evidence that this is the case. By the method of deflection already described, it could be easily ascertained whether the position of the "turning point" was affected by stretching a wire. Now, in every case which I have hitherto investigated, it was found that the critical value of the magnetising force was very approximately equal to twice the magnetising force at the turning point. If therefore it should appear that the position of the turning point was affected by stretching, the presumption would be strong that the critical value would be altered to a corresponding extent.

Four deflection experiments were therefore made. In the first the wire, which was of annealed iron 0·77 mm. in diameter, was supported in a horizontal position inside the coil by means of corks; gradually increasing currents were passed through the coil and the galvanometer deflections noted as in former cases. The wire was then removed, and after being demagnetised was replaced inside the coil, and a weight of 7 lbs. was attached to it by means of a cord passing over a pulley. Once more the deflections accompanying increasing currents were noted.

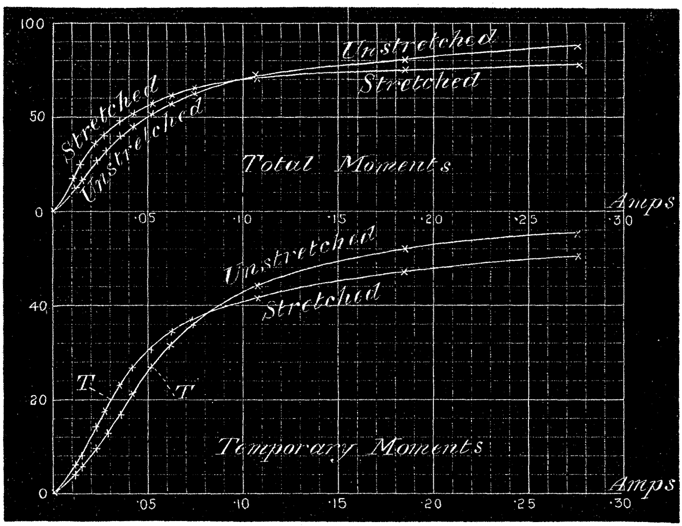
The third and fourth experiments were similar to the first two, except for the fact that the wire was magnetised to saturation before any observations were made. The deflections recorded in the first and second experiments are due therefore to the sum of the permanent

Table V.

Currents.	Magnetometer deflections for total magnetisation.		Magnetometer deflections for temporary magnetisation.	
	Unstretched.	Stretched.	Unstretched.	Stretched.
0·012	12	16	4·5	6·5
0·015	16·5	24	6	8
0·023	25	35	10	15
0·027	31	40	13	18
0·035	38	47	18	23
0·042	44	51	21·5	27
0·051	51	56	27	30·5
0·061	56	59	31	34
0·073	62	64	36	37
0·108	71	70	44	41·5
0·185	80	76	52	47·5
0·274	88·5	79	55	50

and temporary magnetisations, while those in the third and fourth were produced by the temporary magnetisation only. The results are given in Table V and in the subjoined curves (fig. 5). Both series

Fig. 5.



are interesting as affording an illustration of the law which has been fully investigated by Sir William Thomson,* that the magnetisation of a wire is at first increased and afterwards diminished by stretching; but the results of the second series only (in which the ordinates represent the temporary moments) are comparable with those of the former experiments.†

Referring to the curve of temporary magnetisation, it will be seen that the magnetising current at the turning point is reduced by stretching with a weight of 7 lbs. from 0·051 to 0·030: presumably, therefore, the magnetising current for the critical point is at the same time reduced from about 0·102 to 0·060, and a current between these limits would be accompanied by elongation when the wire was unstretched, and by retraction when it was stretched.

