another table, and follows the "march" first indicated by Dr. Hughlings Jackson as existing in epileptic seizures.

This *march* is in accordance with Axiom I, since the shoulder commences the series of movements in the uppermost part of the area, the thumb at the lowest part, and the wrist in the intermediate part.

Summary.—1. That X is the superior frontal sulcus of man.

2. That the movements of the joints are progressively represented in the cortex from above down.

3. Localisation of sequence of movements.

4. Localisation of quality of movements.

5. That there is no absolute line of demarcation between the different centres.

III. "On the Discrimination of Maxima and Minima Solutions in the Calculus of Variations." By E. P. CULVERWELL. Communicated by Professor B. WILLIAMSON. Received June 5, 1886.

(Abstract.)

In the first part of the paper it is shown that the usual investigation by which the second variation of an integral is reduced, requires that the variation given to y (the undetermined function) is such that its differential coefficients, taken with regard to x (the independent variable) are continuous up to the twice-nth order, $\frac{d^ny}{dx^n}$ being the highest differential coefficient of y appearing in the function to be integrated. But it is not necessary that the variation should be continuous beyond its $(n-1)$th differential coefficient, and a method of reducing the variation to Jacobi's form by a process which is not open to the above objection is then given; and the method has the additional advantage that its simplicity enables it to be easily extended to other cases where there are more than two variables.

But in dealing with multiple integrals especially, any method depending on algebraic transformation is necessarily defective, inasmuch as it is invalid unless solutions, which do not become either zero or infinite within the limits of the integration, can be found for a number of simultaneous partial differential equations containing at least as many unknown quantities as equations. It is pointed out that it is not in general possible to obtain such solutions, and that even when the particular problem is assigned, it would be impracticable to ascertain whether there were such solutions.

The method given in the second part of the paper does not depend on or require any algebraic transformation. The second variation is

taken in its unreduced form, and, by considerations founded on the *degree of continuity* required in the variations of the dependent variables, it is shown without difficulty that, *when the range of integration is small*, the sign of the second variation is the same as that of a certain quadratic function. The limits of integration within which this result is applicable are then determined by considerations depending on the continuity of the integrals.

The following is a brief sketch of the method of obtaining the quadratic function on which the sign of the second variation depends. Let the function to be made a maximum be—

$$U=\iint \dots \int\left(x_1, x_2, \dots x_m, y_1, y_2, \dots y_n, \frac{dy_1}{dx_1}, \frac{dy_1}{dx_2}, \dots \frac{d^p y_n}{dx_m^p}\right) dx_1 dx_2 \dots dx_m,$$

(the form of $y_1, y_2, \dots y_n$ as functions of $x_1, x_2, \dots x_m$ having already been determined by making the first variation vanish). Taking the second variation by Taylor's theorem, we may write—

$$\delta^2 U=\tfrac{1}{2}\iint \dots \int \Sigma \frac{d^2 f}{dz dz'} \delta z \delta z' dx_1 dy_2 \dots dx_m,$$

when z and z' typify any of the quantities $y_1, y_2, \dots y_n$, or their differential coefficients, and Σ means that all such terms are to be taken. Restricting ourselves to the case in which the limits are fixed, we have $\delta y_1, \delta y_2, \dots \delta y_n$ zero at the limits, and similarly any differential coefficient of these quantities must be zero at the limits, provided it appears among the limiting terms in the first variation. Now if θ be any function of x which vanishes when $x=x_0$, we have—

$$\theta=\int_{x_0}^{x} \frac{d\theta}{dx} dx.$$

Hence, if $x-x_0$ be a small quantity of order β, $\theta/\frac{d\theta}{dx}$ is also a small quantity of the same *or a smaller order*, and therefore if we require to obtain the sign of a quadratic function of θ and $\frac{d\theta}{dx}$, we may neglect all terms except those involving θ^2. Reasoning of this character is applied to the variations in $\delta^2 U$, and the important terms are thus picked out. In this way a set of inequalities is obtained which enable us to determine the important terms in $\delta^2 U$, and therefore its sign.