the author explain in any way the difference between his own result and that of Maxwell.

[The method followed by Maxwell is liable to be vitiated to a very sensible degree by small errors of level of the movable disks, especially when they are closest to the fixed disks. The final adjustment is stated to have been that of the fixed disks, and no special precautions seem to have been taken to secure the exact horizontality of the movable disks. By a calculation founded on the equations of motion of a viscous fluid, I find that at the closest distance (about the one-sixth of an inch) at which the fixed and movable disks were set, an error of level of only 1° 8′ would suffice to make the internal friction appear 8 per cent. too high.

In Mr. Tomlinson's reductions no allowance has at present been made for the effect of the rotation of the spheres or cylinders about their own axes, which is not quite insensible, as it would be in the case of a ball pendulum. The introduction of a correction on this account would slightly diminish the values resulting from the experiments, especially in the case of the sphere, where it would come to about 4 per cent.—G. G. S.]

January 21, 1886.

Professor STOKES, D.C.L., President, in the Chair.

The presents received were laid on the table, and thanks ordered for them.

The following Papers were read :—

I. "Family Likeness in Stature." By FRANCIS GALTON, F.R.S. With an Appendix by J. D. HAMILTON DICKSON, Fellow and Tutor of St. Peter's College, Cambridge. Received January 1, 1886.

I propose to express by formulæ the relation that subsists between the statures of specified men and those of their kinsmen in any given degree, and to explain the processes through which family peculiarities of stature gradually diminish, until in every remote degree of kinship the group of kinsmen becomes undistinguishable from a group selected out of the general population at random. I shall determine the constants in my formulæ referring to kinship with a useful

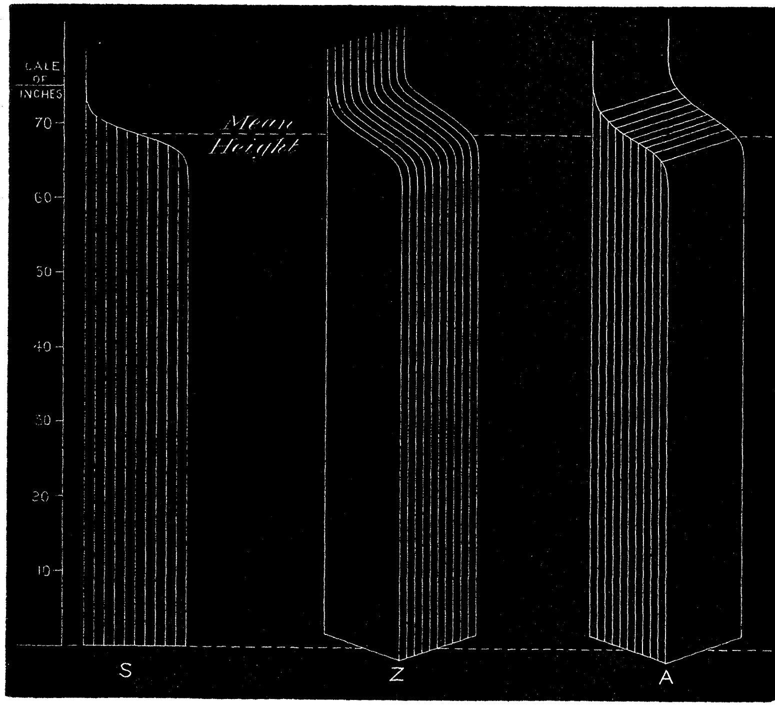
degree of precision. These constants may provisionally and with some reservation be held applicable to other human peculiarities than stature, while the formulæ themselves are, I presume, applicable to every one-dimensioned faculty that all men possess in some degree, but that different men possess in different degrees.

I selected stature for the subject of this inquiry, for reasons fully set forth in two recent publications,* which dealt with one small portion of the ground covered by the present memoir, and from which it will be convenient that I should make as I proceed occasional short extracts, in order to complete the present argument and to save cross-reference. The reasons that combine to render stature an excellent subject for hereditary inquiry are, briefly, the ease and frequency of its measurement, its constancy during adult life, its inconsiderable influence on the death-rate, its dependence on a multiplicity of separate elements, and other points that I shall dwell on as I proceed, namely, the ease with which female statures are transmuted to their male equivalents, and so enabled to be treated on equal terms with male statures, the tendency of the parental statures to blend in inheritance, and the disregard of stature in marriage selection.

Stature-schemes.—It is an axiom of statistics that large samples taken out of the same population at random are statistically similar, and in such inquiries as these which do not aim at minute accuracy, they may be considered identical. Thus the statures in every group, say of 1000 male adults, when distributed in order of their magnitudes at equal distances apart and in a row, will form almost identical figures; it being only towards either end of the long row that irregularities will begin to show themselves. These are unimportant in the present inquiry and I disregard them. The Diagram S, fig. 1, shows the outline of such a group of statures. It is drawn to scale, each of the statures being supposed to have been represented by a vertical line of proportionate length, standing on a horizontal base, the lines being at equal distances apart, and the whole system being compressed into the space between two termini, which may be set at any convenient distance asunder. The vertical lines in the figure do not indicate these statures, but they are divisions, ten in number, between each of which 100 stature lines are compressed. The first and last stature will not touch the termini, but will be removed from them by a half-interval. As it will be convenient to assign a name to this figure, I will call it a

* (1.) "Presidential Address to the Anthropological Section of the British Association in 1885." (2.) "Regression towards Mediocrity in Hereditary Stature." "Journ. Anthrop. Institute," 1885, p. 246. The latter is a reprint of that portion of the former with which I am now concerned, together with some additional matter; it contains tables and diagrams, and should be referred to in preference.

Fig. 1.



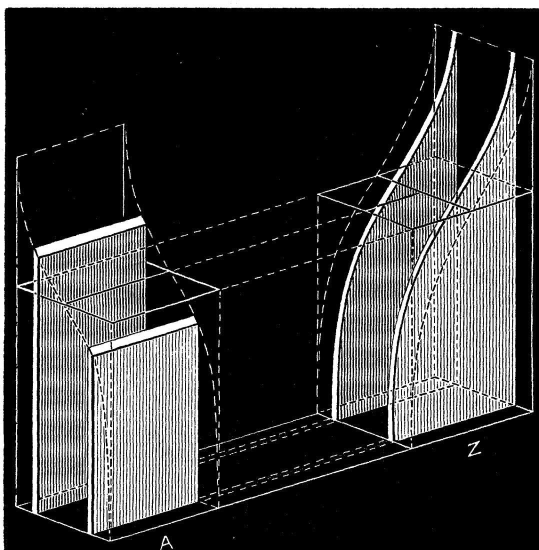
"stature-scheme." The numerous cases near mediocrity that differ little from one another, cause the middle portion of the upper boundary of the stature-scheme to assume a gentle slope, which increases rapidly towards either end, where the increasing rareness of more and more exceptional cases causes that boundary line to slope upwards, as an asymptote to one of the termini, and downwards as an asymptote to the other.

Now suppose that instead of compressing 1000 statures between the termini, I compressed 1000×1000, or a million of them, the stature-scheme would be unaltered, except that such small irregularities as might have been previously seen would become smoothed. The height of the middlemost or median stature-line would remain the same as before, and so would the heights of the lines standing at each quarter, each tenth, and at every other proportionate distance between the termini. Or again, instead of arranging the lines in a single scheme, we might arrange them in a thousand schemes, which as we have seen, would be practically identical in shape, and we may place these schemes side by side, as is done in Z, fig. 1, forming a

"squadron" numbering 1000 statures each way, the whole standing upon a square base. Our squadron may be considered as made up of *ranks* (parallel to the plane of zx) as in Z, or of *files* (parallel to the plane of zy) as in A. The ranks, as we have seen, are all similar stature-schemes, the files are all rectangles which have the same breadths but are of dissimilar heights.

It is now easy to give a general idea, to be developed as we proceed, of the way in which any large sample of a population gives rise to a group of distant kinsmen in any given degree, who are statistically (in all respects except numbers) undistinguishable as regards their statures from themselves. I must suppose for convenience of explanation, that tall, short, and mediocre men are equally fertile (which is not, however, strictly the case, the tall being somewhat less fertile than the short*), and then on referring to fig. 2, the

FIG. 2.



fortunes of the distant descendants of two of the rectangular files of squadron A will be seen traced.

As the number of kinsmen, in any remote degree we please to specify, of the men in each of the two files is about the same; I take 1000 of them in each case. Again, as the stature-schemes of those kinsmen are identical with those of equal numbers of men taken at random, as samples of the general population, it follows that they

* Oddly enough, the shortest couple on my list have the largest family, namely sixteen children, of whom fourteen were measured.

will be identical with one another. Every other rectangular file being similarly represented, a complete squadron Z of the kinsmen is produced. It is obvious, then, that the squadrons A and Z are identical, and as the ranks of Z have proceeded from the files of A, the result is that the two squadrons will stand at right angles to one another. The upper surface of A was curved in rank, but was horizontal in file; that of Z is curved in file, but is horizontal in rank.

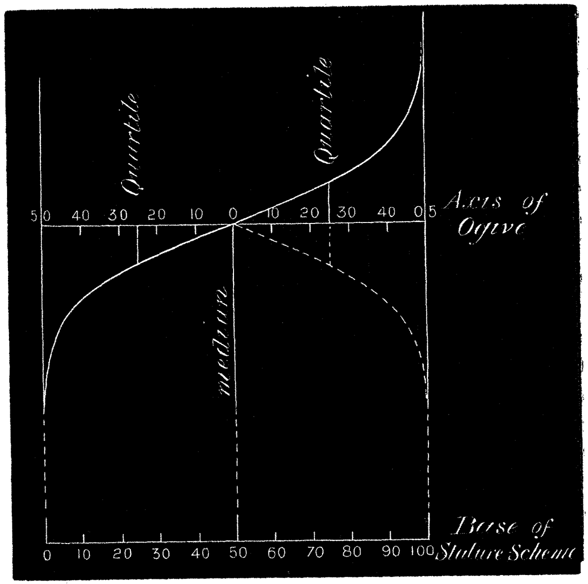
Kinsmen in near degrees are represented by squadrons of intermediate form. These will not have lost the whole of the curvature in rank of A, nor will they have acquired the whole of the curvature in file of Z. Consequently they will be curved moderately in both ways.* Also it will be found that the intersection of their surfaces by the horizontal plane of median height forms in each case an approximately straight line that assumes different and increasing inclinations, in the successive squadrons of intermediate shape between A and Z. These lines are indicated by straight lines on the squares below the squadrons in fig. 4, which represent the square bases upon which the squadrons stand.

I shall now show how these curves in rank and file should be treated. But before doing so, it is necessary to remark that female adult stature (I speak throughout of adults) may be safely transmuted to its male equivalent by multiplying it by a constant constant, which as regards my data is 1·08. After this has been done, the transmuted female statures may be treated on equal terms with the male statures, and the word "men" or other masculine term will include both sexes, unless otherwise stated distinctly. This procedure is adopted in the present memoir.