For a few experiments made with nickel, a strip was used of the following dimensions:—Length 100 mm., breadth 9 mm., thickness

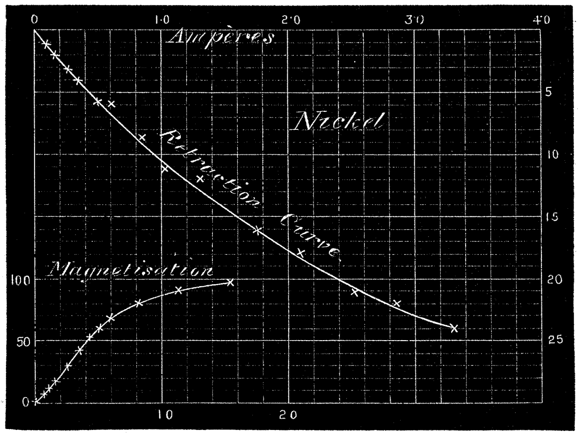
* "Phil. Trans.," 1876 and 1879.

† It should be noticed that the ordinates are on a different scale in the two diagrams, as may be seen by comparing the figures at the side. The distance between the wire and the centre of the galvanometer magnet was 13 cm. in all the experiments.

0·75 mm. Brass wires were soldered to the ends in the usual manner.

The permanent magnetisation induced by the strongest current appeared not to cause a permanent retraction of more than one scale division. The temporary retractions produced by increasing magnetising forces are given in Table II and in fig. 6. The retractions

FIG. 6.



are of much greater extent than the elongations of iron under similar circumstances, and though the curve affords evidence of an approach to a limit, it is nevertheless clear that a considerable further retraction would have occurred if the current had been increased beyond the power of my battery.

I also made a deflection experiment with the nickel, thinking it might possibly be the case that it had no turning point., *i.e.*, that the ratio of the temporary moments to the magnetising forces decreased *ab initio*. It appeared, however, that the turning point was unusually well marked, occurring with a current of 0·042 ampère. The details are given in Table III and fig. 6. The experiment was repeated two or three times with the same result, the nickel having been in each case previously magnetised with a strong current. The distance between the centre of the galvanometer needle and the nickel was 15 cm.

The behaviour of a stretched nickel wire has, I believe, never hitherto been investigated. I therefore made the experiment with a nickel wire 0·5 mm. in diameter, loaded with a weight equivalent to about 2 lbs. The result of magnetisation was a very considerable

retraction, but for reasons already referred to I was unable to measure the amount.

I have not for some weeks occupied myself with the investigation of the singular facts described in this paper without from time to time indulging in speculations as to their physical causes. It is, however, evident enough that the investigation is incomplete, and many more experiments, some of them requiring additional apparatus of a special kind, remain to be tried. I hope to return to the subject on a future occasion, and in the meantime refrain from theorising as to the causes of the phenomena.

Summary.

The experiments have not been sufficiently numerous to render generalisation in all cases perfectly safe; but, so far as they go, they indicate the following laws :—*

I. *Iron.*

1. The length of an iron rod is increased by magnetisation up to a certain critical value of the magnetising force, when a maximum elongation is reached.

2. If the critical value of the magnetising force is exceeded, the elongation is diminished, until, with a sufficiently powerful force, the original length of the rod is unaffected, and if the magnetising force is still further increased the rod undergoes retraction.

3. Shortly after the critical point is passed, the elongation diminishes in proportion as the magnetising force increases. The greatest actual retraction hitherto observed was equal to about half the greatest elongation; but there was no indication of a limit, and a stronger magnetising force would have produced further retraction.

4. The value of the external magnetising force corresponding to maximum elongation is nearly equal to twice its value at the "turning point."

Definition.—The turning point in the magnetisation of an iron bar is reached when the temporary moment begins to increase less rapidly than the external magnetising force.

5. The external magnetising force corresponding to the point of maximum elongation is greater for thick rods than for thin rods.

6. The amount of the maximum elongation varies inversely as the square root of the diameter of the rod.

7. The turning point, and therefore presumably the point of maximum elongation, occurs with a smaller magnetising force when the rod is stretched than when it is unstretched.

* The elongations and magnetisations referred to are temporary only.

II. *Steel.*

The behaviour of steel varies greatly with the hardness and temper of the metal. More experiments than I have hitherto made would be necessary to establish the general laws with certainty; but my results are consistent with the following conclusions:—

1. In soft steel magnetisation produces elongation, which increases up to a certain value of the magnetising force, and afterwards diminishes. The maximum elongation is less than in the case of iron, and the rate of diminution after the maximum is passed is also less.

2. The critical value of the magnetising force for a steel bar diminishes with increasing hardness of the steel up to a certain point corresponding to a yellow temper, after which it again increases, and with very hard steel becomes very high.

3. In soft steel a strong magnetising force subsequently diminished may cause a greater temporary elongation than the diminished force is capable of producing if applied in the first place.

4. A temporary elongation when once produced in soft steel may be maintained by a magnetising force which is itself too small to originate any perceptible elongation.

III. *Nickel.*

1. Nickel continues to retract with magnetising forces far exceeding those which produce the maximum elongation of iron. The greatest retraction of nickel hitherto observed is more than three times as great as the maximum elongation of iron, and the limit has not yet been reached.

2. A nickel wire stretched by a weight undergoes retraction when magnetised.

FIG. 1.

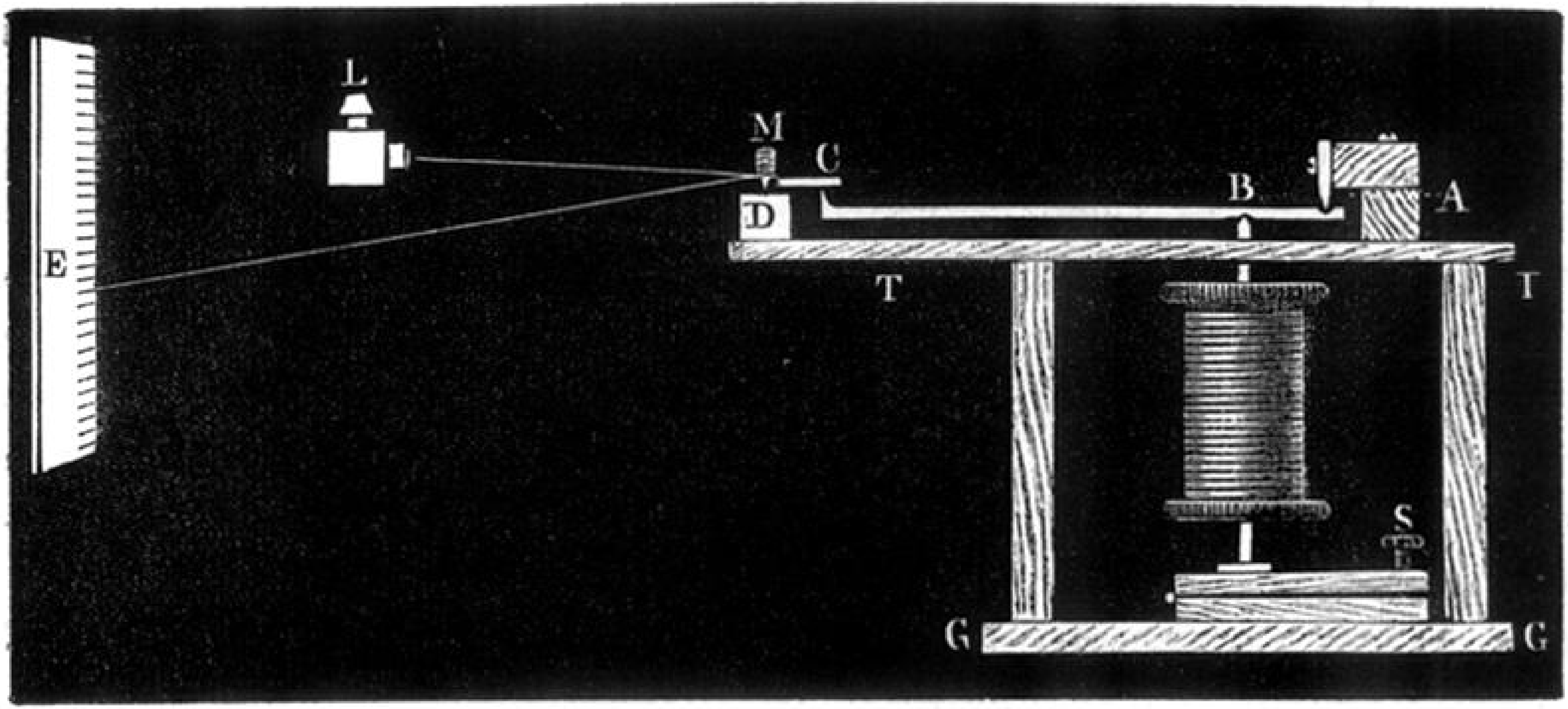
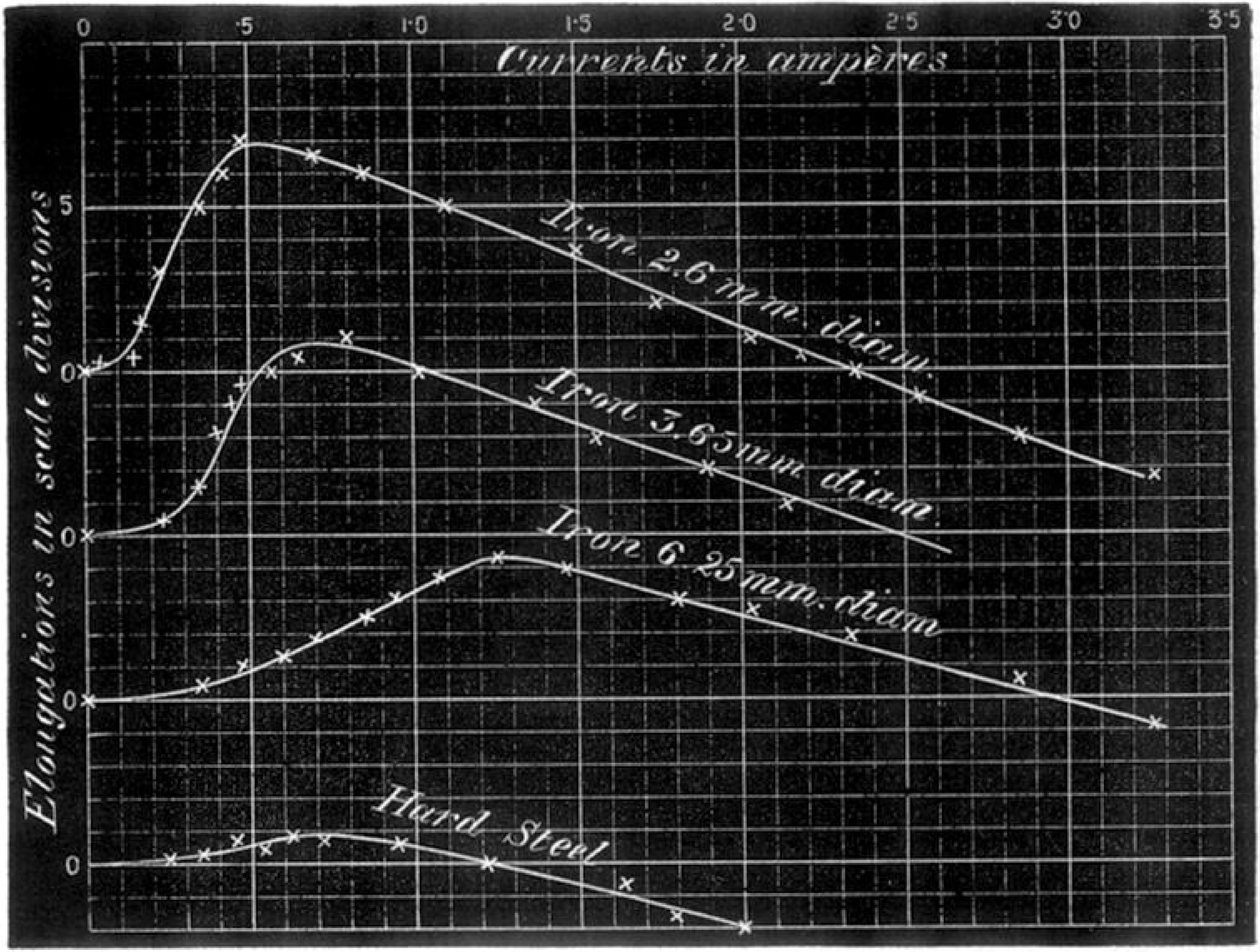
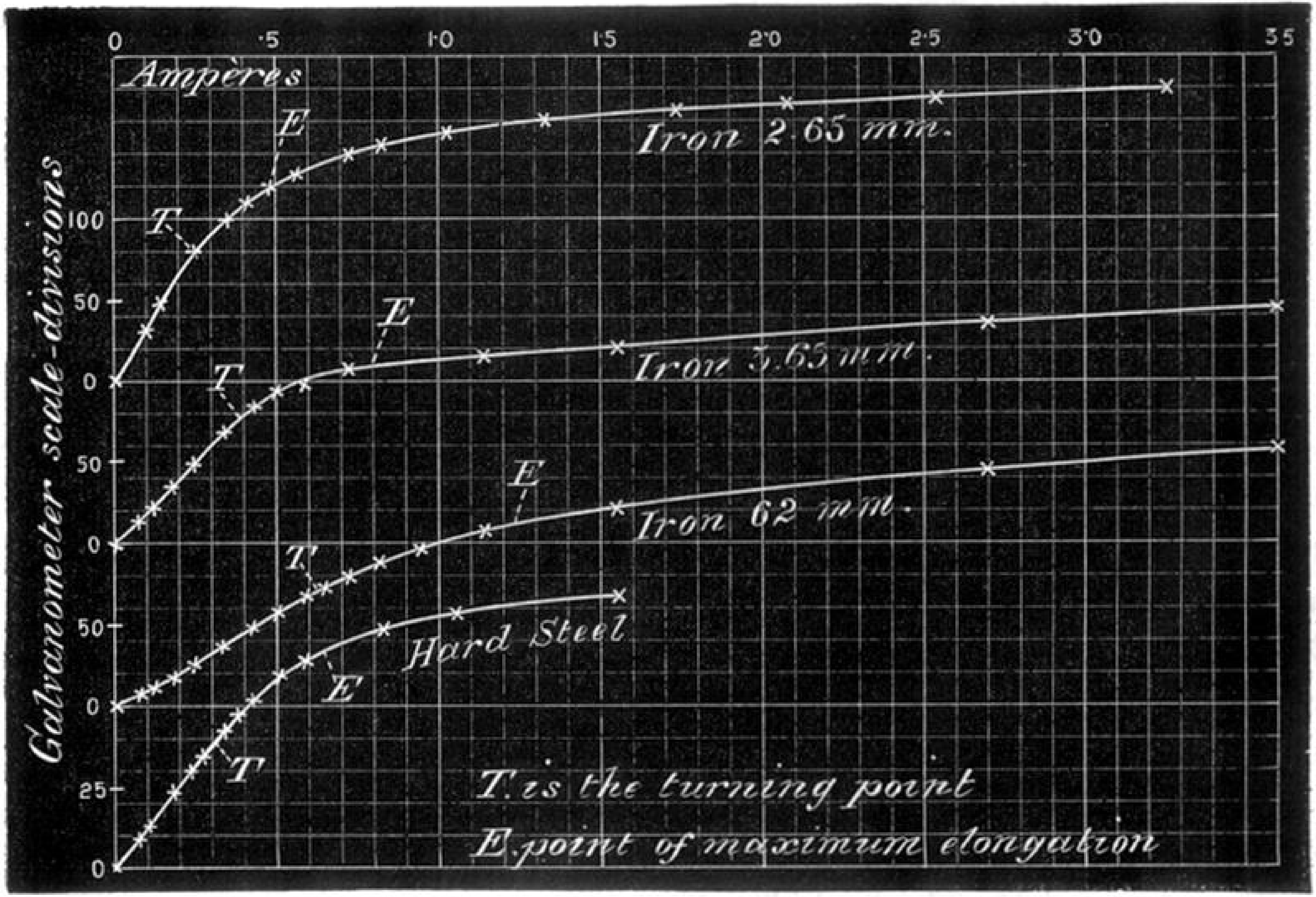