It is now generally recognised that the statures in every ordinary population are distributed in approximate conformity with what might have been inferred, if it were known that their variations were governed by such conditions as those upon which the exponential law of frequency of error is based. Therefore the upper boundary of the stature-scheme is approximately a curve (I call it an "ogive") that admits of mathematical expression. The abscissæ of the normal ogive (fig. 3) are values of the probability integral $\frac{1}{\sqrt{\pi}}\int_0^t e^{-t^2}dt$, and the ordinates are the corresponding values of t. These are given in column A of Table I. Column B contains the same values divided by 0·477, by which means they are expressed in units of the probable error. I find it convenient to call the ordinates to an ogive (drawn

* A plaster model of one of these intermediate forms was exhibited at the meeting by Mr. J. D. H. Dickson, who stated that his recent mathematical investigation of the properties of their surfaces, had shown that no strictly straight line could be drawn upon them.—F. G.

Fig. 3.



from its axis) by the name of "deviates," and to describe either of those two symmetrical deviates of the normal ogive that stand at $\pm$ 25° by the name of "quartile deviate," or, more briefly, "quartile." I also give this name to the mean length of the upper and lower quartile, in those ogives which are drawn from observed data, and which are not strictly symmetrical. The numerical value of the quartile is identical with that of the well-known but here inappropriate term of "probable error."

Construction of Stature-Schemes and of Ogives from Observations.— The method of drawing an ogive from observations of stature is as follows. The observations (see Tables III, IV, and V, and compare with VI and VII) are sorted into grades, such as ". . . cases of 60 inches and under 61," ". . . cases of 61 inches and under 62," &c. If we are constructing a stature-scheme, or desire to obtain the median value of the series, we have to consider these values of inches, but in constructing no more than an ogive, which is only the upper boundary of a stature-scheme, it suffices to consider them as successive grades of 1 inch each, and I reckon the first grade not as 0, but as 1. This has been done in column A, Table VI, for the sake of treating different groups on a uniform plan. The number of cases in these grades are then summed from the beginning, and the sum, up to each grade inclusive, is written down, as shown

in column B in Table VI. The percentage values of these, taking the total number of observations as 100, are written in column C. A series is there obtained which shows how many per cent. of the statures fall short of the parting value that separates each pair of adjacent grades. Thus if n per cent. of the statures fall within the first r grades, that is to say, are less than the value of the rth parting line, then $100-n$ per cent. of them will exceed that value. Consequently, if the observations are read off and recorded to the utmost nicety, r will be the value of the ordinate representing the stature which has to be erected on a base line at n per cent. of its length from one of its ends. In short, a base line of any convenient length has to be divided into 100 parts, and an ordinate of a length proportionate to r erected at the division n. As observations are never read off and recorded with perfect accuracy, a correction has here to be applied according to the circumstances of the particular case, whenever we are drawing a stature-scheme, and not merely an ogive. If the records are kept to the nearest mth part of an inch, the phrase "exceeding r inches" would really mean exceeding $r-1/m$ inches." This then is the true parting value corresponding to the nominal r. In drawing ogives, and not stature-schemes, this correction may of course be disregarded. Having erected ordinates corresponding to each value of r, their tops are connected by straight lines forming a polygonal boundary that approximates to the curvature of an ogive, and would become one if it were corrected with a free hand, or otherwise smoothed. The centre of the ogive lies at the intersection of the curve with the ordinate drawn from the base at the fiftieth division, and the horizontal axis of the ogive runs through that point of intersection (see fig. 3).

A half-ogive, whose ordinates are the mean lengths of the symmetrically disposed ordinates of the complete ogive, is constructed on the same general principles, but more simply, because the base from which it is plotted coincides with the axis of the ogive, and the graduations run alike, viz., from 0° to 50°.

In Table VII, the entries in the first lines of each of the three groups it contains, are the lengths of the ordinates that have been measured from the bases of ogives constructed from the data in Table VI. The abscissæ corresponding to the measured ordinates, are in every case the same fractional lengths of the bases. The entries in the second lines are the differences between these several ordinates and the median ordinate; they are, therefore, the deviates. The entries in the third lines are the negative deviates written under the corresponding positive ones. The entries in the fourth lines are the means of the values of the positive and negative deviates, disregarding their signs.

Comparison of Ogives.—The ogive being drawn according to the

observations, its axis is divided into 100 parts, the fiftieth division being reckoned as 0°, then the deviates standing at the ± graduations of 10°, 20°, 25°, 30°, 40°, and 45° are measured. The mean of each pair of lengths, not regarding signs, has then to be divided by the mean lengths of the deviates at ± 25°, that is by the quartile deviate, and so is made to yield a series that is directly comparable with column B in Table I. The closeness with which it conforms to that standard series is the test of the closeness with which the observations conform to the law of frequency of error.

Table II effects this comparison for all the series that I have to deal with in the present paper. The values are entirely unsmoothed, except in two named instances, being taken from measurements made to the above-mentioned polygonal boundary. I thought it best to give these interpolated values in this, their rudest form, leaving it to be understood that with perfectly legitimate correction the accordance would become still closer. I do not carry the comparison beyond 45°, partly because my cases are not numerous enough to admit of a fair comparison being made, and chiefly because I am well aware that conformity is not to be expected towards the end of any series. I am content to deal with nine-tenths of the observations, namely, those between 0° and 45°, and to pay little heed to the remaining tenth, between 45° and 50°. It will be seen that the conformity of more than one half of each series is closer than to the first decimal place, and that in absolute measurement it is closer than to one-tenth of an inch.

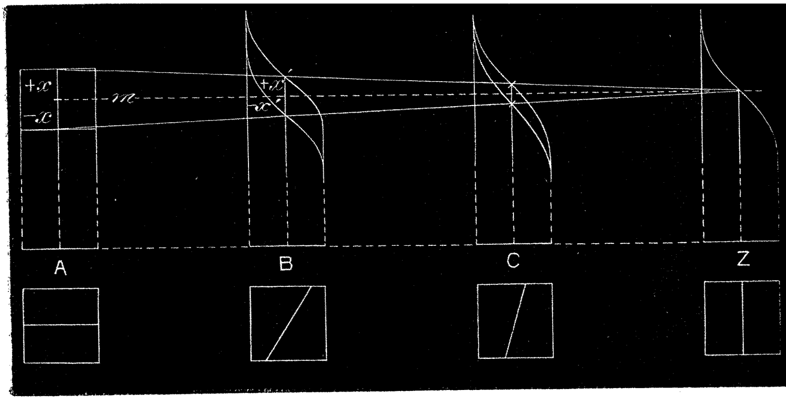
Arithmetic and Geometric Means.—I use throughout this inquiry the ordinary law of frequency of error, which being based on the assumption of entire ignorance of the conditions of variability, necessarily proceeds on the hypothesis that *plus* and *minus* deviations of equal amounts are equally probable. In the present subject of discussion our ignorance is not so complete; there is good reason to suppose that *plus* and *minus* deviations, of which the probability is equal, are so connected together that the ratio between the lower observed measurement and the truth is equal to that between the truth and the upper observed measurement. My reasons for this were explained some years ago, and were accompanied by a memoir by Mr. Donald Macalister, showing how the law of frequency of error would be modified if based on the geometric, instead of on the arithmetic mean.* Though in the present instance the former process is undoubtedly the more correct of the two, the smallness of the error here introduced by using the well known law is so insignificant that it is not worth regarding. Thus the mean stature of the population is about 68·3 inches, and the quartile of the stature-scheme (the probable error) is 1·7 inch, or only about one-fortieth of its amount, and the

* "Proc. Roy. Soc.," vol. 29 (1879), pp. 365, 367.

difference between $40^2/39$ and 41 is that between about 41·025 and 41·000, or only about 6 per thousand.

Regression.—It is a universal rule that the unknown kinsman in any degree of any specified man, is probably more mediocre than he. Let the relationship be what it may, it is safe to wager that the unknown kinsman of a person whose stature is $68\frac{1}{4} \pm x$ inches, is of some height $68\frac{1}{4} \pm x'$ inches, where x' is less than x. The reason of this can be shown to be due to the combined effect of two causes: (1) the statistical constancy during successive generations of the statures of the same population who live under, generally speaking, uniform conditions; (2) to the reasonable presumption that a sample of the original population and a sample of their kinsmen in any specified degree are statistically similar in the distribution of their statures. To fix the ideas, let us take an example, namely, that of the relation between men and their nephews:—(*a.*) A sample of men, and a sample of the nephews of those men, are presumed to be statistically alike in stature, that is to say, their mean heights and their quartile deviates of height will be of the same value. I will call the value of this quartile p. (*b.*) Each family of nephews affords a series of statures that are distributed above and below the common mean of them. They are deviations from a central family value, or, as we may phrase it, from a nepotal centre, and it will be found as we proceed (it results from what appears in Tables III, IV, and V) that these deviations are in conformity with the law of error, and that the quartile values (probable errors) of these systems of deviations, which we will call f, are practically uniform, whatever the value of the central nepotal family stature may be. (*c.*) It will be found, as it is reasonable enough to anticipate, that the system of nepotal centres is distributed above and below the median stature of the population, in conformity with the law of frequency of error, and with a quartile value that we will call d. It follows from (a) that we possess data for an equation between p, f, and d, which, from a well-known property of the law of error, assumes the form $d^2+f^2=p^2$. Now the unknown nephew is more likely to be of the stature of his nepotal centre than any other stature that can be named. But the system of statures of nepotal centres is more concentrated than that of the general population (d^2 is less than p^2). That is to say, the unknown nephew is likely to be more mediocre than the known man of whom he is the nephew. What I shall have to show is expressed in fig. 4, where A and Z are side views of squadrons such as A and Z in fig. 2. [They are drawn shorter than the stature-schemes in fig. 1, and therefore out of scale, to save space, which is an unimportant change, as it is only the variation in the ogives we are now concerned about.] Let m represent the level of mediocrity above the ground, $m+x$ and $m-x$ the heights of any two rectangular files in

FIG. 4.



the squadron of known men. We have seen that x becomes 0 in remote degrees of kinship, and I shall show that in intermediate degrees the value of x'/x is constant for all statures in the same degree of kinship. This fraction is what I call the ratio of regression, and I designate it by w. Consequently the above formula becomes $w^2p^2+f^2=p^2$, which is universally applicable to all degrees of kinship between man and man, so long as the statistics of height of the population remain unchanged.

Hence in the squadrons, the curvature in rank is an ogive with the quartile value of wp, and in file with one having the quartile value of f, these two values being connected by the above formula. If the squadron is resolved into its elements, and those elements are redistributed into an ordinary stature-scheme, the quartile of the latter will be p.

Another way of explaining the universal tendency to regression may be followed by showing that this tendency necessarily exists in each of the three primary relationships, fraternal, filial, and parental, and therefore in all derivative kinships. Fraternal regression may be ascribed to the compromise of two conflicting tendencies on the part of the unknown brother, the one to resemble the given man, the other to resemble the mean of the race, in other words to be mediocre. It will be seen that this compromise results in a probable fraternal stature that is expressed by the formulæ $(p^2-b^2)/p^2$, in which b is a constant as well as p, therefore the ratio of fraternal regression is also a constant. Filial regression is due (as I explained more fully than I need do here, in the publications alluded to in the second paragraph) to the concurrence of atavism with the tendency to resemble the parent. The remote ancestry in any mixed population resembles, as has been

already said, any sample taken at random out of that population, therefore their mean stature is mediocre; consequently the parental peculiarities are transmitted in a diluted amount. Parental regression is shown to be the necessary converse of filial regression by mathematical considerations, kindly investigated for me by Mr. Dickson, in the Appendix to this memoir in Problem 1. It is easy in a general way to see that this would be the case, but I find it not easy otherwise to prove it. Still less would it be easy to prove the connexion between filial and mid-parental regression, which depend on considerations that are thoroughly investigated in the Appendix.

Data.—I will now describe the data from which I obtain my conclusions. They consist of two sets of practically independent observations, though they do in some small degree overlap.

(1.) Special observations. These concern variation in height among brothers. I circulated cards of inquiry among trusted correspondents, stating that I wanted records of the heights of brothers who are more than 24 and less than 60 years of age; not necessarily of all the brothers of the same family, but of as many of them as could be easily and accurately measured, the height of even two brothers being acceptable. If more than one set of brothers were entered on the same card, the entries were of course to be kept separate. The back of the card was ruled vertically in three parallel columns: (*a*) family name of each set of brothers; (*b*) order of birth in each set; (*c*) height, without shoes, in feet and inches. A place was reserved at the bottom for the name and address of the sender. The circle of inquiry widened, and I closed it when I had obtained returns of 295 families, containing in the aggregate 783 brothers.

I look upon these returns as quite as trustworthy as any such returns are likely to be. They bear every internal test that I can apply to them very satisfactorily. They are commonly recorded to quarter and half inches.

(2.) R.F.F. data. By this abbreviation I refer to the Records of Family Faculties that I obtained in the summer of 1884, in reply to an offer of prizes. I have been able to extract from these the heights of 205 couples of parents, with those of an aggregate of 930 adult children of both sexes. I have transmuted all the female heights to their male equivalents, and have treated them thus transmuted on equal terms with the measurement of males, except where otherwise expressed. These data have by no means the precision of the special observations. There is in many cases considerable doubt whether the measurements refer to the height with the shoes on or off; many entries are, I fear, only estimates, and the heights are commonly given only to the nearest inch. Still, speaking from a knowledge of many of the contributors, I am satisfied that a fair share of these

returns are undoubtedly careful and thoroughly trustworthy, so that I have reason to place confidence in mean results. They bear those internal tests that I apply to them better than I should have expected, and when taken in connexion with and checked by the special data, and used with statistical caution, they have proved very valuable to me.

I have discussed these materials in a great variety of ways to guard myself against rash conclusions, but I shall not present more than three primary tables, which contain sufficient materials for determining the constants of the formulæ to be used.

The first of them (Table III) refers to the children of what I call "mid-parents" of various statures. A mid-parent is the imaginary mean of the two parents, after the female measurements have been transmuted to their male equivalents, so that a mid-parent of 70 inches in height refers to a couple whose mean stature under the above reservations is 70 inches. I have given data in the "Journ. Anthrop. Inst." (*loc. cit.*) to show that we need not regard differences in stature between the parents, inasmuch as the distribution of heights among the children proves to be statistically the same, so long as the mid-parentages are alike, whether the two parents are the same or of different statures. This blending of paternal and maternal qualities in the stature of the offspring is one great advantage in selecting stature as a subject for the present inquiry.

General Population.—(1.) Its variability. The value of the quartile deviate in the population ogive (that is to say, the probable error) may be deduced from the bottom lines of any one of the three Tables III, IV, and V. Those in III and IV refer to data that are in part but by no means wholly the same, that of V refers to almost totally distinct data. The work is shown in Tables VI and VII; in the former the ordinates are calculated whence the ogive is drawn, in the latter I have given the values of the measured ordinates at the same points along its axis as those to which the ordinates given in Table I refer. The values of the quartile that I obtain in this way from the three cases are 1·65, 1·7, and 1·7. I should say that the more careful treatment that I originally adopted happened to make the first of these values also 1·7, so I have no hesitation in accepting 1·7 as the proper value of p for all my data.

(2.) Variability of system of mid-parents. I have published data in the memoir already alluded to, to show that marriage selection takes small account of stature, which is another great merit in stature as a subject for this inquiry. Some further proof of this may be got by comparing the variability of the system of mid-parents with that of the general population. If the married couples had paired together regardless of stature, their mean heights would be elements of a statistical system identical with one in which the pairs had been

selected at random. In this latter case the quartile value of the system of mid-parents would be $1/\sqrt{2}\,.\,p = 1{\cdot}21$ inch. Now, I find the quartile of the series of the mid-parental system obtained from the two columns in Table III, that are headed respectively "Heights of the mid-parents" and "Total number of mid-parents," to be 1·19 inches,* which is an unexpectedly exact accordance.

(3.) Median Stature. I obtain the values 68·2, *68·5*, 68·4, from the three series mentioned above, but the middle value, printed in *italics*, is a smoothed value. This is one of the only two smoothed values in the whole work, and has been justifiably corrected because the one ordinate that happens to accord closely with the median is out of harmony with all the rest of the curve. This fortuitous discrepancy amounts to more than 0·15 inch. It does not affect the quartile value, because neither the upper nor the lower quartile is touched, and, therefore, the half-interquartile remains unchanged. It must be recollected that the series in question refers to R.F.F. brothers, which are a somewhat conditioned selection from the general R.F.F. population, and could not be expected to afford as regular an ogive as that made from observations of men selected from the population at hazard. It is undoubtedly in this group that the least accuracy was to have been expected.

Mean Ratios of Regression in the Primary Degrees of Kinship.—(1.) From the stature of mid-parents of the same height, to the mean of the statures of all their children. I have already (*loc. cit.*) published the conclusions to which I arrived about this, but it is necessary to enter here into detail. The data are contained in Table III, where each line exhibits the distribution of stature among the children of all the mid-parents in my list, who were of the stature that forms the argument to that line. The median stature in each successive line is the mean stature of all the children, and is given at the side in the column headed "Medians." Their values are graphically represented in fig. 5. It will be there seen that these value are disposed about a straight line. If the median statures of the children had been the same as those of their mid-parents this line would have accorded with the line AB, which, from the construction of the table, is inclined at an angle of 45° to the line "Mean Stature of Population," which represents the level of mediocrity. However, it does not do this, but its position is inclined at a smaller angle, θ, such that

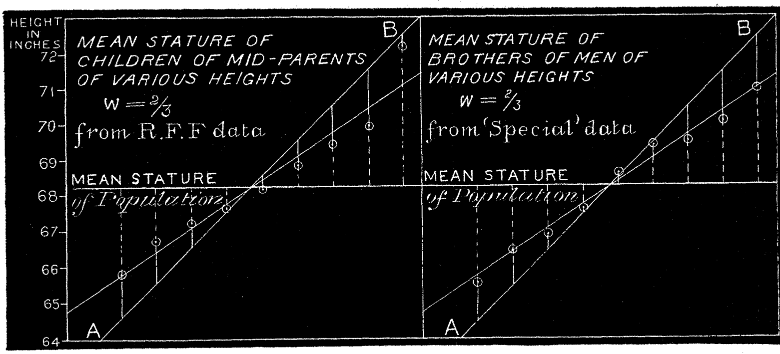
$$\tan\theta : \tan 45 \;::\; 2 : 3.$$

This gives us the ratio of regression ($=w$) in the present case; and, therefore, in the notation I adopt $w = \frac{2}{3}$.

(2.) From the stature of men of the same height, to the mean of the statures of all their children. We have just seen that when both

* In all my measurements the second decimal is only approximately correct.

FIGS. 5 and 6.



parents have a deviate of $\pm x$, the mean of the deviates of all of their family centres will be $\pm \frac{2}{3}x$. It follows that if one parent only has that deviate $\pm x$, and if the stature of the other parent is unknown, and, therefore, on the average, mediocre, the mean of the statures of their children will be half the above amount, or $\frac{1}{3}$. I cannot test this conclusion very satisfactorily by direct observation, for my data are barely numerous enough to enable me to deal even with the mid-parentages. They are consequently insufficient to deal with a question involving the additional large uncertainty of the stature of one of the parents. I have, however, tabulated the data, but do not think it worth while to give them. They yield a ratio of regression of 0·40 instead of 0·33 as above. I disregard it, and adopt the latter, namely, $w=\frac{1}{3}$.

(3). From the stature of men of the same height to the mean of the statures of their mid-parents. By treating the vertical columns of Table III in the same way as we have just dealt with the horizontal lines, we obtain results of the same general form as in the last paragraph but one, though of different values.

Taking the height of a group of men of the same stature (viz., the "Adult Children") as given in the line that forms the heading to the table, we find the median stature of all their mid-parents, whence I deduce in this case $w=\frac{1}{3}$. The apparent paradox that the same table should give results by no means converse in their values for converse degrees of kinship, will be more conveniently examined later on.

(4.) From the stature of men of the same height to the mean of the statures of all their brothers. In seeking for this I shall at first confine myself to the more accurate special data, reserving to the end a comparison between their results and those derived from the R.F.F. The entries in the column headed "medians" in Table V are

graphically represented in fig. 6, whence I deduce the value of $w = \frac{2}{3}$.

Variability of Statures of "Co-kinsmen" about their common mean Value.—By "co-kinsmen" I desire to express the group distributed in any one line of Tables III, IV, V, or of other tables constructed on a like principle. They are the kinsmen in a specified degree, not of a single person, but of a group of like persons, who probably differ both in ancestry and nurture. For example, the persons to whom the entries opposite 68·5 in Table III refer are not brothers, but they are what I call "co-fraternals," or from another point of view, "co-filials," namely, the children of numerous mid-parentages, differing variously in their antecedents, and alike only in their personal statures.

Co-filial Variability.—It appears from Table III that the mean of the quartiles derived from the successive lines, and which I designate by f, is 1·5 inch; also that the quartiles are of nearly the same value in all of the lines, allowance being made for statistical irregularities. A protraction on a large sheet of the individual observations in their several exact places, gave the result that the quartile was a trifle larger for the children of tall mid-parentages than for those of short ones. This justifies what was said some time back about the use of the geometric mean; it also justifies the neglect here of the method founded upon it, on the ground that it would lead to only an insignificant improvement in the results.