FIG. 2.



0
·5
1·0
1·5
2·0
2·5
3·0
3·5
Currents in ampères
Elongations in scale divisions
5
0
0
0
0
Iron 2.6 mm. diam.
Iron 3.65 mm. diam.
Iron 6.25 mm. diam.
Hard Steel

Fig. 3.



0
·5
1·0
1·5
2·0
2·5
3·0
3·5
Ampères
Galvanometer scale-divisions
100
50
0
50
0
50
0
25
0
E
T
Iron 2.65 mm.
E
T
Iron 3.65 mm.
E
T
Iron 62 mm.
Hard Steel
E
T
T. is the turning point
E. point of maximum elongation

FIG. 4.

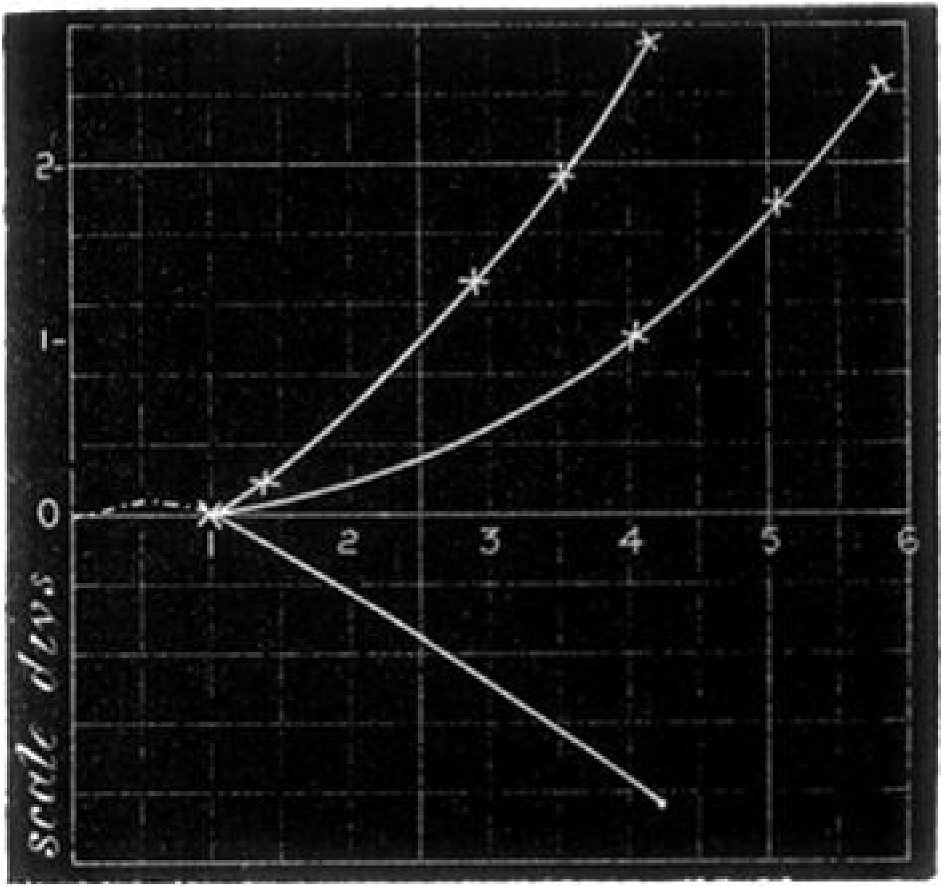
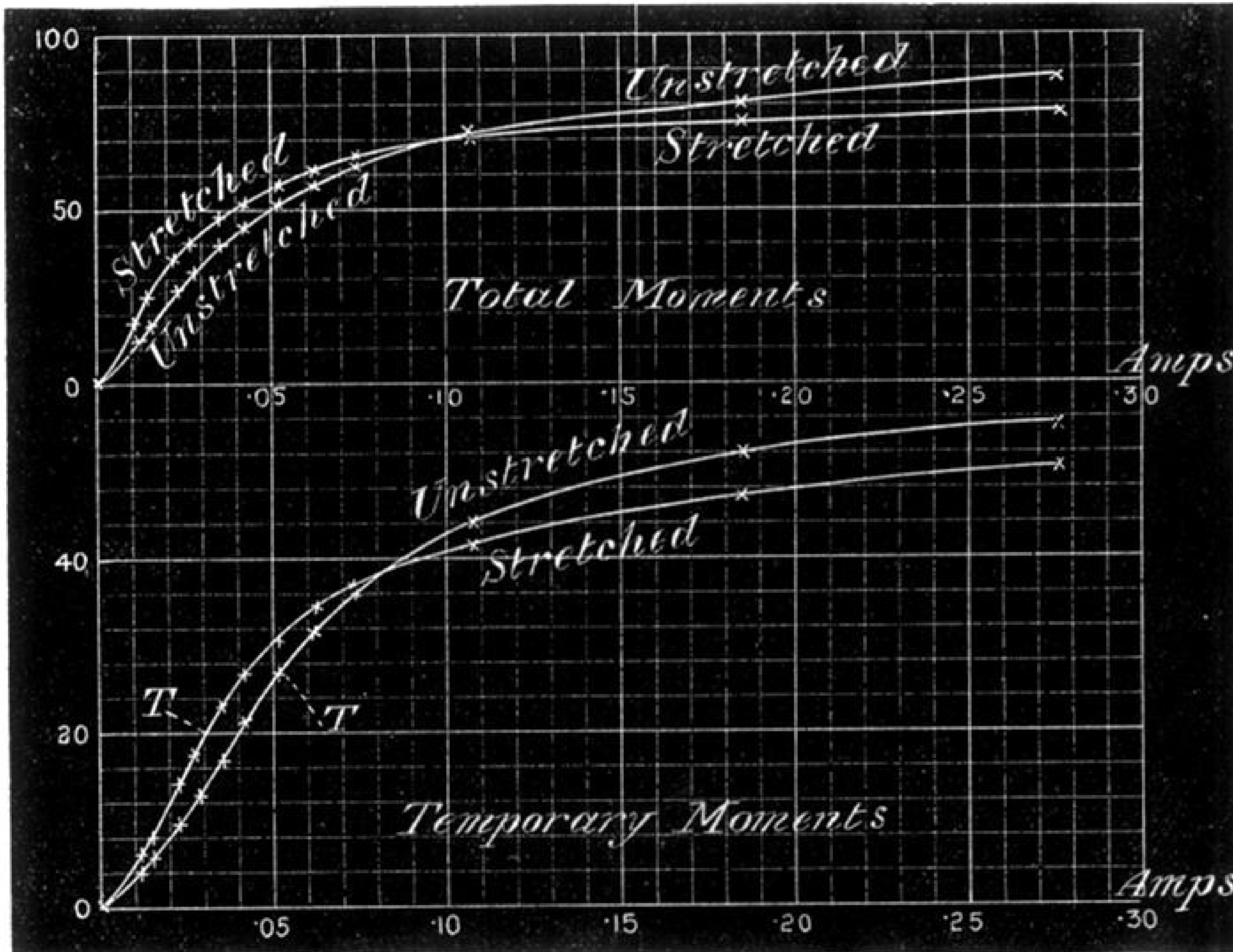
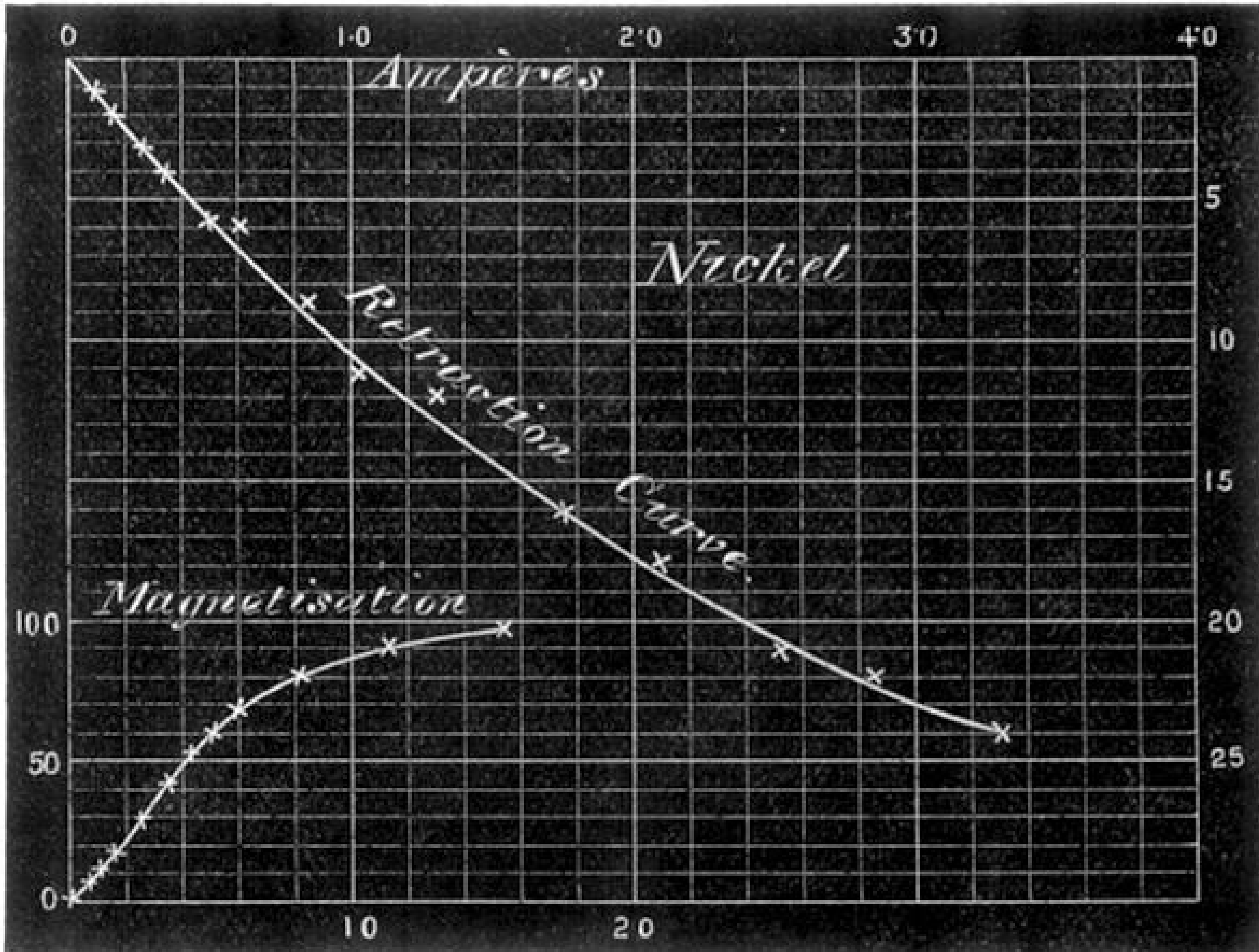


FIG. 5.



100
50
0
Stretched
Unstretched
Unstretched
Stretched
Total Moments
Amps
·05
·10
·15
·20
·25
·30
Unstretched
Stretched
40
20
0
T
T
Temporary Moments
Amps
·05
·10
·15
·20
·25
·30

FIG. 6.



0
1·0
2·0
3·0
4·0
Ampères
5
Nickel
Retraction
10
Curve.
15
Magnetisation
100
20
50
25
0
10
20