We have now obtained the values of the three constants in the general equation $w^2p^2 + f^2 = p^2$, when it is used to express the relation between mid-parentages and cofilials. Thus the quartile of the population being $p = 1{\cdot}7$, it was shown both by observation and by calculation, that the quartile of the mid-parental system was $1/\sqrt{2}\,.\,p$, or 1·21. It was also shown that the ratio of regression in that case was $\frac{2}{3}$, consequently the general equation becomes $(\frac{2}{3} \times 1{\cdot}21)^2 + (1{\cdot}5)^2 = (1{\cdot}7)^2$, or $0{\cdot}64 + 2{\cdot}25 = 2{\cdot}89$, which is an exact accordance, satisfactorily cross-testing the various independent estimates.

Converse Ratios of Regression.—We are now sufficiently advanced to be able to examine more closely the apparent paradox that the ratio of regression from the stature of mid-parents of the same height to the mean of the statures of their sons should be $\frac{2}{3}$, while that of men of the same stature to the mean of the statures of their several mid-parents should be, not the numerical converse of this, but $\frac{1}{3}$. We may look upon the entries in Table III as the values of (vertical) ordinates in z to be erected upon it at the points where those entries lie, and which are specified by the arguments of "heights of mid-parents" written along the side, as values of ordinates in y, and of "heights of adult children" written along the top, as values of ordinates in x. The smoothed result would form a curved surface of frequency. I accordingly smoothed the table by writing at each

intersection of the lines that separated the vertical columns with those that separated the horizontal lines, the sums of the four adjacent entries. Then I drew lines with a free hand through all entries, or interpolations between entries, that were of the same value. These lines formed a concentric series of elliptical figures, passing through values of z that diminished, going outwards. Their common centre at which z was the greatest, and which therefore was the portion of maximum frequency, lay at the point where both x and y were of the same value of $68\frac{1}{4}$ inches, that is, of the value of the mean stature of the population. The line in which the major axes of the ellipses lay was inclined nearer to the axis of x than that of y. It was evident from the construction that the median value of the entries, whether in each line or in each column of the table, must lie at the point where that line or column was touched by the projection of one of these ellipses. It was easy also to believe that the equation to the surface of frequency and the lines of loci of the above-mentioned points of contact, admitted of mathematical expression. Also that the problem to be solved might be expressed in a form that had no reference to heredity. In such a form I submitted it to Mr. J. Hamilton Dickson, who very kindly undertook its solution, which appears as an Appendix to this paper, and which helps in various ways to test and confirm the approximate and uncertain conclusions suggested by the statistical treatment of the observations themselves. I shall make frequent use of his mathematical results, both in respect to this problem and to another one (also given in the Appendix), in the course of my further remarks.

As regards the present subject of the connexion between the regression in direct and in converse kinships, it appears that it wholly depends on the relation between the quartiles of the two series of "arguments," and is expressed by the formula $c^2w = p^2w'$. In this case $c^2 = (1{\cdot}21)^2 = 1{\cdot}46$, and $p^2 = 2{\cdot}89$; also $w = \frac{2}{3}$; therefore $w' = \frac{1}{3}$ nearly.

It will be observed that in all cases of converse kinship, from man to man—as from man to brother, and conversely; from man to nephew, and conversely; from father to son, and conversely; $c = p$, therefore in these the ratio of regression is the same in the converse as in the direct kinship.

Brotherly Variability.—The size of human families is much too small to admit of the quartile of brotherly variability being determined in the same way as that of the population, namely, by finding the quartiles in single families, but there are four indirect ways of finding its value, which I will call b.

(1.) A collection of differences (see Table VIII) between the statures of individual brothers, in families of n brothers, and the mean of all the n statures in the same family, gives a quartile value, which I will

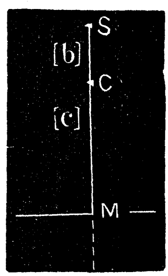
call d, whence b may be deduced as follows:—Suppose an exceedingly large family (theoretically infinitely large) of brothers; their quartile would be b. Then if we select from it, at random, numerous groups of n brothers in each, the means of the mid-deviates of the several groups would form a series whose quartile is $1/\sqrt{n} \times b$. Hence b is compounded of this value and of d; that is to say,

$$b^2 = d^2 + 1/n \times b^2 \quad \text{or} \quad b^2 = \frac{n}{n-1} d^2.$$

I treated in this way four groups of families, in which the values of n were 4, 5, 6, and 7 respectively, as shown in Table VIII, whence I obtained for b the four values of 1·01, 1·01, 1·20, and 1·08, whose mean is 1·07.

(2.) Let c be the quartile of a series of brotherly centres whose quartile is unknown and has to be determined, and that the statures of the individual brothers diverge from their several family centres $C_1 C_2$ . . ., with a quartile b, the whole group of brothers thus forming a sample of the ordinary population; consequently $c^2 = p^2 - b^2$. Now in fig. 7, MS represents the deviate in stature of a group of like persons

Fig. 7.



who are not brothers, and MC represents the mean of the mid-deviates of their respective families of brothers. It can be shown (see Appendix, Problem 2) that if the position of c varies with respect to M with a quartile $= \sqrt{p^2 - b^2}$, and if S varies with respect to c with a quartile $= b$, then, when S only is observed, the most probable value of CM is such that $\frac{\text{CM}}{\text{SM}} (= w) = \frac{p^2 - b^2}{p^2}$,

$$\text{or } b^2 = p^2(1 - w).$$

Substituting 1·7 for p, and $\frac{2}{3}$ for w,

$$b = 0{\cdot}98 \text{ inch.}$$

(3.) It can also be shown (see Appendix, Problem 2) that the variability of particular mid-brotherly deviates, $C_1 C_2$ . . ., about C, the

mean of all them, is such that its quartile $= \frac{cb}{\sqrt{(c^2+b^2)}}$. Now the distribution of values in each line of Table V, whose quartile $= f$, is due to the combination of two variables. The one is the variability of $C_1C_2 \ldots$, about C; the other is the variability of the individual brothers in each family, about C_1, C_2, &c., respectively. Therefore $f^2 = \frac{c^2b^2}{c^2+b^2}+b^2$. Substituting for c^2 its value p^2-b^2, we obtain

$$b^2=p(p-\sqrt{(p^2-f^2)}).$$

The observed value of f in Table V is 1·24, whence we obtain $b=1·10$.

(4.) Pairs of brothers may be taken at random, and the differences noted between their statures; then under the following reservation, as regards the differences to be taken, we should expect the observed quartile of the differences to be $=\sqrt{2}\times b$. The reservation is, that only as many differences should be taken out of each family as are independent. A family of n brothers admits of $n \, . \, n-1/2$ possible pairs, but no more than $n-1$ of these are independent and only these should be taken. I did not appreciate this necessity at first, and selected pairs of brothers on an arbitrary system, which had at all events the merit of not taking more than four pairs of differences from any family, however numerous. It was faulty in taking three differences instead of only two from a family of three, and four differences instead of only three from a family of four, and therefore giving an increased weight to those families, but in other respects the system was hardly objectionable. On the whole the introduced error would be so slight as scarcely to make it worth while now to go over the work again. By the system adopted I found a quartile value of 1·55, which divided by $\sqrt{2}$ gives $b=1·10$ inch.

Thus far we have dealt with the special data only. The less trustworthy R.F.F. give larger values of b. An epitome of all the results appears in the following table.

	Values of b obtained by different methods and from different data.	
	Specials.	R.F.F.
(1.) From families	1·07	1·38
(2.) From w (Tables V and IV)	0·98	1·31
(3.) From f (Tables V and IV)	1·10	1·14
(4.) From pairs of brothers	1·10	1·35
Mean	1·06	

The R.F.F. results refer to brothers only and not to transmuted sisters, except in method (2), where the paucity of the data compelled me to include them. I should point out that the data used in these four methods differ. In (1) I did not use families under four. In (2) and (3) I did not use large families. In (4) the method of selection was as we have seen, again different. This makes the accordance of the results still more gratifying. I gather from the above that we may securely consider the value of b to be less than 1·10, and allowing for some want of precision in the special data, the very convenient value of 1·0 inch may reasonably be adopted.

We are now able to deal completely with the distribution of statures in every degree of kinship of the kinsmen of those whose statures we know, but whose ancestral statures we are ignorant of or do not take into account. We are, in short, able to construct tables on the form of III, IV, and V, for every degree of kinship, and to reconstruct those tables in a way that shall be free from irregularities. The fraternal relation as distinguished from the co-fraternal has also been clearly explained.

In constructing a table of the form of III, IV, and V, we first find the value of w for the degree of kinship in question, thence we deduce f by means of the general equation $w^2p^2+f^2=p^2$ (p is supposed to be known, or for the general purpose of comparing the relative nearness of different degrees of kinship as tested by family likeness in stature, it may be taken as unity). The entries to be made in the several lines are then to be calculated from the ordinary tables of the "probability integral."

As an example of the first part of the process, suppose we are constructing a table of men and their nephews. A nephew is the son of a brother, therefore in his case we have $w=\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}$; and $f=p\sqrt{(1-w^2)},=1{\cdot}66$.

Form of Data for calculating Tables of Distribution of Stature among Kinsmen.

From any group of persons of the same height, to their kinsmen as below.	Mean regression w.	Quartile of individual variability, $f(=p\times\sqrt{(1-w^2)}$.
Mid-parents	2/3	1·27
Brothers	2/3	1·27
Fathers or sons	1/3	1·60
Uncles or nephews	2/9	1·66
Grandfathers or grandsons	1/9	1·69

Trustworthiness of the Constants.—There is difficulty in correcting the results obtained solely from the R.F.F. data, by help of the knowledge of their general inaccuracy as compared with the

special data. The reason is that this inaccuracy cannot be ascribed to an uncertainty of equal $\pm$ amount in every entry, such as might be due to a doubt of "shoes off" or "shoes on." If it were so, the quartile deviate of the R.F.F. would be greater than that of the specials, whereas it proves to be the same. It is likely that the inaccuracy is a result of the uncertainty above mentioned, which would increase the value of the quartile deviate, combined with a tendency on the part of my correspondents to record medium statures when they were in doubt, and which would reduce the quartile deviate. What the effect of all this might be on the value of w in Table IV, which is a datum of primary importance, I am not prepared to say, except that it cannot be great. While sincerely desirous of obtaining a revised value of w from new and more accurate data, the provisional value I have adopted may be accepted as quite accurate enough for the present.

Separate Contribution of each Ancestor to the Heritage of the Child.— I here insert a short extract from my paper in the "Journ. Anthrop. Inst.," with slight revision, as this memoir would be incomplete without it.

When we say that the mid-parent contributes two-thirds of his peculiarity of height to the offspring, it is supposed that nothing is known about the previous ancestor. But though nothing is known, something is implied, and this must be eliminated before we can learn what the parental bequest. pure and simple, may amount to. Let the deviate of the mid-parent be x (including the sign), then the implied deviate of the mid-grandparent will be $\frac{1}{3}x$, of the mid-ancestor in the next generation $\frac{1}{9}x$, and so on. Hence the sum of the deviates of all the mid-generations that contribute to the heritage of the offspring is $x(1+\frac{1}{3}+\frac{1}{9}+\&c.)=x\frac{3}{2}$.

Do they contribute on equal terms, or otherwise? I have not sufficient data to yield a direct reply, and must, therefore, try the effects of limiting suppositions. First, suppose the generations to contribute in proportion to the values of their respective mid-deviates; then as an accumulation of ancestral deviates whose sum amounts to $x\frac{3}{2}$, yields an effective heritage of only $x\frac{2}{3}$, it follows that each piece of heritable property must be reduced, as it were, by a succession tax, to $\frac{4}{9}$ of its original amount, because $\frac{3}{2}\times\frac{4}{9}=\frac{2}{3}$.

Another supposition is that of successive proportionate diminutions, the property being taxed afresh in each transmission to $1/r$ of its amount, so that the effective heritage would be—

$$x\left(\frac{1}{r}+\frac{1}{3r^2}+\frac{1}{3^2r^3}+-\right)=x\left(\frac{3}{3r-1}\right)$$

and this must, as before, be equal to $x\frac{2}{3}$, whence $\frac{1}{r}=\frac{6}{11}$.

A third possible supposition of the mid-ancestral deviate in any one remote generation contributing more than would be done by an equal mid-parental deviate, is notoriously incorrect. Thus the descendants of "pedigree wheat" in the (say) twentieth generation show no sign of the remarkable size of their mid-ancestors in that degree, but the offspring in the first generation do so unmistakably.

The results of our only two valid limiting suppositions are therefore (1) that the mid-parental deviate, pure and simple, influences the offspring to $\frac{4}{9}$ of its amount; (2) that it influences it to the $\frac{6}{11}$ of its amount. These values differ but slightly from $\frac{1}{2}$, and their mean is closely $\frac{1}{2}$, so we may fairly accept that result. Hence the influence, pure and simple, of the mid-parent may be taken as $\frac{1}{2}$, of the mid-grandparent $\frac{1}{4}$, of the mid-great-grandparent $\frac{1}{8}$, and so on. That of the individual parent would therefore be $\frac{1}{4}$, of the individual grandparent $\frac{1}{16}$, of an individual in the next generation $\frac{1}{64}$, and so on.

[I do not propose here to discuss the reason why the effective heritage of the child should be less than the accumulated deviates of his ancestors. It is obviously connected with considerations that bear on stability of type.]

Pure breed.—In a perfectly pure breed, maintained during an indefinitely long period by careful selection, w would become $=0$, and the value of b would be changed, but apparently only a little. Call its new value β. It may be roughly estimated as follows. In mixed breeds the value of b includes the probable uncertainty of the implied value of the contributions inherited from the mid-grandparents, and from the mid-ancestry of each preceding generation. This can be but a trifle. Suppose the quartile of the uncertainty in the implied stature of each grandparent to be even as much as 1·7 inch (we need not wait to discuss its precise value), then the quartile of the uncertainty as regards the implied mid-grandparental stature would be $1/\sqrt{4}\times$ that amount, or say 0·8. The proportion of this, which would on the average be transmitted to the child, would be only $\frac{1}{4}$ as much, or 0·2. From all the higher ancestry put together, the contribution would be much less than this, and we may disregard it. The result then is $b^2=\beta^2+0{\cdot}04$. Taking $b=1{\cdot}07$, this gives $\beta=1{\cdot}05$ inch.

Probable Stature of the Child when the Statures of several of his Kinsmen are known.—First we have to add their several contributions as assessed in the last paragraph but one, and to these we have to add whatever else may be implied. A just estimate of the latter requires the solution of a very complex problem. Thus:—a tall son has a short father; this piece of knowledge makes us suspect that the mother was tall, and we should do wrong to set down her unknown stature as mediocre. Our revised estimate would be further modified if we knew the stature of one of her brothers, and so on. Moreover, the general equation $w^2p^2+f^2=p^2$ may cease to hold good. The pos-

sible problems are evidently very various and complicated, I do not propose to speak further about them now. It is some consolation to know that in the commoner questions of hereditary interest, the genealogy is fully known for two generations, and that the average influence of the preceding ones is small.

In conclusion, it must be borne in mind that I have spoken throughout of heredity in respect to a quality that blends freely in inheritance. I reserve for a future inquiry (as yet incomplete) the inheritance of a quality that refuses to blend freely, namely, the colour of the eyes. These may be looked upon as extreme cases, between which all ordinary phenomena of heredity lie.

Appendix. By J. D. Hamilton Dickson.

Problem 1.

A point P is capable of moving along a straight line P′OP, making an angle $\tan^{-1}\frac{2}{3}$ with the axis of y, which is drawn through O the mean position of P; the probable error of the projection of P on Oy is 1·22 inch: another point p, whose mean position at any time is P, is capable of moving from P parallel to the axis of x (rectangular co-ordinates) with a probable error of 1·50 inch. To discuss the "surface of frequency" of p.

1. Expressing the "surface of frequency" by an equation in x, y, z, the exponent, with its sign changed, of the exponential which appears in the value of z in the equation of the surface is, save as to a factor,

$$\frac{y^2}{(1\cdot22)^2}+\frac{(3x-2y)^2}{9(1\cdot50)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

hence all sections of the "surface of frequency" by planes parallel to the plane of xy are ellipses, whose equations may be written in the form,

$$\frac{y^2}{(1\cdot22)^2}+\frac{(3x-2y)^2}{9(1\cdot50)^2}=\text{C, a constant} \quad . \quad . \quad . \quad . \quad (2)$$

2. Tangents to these ellipses parallel to the axis of y are found, by differentiating (2) and putting the coefficient of dy equal to zero, to meet the ellipses on the line,

$$\left.\begin{aligned} &\frac{y}{(1\cdot22)^2}-2\frac{3x-2y}{9(1\cdot50)^2}=0, \\ \text{that is} \quad &\frac{y}{x}=\frac{\dfrac{6}{9(1\cdot50)^2}}{\dfrac{1}{(1\cdot22)^2}+\dfrac{4}{9(1\cdot50)^2}}=\frac{6}{17\cdot6} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or, approximately, on the line $y=\frac{1}{3}x$. Let this be the line OM.

From the nature of conjugate diameters, and because P is the mean position of p, it is evident that tangents to these ellipses parallel to the axis of x meet them on the line $x=\frac{2}{3}y$, viz., on OP.

3. Sections of the "surface of frequency" parallel to the plane of xz, are, from the nature of the question, evidently curves of frequency with a probable error 1·50, and the locus of their vertices lies in the plane zOP.

Sections of the same surface parallel to the plane of yz are got from the exponential factor (1) by making x constant. The result is simplified by taking the origin on the line OM. Thus putting $x=x_1$, and $y=y_1+y'$, where by (3)

$$\frac{y_1}{(1\cdot22)^2}-2\frac{3x_1-2y_1}{9(1\cdot50)^2}=0$$

the exponential takes the form

$$\left\{\frac{1}{(1\cdot22)^2}+\frac{4}{9(1\cdot50)^2}\right\}y'^2+\left\{\frac{y_1^2}{(1\cdot22)^2}+\frac{(3x_1-2y_1)^2}{9(1\cdot50)^2}\right\} \quad . \quad . \quad (4)$$

whence, if e be the probable error of this section,

$$\left.\begin{aligned} \frac{1}{e^2}&=\frac{1}{(1\cdot22)^2}+\frac{4}{9(1\cdot50)^2} \\ \text{or [on referring to (3)]} \quad e&=1\cdot50\sqrt{\frac{9}{17\cdot6}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

that is, the probable error of sections parallel to the plane of yz is nearly $\frac{1}{\sqrt{2}}$ times that of those parallel to the plane of xz, and the locus of their vertices lies in the plane zOM.

It is important to notice that all sections parallel to the same coordinate plane have the same probable error.

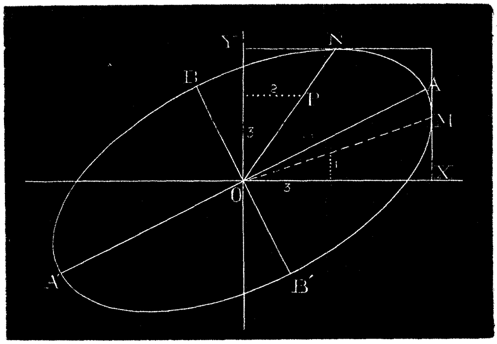
4. The ellipses (2) when referred to their principal axes become, after some arithmetical simplification,

$$\frac{x'^2}{20\cdot68}+\frac{y'^2}{5\cdot92}=\text{constant}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

the major axis being inclined to the axis of x at an angle whose tangent is 0·5014. [In the approximate case the ellipses are $\frac{x'^2}{7}+\frac{y'^2}{2}=$ const., and the major axis is inclined to the axis of x at an angle $\tan^{-1}\frac{1}{2}$.]

5. The question may be solved in general terms by putting YON$=\theta$, XOM$=\phi$, and replacing the probable errors 1·22 and 1·50 by a and b respectively: then the ellipses (2) are

FIG. 8.



$$\frac{y^2}{a^2}+\frac{(x-y\tan\theta)^2}{b^2}=C, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

equation (3) becomes

$$\left.\begin{aligned}&\frac{y}{a^2}-\tan\theta\frac{x-y\tan\theta}{b^2}=0\\ \text{or}\qquad &\frac{y}{x}=\tan\phi=\frac{a^2\tan\theta}{b^2+a^2\tan^2\theta}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and (5) becomes

$$\frac{1}{e^2}=\frac{1}{a^2}+\frac{\tan^2\theta}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

whence

$$\frac{\tan\phi}{\tan\theta}=\frac{e^2}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If c be the probable error of the projection of p's whole motion on the plane of xz, then

$$c^2=a^2\tan^2\theta+b^2,$$

which is independent of the distance of p's line of motion from the axis of x. Hence also

$$\frac{\tan\phi}{\tan\theta}=\frac{a^2}{c^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Problem 2.

An index q moves under some restraint up and down a bar AQB, its mean position for any given position of the bar being Q; the bar, always carrying the index with it, moves under some restraint up and down a fixed frame YMY′, the mean position of Q being M: the movements of the index relatively to the bar and of the bar relatively to the frame being quite independent. For any given observed position of q, required the most probable position of Q (which cannot be observed); it being known that the probable error of q relatively to

Q in all positions is b, and that of Q relatively to M is c. The ordinary law of error is to be assumed.

If in any one observation, $MQ=x$, $Qq=y$, then the law of error requires

$$\frac{x^2}{c^2}+\frac{y^2}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

to be a minimum, subject to the condition

$$x+y=a, \text{ a constant.}$$

Hence we have at once, to determine the most probable values of x', y',

$$\frac{x'}{c^2}=\frac{y'}{b^2}=\frac{a}{b^2+c^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

and the most probable position of Q, measured from M, when q's observed distance from M is a, is

$$\frac{c^2}{b^2+c^2}a.$$

It also follows at once that the probable error v of Q (which may be obtained by substituting $a-x$ for y in (12)) is given by

$$\frac{1}{v^2}=\frac{1}{c^2}+\frac{1}{b^2}, \text{ or } v=\frac{bc}{\sqrt{b^2+c^2}} \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

which, it is important to notice, is the same for all values of a.

Throughout this discussion the technical term "probable error" has been used; it may in every instance be replaced by Mr. Galton's very apt name "quartile," in which case the results of these problems may be read in conjunction with Mr. Galton's papers.

Table I.

Ogive, or Normal Curve of Distribution of Error.

Abscissæ reckoned from 0° to ± 50° (value of the probability integral).		Corresponding ordinates (or deviates). Value of the deviate when modulus = 1, A.	Value of deviate reduced proportionately to quartile = 1, B.
	10	0·179	0·38
	20	0·371	0·78
Quartile	25	0·477	1·00
	30	0·595	1·25
	40	0·906	1·90
	45	1·163	2·44

Table II.

Comparison of observed Ogives with the Normal.

	Abscissæ of the half-ogive.						Value of the unit in inches.
	10.	20.	25.	30.	40.	45.	
Normal ogive, from Table I....	0·38	0·78	1·00	1·25	1·90	2·44	1·00
General population, R.F.F.....	0·33	0·74	1·00	1·23	2·06	2·62	1·7
Population of brothers, R.F.F..	0·36	0·78	1·00	1·41	1·95	2·12	1·7
„ „ specials.	0·38	0·79	1·00	1·25	1·92	2·46	1·7
Mid-parentages	0·35	0·79	1·00	1·28	2·12	2·78	1·2
Brothers in random pairs, R.F.F.	0·47	0·84	1·00	1·29	2·11	2·64	1·4
„ „ specials.	0·42	0·78	1·00	1·25	1·88	2·44	1·4

Note.—The second decimal is only approximate.

Table III (R.F.F. Data).

Number of Adult Children of various Statures born of 205 Mid-parents of various Statures. (All Female Heights have been multiplied by 1·08).

Height of the mid-parents in inches.	Heights of the adult children.														Total number of		Medians.
	Below	62·2	63·2	64·2	65·2	66·2	67·2	68·2	69·2	70·2	71·2	72·2	73·2	Above.	Adult children.	Mid-parents.	
Above	..	..	..	..	..	..	..	..	..	..	..	1	3	..	4	5	
72·5....	..	..	..	..	..	..	..	1	2	1	2	7	2	4	19	6	72·2
71·5....	..	..	..	..	1	3	4	3	5	10	4	9	2	2	43	11	69·9
70·5....	1	..	1	..	1	1	3	12	18	14	7	4	3	3	68	22	69·5
69·5....	..	..	1	16	4	17	27	20	33	25	20	11	4	5	183	41	68·9
68·5....	1	..	7	11	16	25	31	34	48	21	18	4	3	..	219	49	68·2
67·5...	..	3	5	14	15	36	38	28	38	19	11	4	..	..	211	33	67·6
66·5....	..	3	3	5	2	17	17	14	13	4	..	..	..	..	78	20	67·2
65·5....	1	..	9	5	7	11	11	7	7	5	2	1	..	..	66	12	66·7
64·5....	1	1	4	4	1	5	5	..	2	..	..	..	..	..	23	5	65·8
Below	1	..	2	4	1	2	2	1	1	..	..	..	..	..	14	1	
Totals	5	7	32	59	48	117	138	120	167	99	64	41	17	14	928	205	
Medians ..	..	..	66·3	67·8	67·9	67·7	67·9	68·3	68·5	69·0	69·0	70·0					

Note.—In calculating the medians, the entries have been taken as referring to the middle of the squares in which they stand. The reason why the headings run 62·2, 63·2, &c., instead of 62·5, 63·5, &c., is that the observations are unequally distributed between 62 and 63, 63 and 64, &c., there being a strong bias in favour of integral inches. After careful consideration, I concluded that the headings, as adopted, best satisfied the conditions. This inequality was not apparent in the case of the mid-parents.

Table IV (R.F.F. Data).

Relative number of Brothers of various Heights to Men of various Heights, Families of Six Brothers and upwards being excluded.

Heights of the men in inches.	Heights of their brothers in inches.														Total cases.	Medians.
	Below 61·7	62·2	63·2	64·2	65·2	66·2	67·2	68·2	69·2	70·2	71·2	72·2	73·2	Above 73·7		
Above 73·7	..	..	..	..	1	..	1	..	1	4	3	3	3	2	18	
73·2	1	..	..	..	..	..	1	1	2	1	3	4	..	3	16	
72·2	1	..	1	2	1	1	..	8	6	8	11	5	4	3	51	70·3
71·2	..	..	..	4	4	4	9	11	15	12	8	11	3	3	84	69·3
70·2	1	..	2	4	3	7	6	12	25	18	11	8	1	3	101	69·3
69·2	..	..	4	6	13	12	18	29	29	24	15	6	2	1	159	68·6
68·2	1	..	..	3	6	7	15	16	29	12	11	8	1	..	109	69·9
67·2	1	..	4	3	8	14	21	15	19	6	9	..	1	1	102	67·7
66·2	..	..	1	7	10	12	14	7	12	7	4	1	..	..	75	67·2
65·2	..	1	1	4	13	9	8	6	13	3	4	1	..	1	64	67·2
64·2	..	1	..	6	4	7	3	3	6	4	4	2	..	..	40	67·3
63·2	..	..	..	..	1	1	4	..	4	2	..	1	..	..	13	
62·2	..	..	..	..	1	..	..	..	..	..	..	..	..	..	1	
Below 61·7	..	..	..	..	..	..	1	1	..	1	..	1	1	..	5	
..	5	2	13	39	65	74	101	109	161	102	83	51	16	17	838	

Table V (Special Data).

Relative number of Brothers of various Heights to Men of various Heights, Families of Five Brothers and upwards being excluded.

Heights of the men in inches.	Heights of their brothers in inches.													Total cases.	Medians.
	Below 63	63·5	64·5	65·5	66·5	67·5	68·5	69·5	70·5	71·5	72·5	73·5	Above 74		
74 and above	1	1	..	..	..	..	..	1	1	..	5	3	12	24	
73·5........	..	..	..	..	..	1	3	4	8	3	3	2	3	27	
72·5........	..	..	..	..	1	1	6	5	9	9	8	3	5	47	71·1
71·5........	..	1	..	1	2	8	11	18	14	20	9	4	..	88	70·2
70·5........	..	..	1	1	7	19	30	45	36	14	9	8	1	171	69·6
69·5........	..	1	2	1	11	20	36	55	44	17	5	4	2	198	69·5
68·5........	..	1	5	9	18	38	46	36	30	11	6	3	..	203	68·7
67·5........	2	4	8	26	35	38	38	20	18	8	1	1	..	199	67·7
66·5........	4	3	10	33	28	35	20	12	7	2	1	..	..	155	67·0
65·5........	3	3	15	18	33	36	8	2	1	1	..	..	..	110	66·5
64·5........	3	8	12	15	10	8	5	2	1	..	..	..	..	64	65·6
63·5........	5	2	8	3	3	4	1	1	..	1	..	..	1	20	
Below 63	5	5	3	3	4	2	..	..	..	..	..	..	1	23	
Totals	23	29	64	110	152	200	204	201	169	86	47	28	25	1329	

Table VI.

Construction of Ogives from Observations.

(The Statures are here distributed in grades of one inch each.)

The number of the grade.	General population. (R.F.F.)			Population of brothers. (R.F.F.)			Population of brothers. (Specials.)		
	Number of cases in each grade.	Sums from the beginning.	Per cents.	Number of cases in each grade.	Sums from the beginning.	Per cents.	Number of cases in each grade.	Sums from the beginning.	Per cents.
	A.	B.	C.	A.	B.	C.	A.	B.	C.
1........	5	5	0·5	5	5	0·6	23	23	1·7
2........	7	12	1·3	2	7	0·7	29	52	3·9
3........	32	44	4·8	13	20	2·4	64	116	8·7
4........	59	103	11·1	39	59	7·1	110	226	16·9
5........	48	151	16·3	65	124	14·8	152	378	28·2
6........	117	268	28·9	74	198	23·7	200	578	43·1
7........	138	406	43·8	101	299	35·7	204	782	58·4
8........	120	526	63·1	109	408	48·7	201	983	73·4
9........	167	693	74·7	161	569	68·0	169	1152	86·0
10........	99	792	85·4	102	671	80·2	86	1238	92·4
11........	64	856	92·2	83	754	90·1	47	1285	95·9
12........	41	897	96·7	51	805	96·2	28	1313	98·8
13........	17	914	98·5	16	821	98·1	25	1338	100·0
14........	14	928	100·0	17	838	100·0	..	..	..

Table VII.

Measurement of Ogives.

The Entries are Ordinates to the Curves constructed from Table VI, at points which are situated in every case, at the same fractional divisions, either of their bases as in the first lines of each of the three groups, or of their axes as in the other lines.

	Abscissæ, reckoned from the middle of the ogive, in percentages of the length of its axis.												
	−45	−40	−30	−25	−20	−10	0	+10	+20	+25	+30	+40	+45
General population (R.F.F.)	2·85	3·75	5·27	5·68	6·10	6·75	7·32	7·85	8·60	9·05	9·50	10·67	11·65
	4·47	3·57	1·95	1·64	1·22	0·57	0·00	0·53	1·28	1·73	2·18	3·35	4·33
							0·00	0·57	1·22	1·64	1·95	3·57	4·47
Means.............							0·00	0·55	1·25	1·68	2·07	3·46	4·40
Population of brothers (R.F.F.) ..	3·55	4·38	5·60	6·12	6·54	7·35	7·95*	8·59	9·17	9·63	10·00	11·00	11·80
	4·40	3·57	2·35	1·73	1·41	0·60	0·00	0·64	1·22	1·68	2·05	3·05	3·85
							0·00	0·60	1·41	1·73	2·35	3·57	4·40
Means.............							0·00	0·62	1·32	1·70	2·40	3·31	3·61
Population of brothers (specials) ...	2·45	3·15	4·28	4·72	5·12	5·76	6·45	7·10	7·77	8·10	8·52	9·65	10·75
	4·00	3·30	2·17	1·73	1·33	0·69	0·00	0·55	1·32	1·65	2·07	3·20	4·30
							0·00	0·69	1·33	1·73	2·17	3·30	4·00
Means.............							0·00	0·64	1·33	1·69	2·12	3·25	4·15

* The values are unsmoothed with this one exception. Its correction in no way affects the line headed "means."

Table VIII. (Special Data.)

Number of cases in which the Stature of individual Brothers was found to deviate to various amounts from the Mean Stature of their respective families.

Number of brothers in each family	4	5	6	7
Number of families	39	23	8	6
Amount of deviation.	Number of cases.	Number of cases.	Number of cases.	Number of cases.
Under 1 inch	88	62	20	21
1 and under 2	49	30	18	14
2 and under 3	15	17	5	6
3 and under 4	4	3	3	1
4 and above	..	3	2	..

II. "The Early Development of *Julus terrestris*."* By F. G. HEATHCOTE, M.A., Trin. Coll. Cam. Communicated by Professor M. FOSTER, Sec. R.S. Received January 6, 1886.

The following are the principal results of my investigations on the early development of *Julus terrestris* since June, 1882.

When laid the eggs are oval in shape, white, and covered with a thick chitinous chorion. The nucleus is situated in a mass of protoplasm in the centre of the ovum. This mass of protoplasm is of irregular shape, but its long axis corresponds with that of the ovum. From it, anastomosing processes radiate in all directions, forming a network throughout the egg. The yolk-spherules are contained within the meshes of this network. The nucleus is not a distinct vesicle but its position is marked by chromatin granules. There is no nucleolus.

Early on the second day the nucleus and the central mass of protoplasm apparently divide into two parts. But this division is not complete, the two resulting masses with their nuclei remaining connected by a network of protoplasm. Each of these divides in the same incomplete manner, so that we now have four segments all connected together. This process is continued until there are a considerable number of segmentation masses present, and early on the

* Mr. J. D. Gibson Carmichael, F.L.S., has kindly identified the species for me as *Julus terrestris*, Leach, 1814.

FIG. 1.

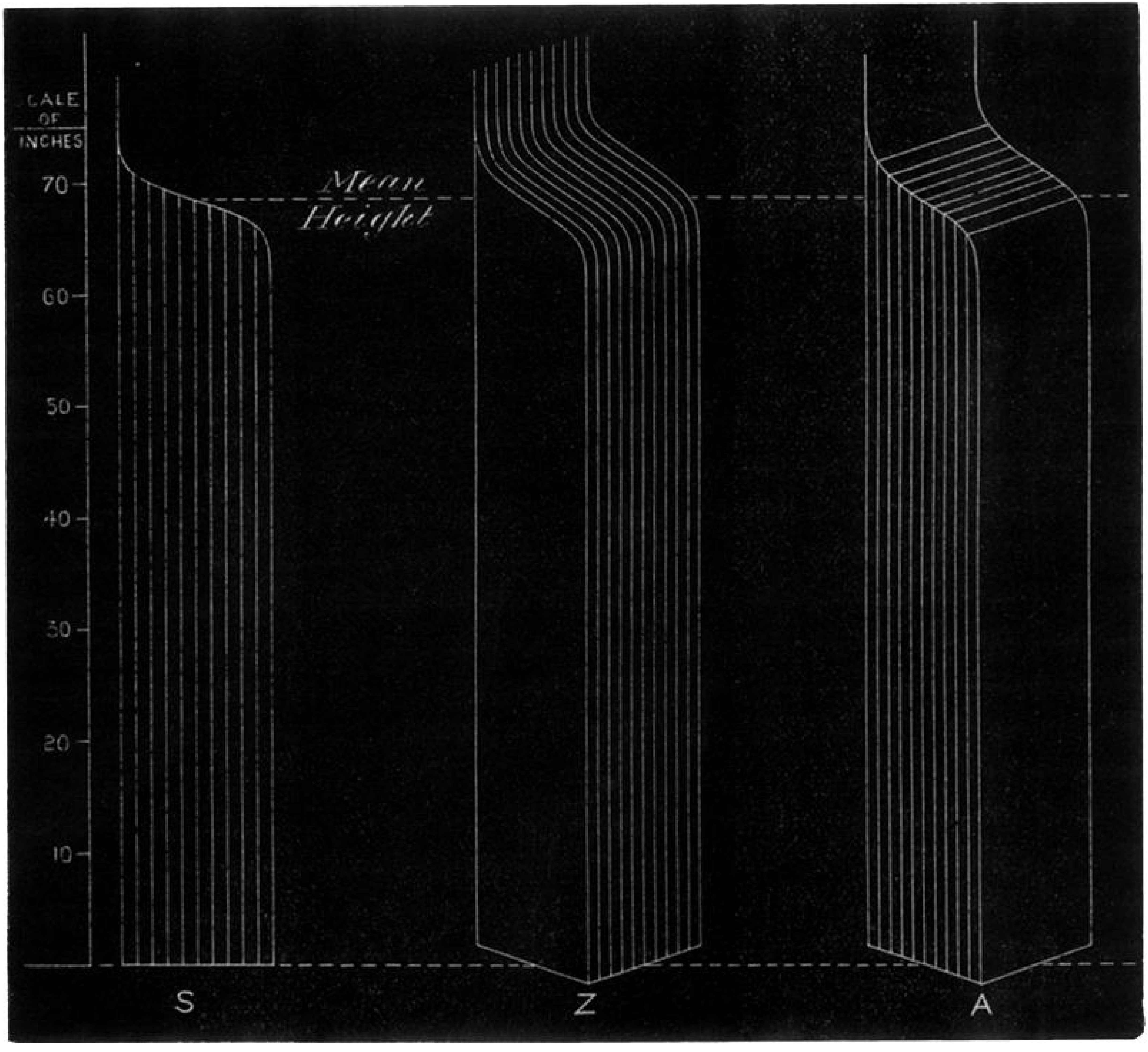


FIG. 2.

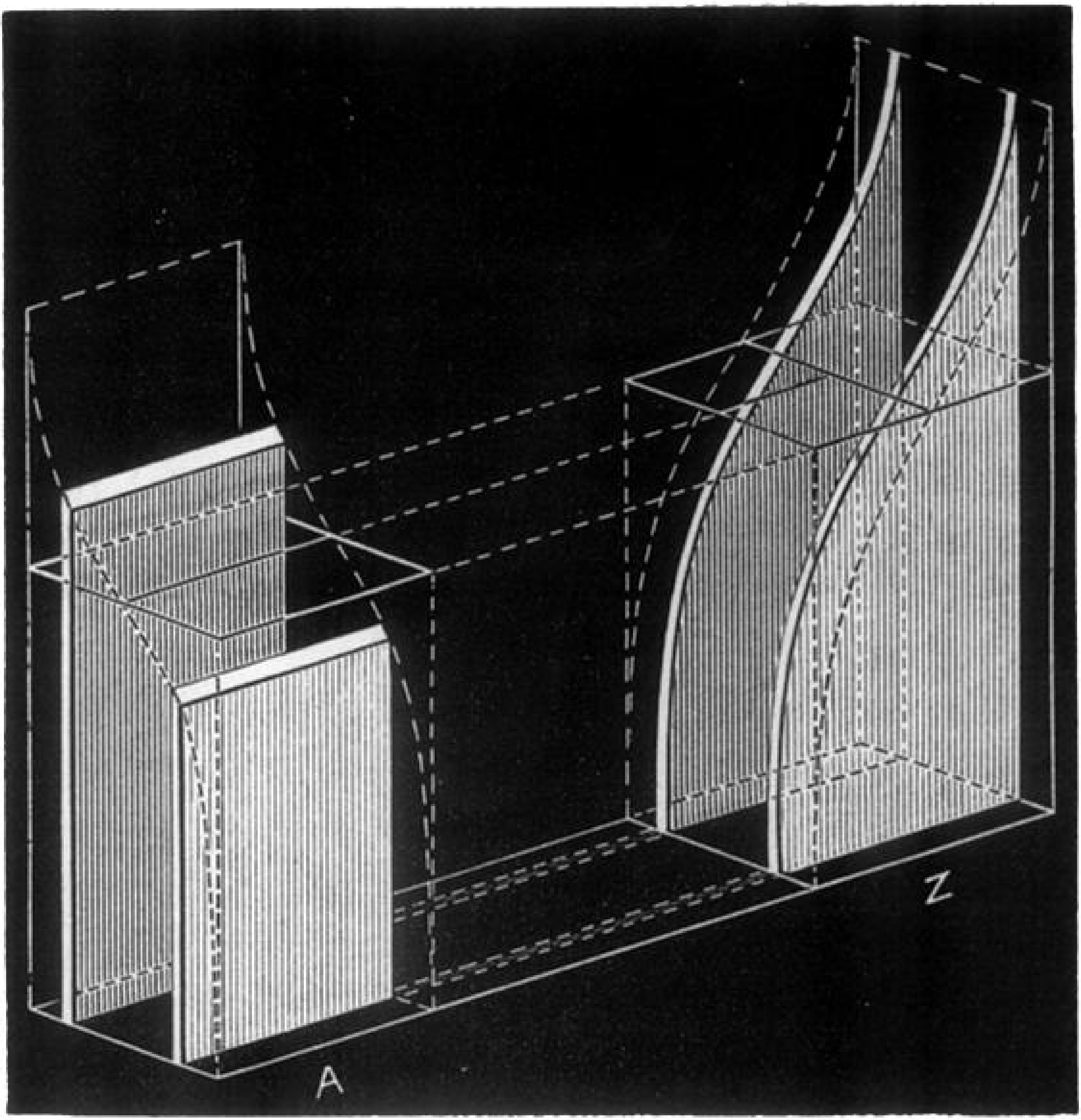
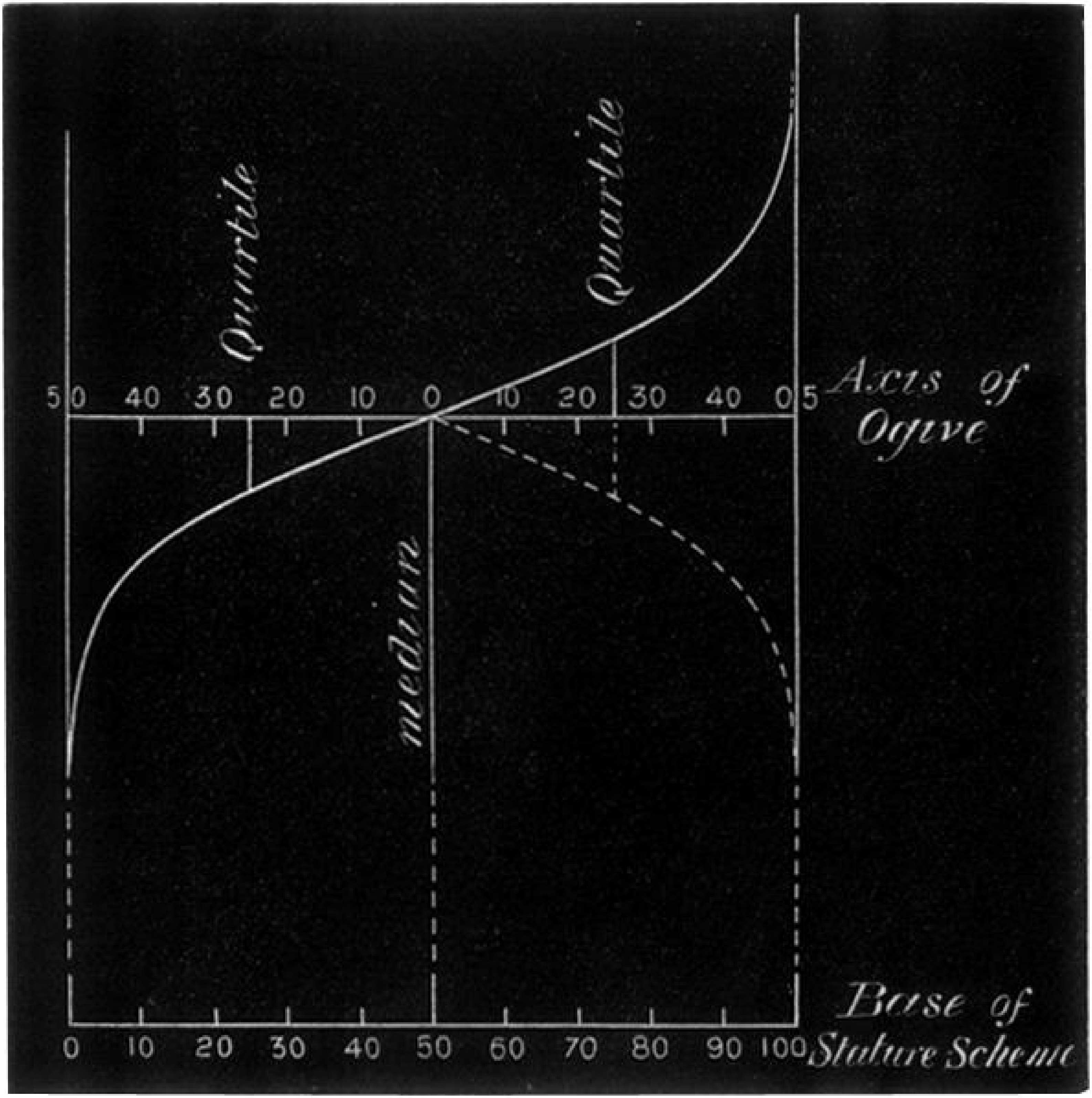


FIG. 3.



Quartile
Quartile
50 40 30 20 10 0 10 20 30 40 05
Axis of Ogive
median
Base of Stature Scheme
0 10 20 30 40 50 60 70 80 90 100

FIG. 4.

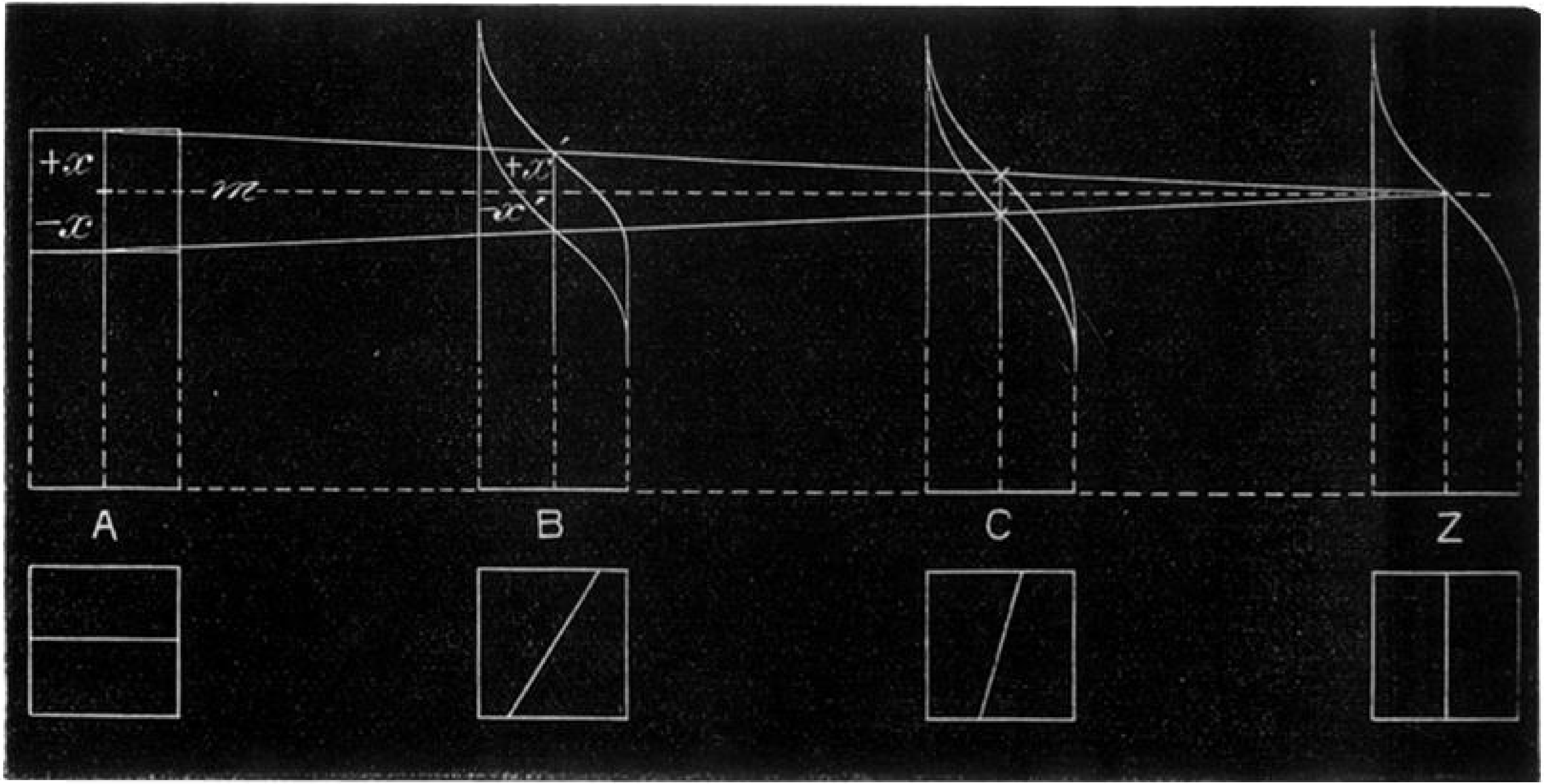


FIG. 5 and 6.

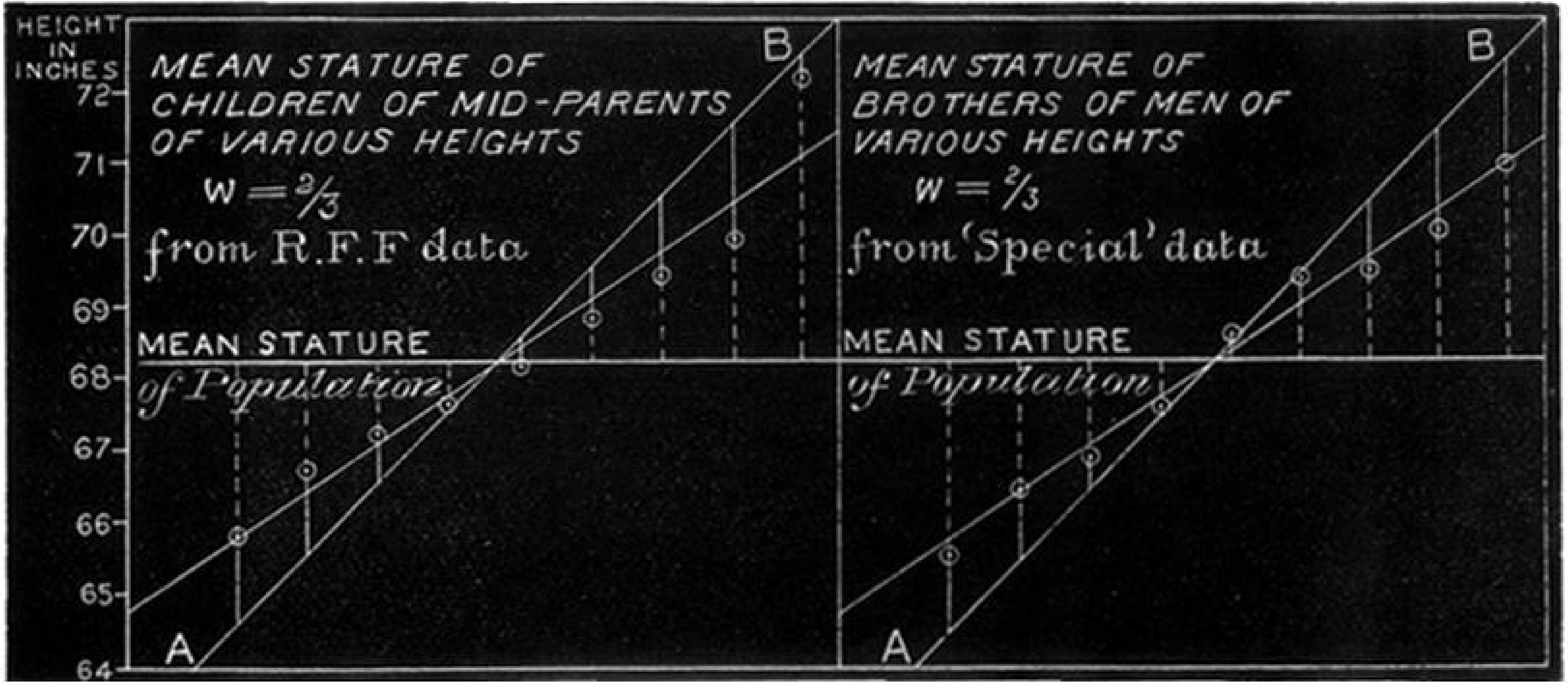

HEIGHT IN INCHES
72
71
70
69
68
67
66
65
64
MEAN STATURE OF CHILDREN OF MID-PARENTS OF VARIOUS HEIGHTS
W = 2/3
from R.F.F data
MEAN STATURE
of Population
A
B
MEAN STATURE OF BROTHERS OF MEN OF VARIOUS HEIGHTS
W = 2/3
from 'Special' data
MEAN STATURE
of Population
A
B

FIG. 7.

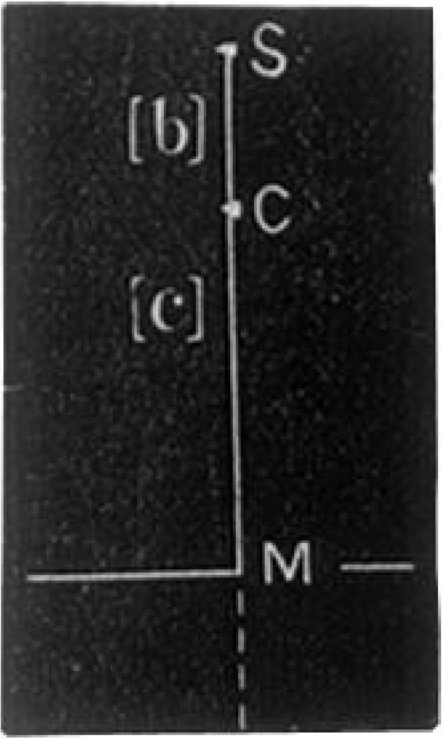


FIG. 8.