has been reached where further exhaustion does not affect the radiation observed. In this way a condition seems to be reached asymptotically, in which the radiation is independent of anything removable by the Sprengel pump. The value of the radiation found is, for the particular bright platinum wire used:—

At 408° C. ...... $378{\cdot}8 \times 10^{-4}$ gram water centigrade units per square centim. per sec.
„ 505° C. ...... $726{\cdot}1 \times 10^{-4}$ gram water centigrade units per square centim. per sec.

the temperature of the envelope being about 15° C.

Comparatively little has been done up to the present as to radiation from the same body with the surface in different conditions. The important results of Mr. Mortimer Evans, 'Roy. Soc. Proc.,' 1886, as to the energy required to maintain a given candle power in incandescent lamps, with dull and with polished filaments, have been confirmed. It is proposed to carry out further experiments on the influence of the surface of radiating bodies.

VI. "On Figures of Equilibrium of Rotating Masses of Fluid." By G. H. Darwin, M.A., LL.D., F.R.S., Fellow of Trinity College and Plumian Professor in the University of Cambridge. Received April 28, 1887.

(Abstract.)

The intention of this paper is, first, to investigate the forms which two masses of fluid assume when they revolve in close proximity about one another, without relative motion of their parts; and secondly, to obtain a representation of the single form of equilibrium which must exist when the two masses approach so near to one another as just to coalesce into a single mass.

When the two masses are far apart the solution of the problem is simply that of the equilibrium theory of the tides. Each mass may, as far as the action on the other is concerned, be treated as spherical. When they are brought nearer to one another this approximation ceases to be sufficient, and the departure from sphericity of each mass begins to exercise a sensible deforming influence on the other.

The actual figure assumed by either mass may be regarded as a deformation due to the influence of the other considered as a sphere, on which is superposed the sum of an infinite series of deformations of each due to the deformation of the other and of itself.

But each mass is deformed, not only by the tidal action of the other, but also by its own rotation about an axis perpendicular to its

orbit. The departure from sphericity of either body due to rotation also exercises an influence on the other and on itself, and thus there arises another infinite series of deformations.

It is shown in the paper how the summations of these two kinds of reflected influences are to be made, by means of the solution of certain linear equations for finding three sets of coefficients.

The first set of coefficients are augmenting factors, by which the tide of each order of harmonics is to be raised above the value which it would have if the perturbing mass were spherical. The second set correspond to one part of the rotational effect, and belong to terms of exactly the same form as the tidal terms, with which they ultimately fuse. The third set correspond to the rest of the rotational effect, and appertain to a different class of deformation, which are in fact sectorial harmonics of different orders. The term of the second order represents the ellipticity of the mass due to rotation, augmented, however, by mutual influence. All the terms of this class, except the second, are very small; their existence is, however, interesting.

From the consideration that the repulsion due to centrifugal force shall exactly balance the attraction between the two masses, the angular velocity of the system is found. It is greater than would be the case if the masses were spherical.

The theory here sketched is applied in the paper numerically, and illustrated graphically in several cases.

When the masses are equal to one another they are found to be shaped like flattened eggs, and the two small ends face one another. Two figures are given, in one of which the small ends nearly touch, and in the other where they actually cross. In the latter case, as two portions of matter cannot occupy the same space, the reality must consist of a single mass of fluid consisting of two bulbs joined by a neck, somewhat like a dumb-bell. In the figure conjectural lines are inserted to show how the overlapping of the masses must be replaced by the neck of fluid.

A comparison is also made between the Jacobian ellipsoid of equilibrium with three unequal axes and the dumb-bell. It appears that with the same moment of momentum the angular velocity is nearly the same in the two figures, but the kinetic energy is a little less in the dumb-bell. The intrinsic energy of the dumb-bell is, however, greater than that of the ellipsoid, so that the total energy of the dumb-bell is slightly greater than that of the ellipsoid.

Sir William Thomson has remarked on the "gap between the unstable Jacobian ellipsoid . . . . . and the case of the smallest moment of momentum consistent with stability in two equal detached portions." "The consideration," he says, "of how to fill up this gap with intermediate figures is a most attractive question,

towards answering which we at present offer no contribution."* This paper is intended to be such a contribution, although an imperfect one.

M. Poincaré has made an admirable investigation of the forms of equilibrium of a single rotating mass of fluid, and has specially considered the stability of Jacobi's ellipsoid.† He has shown by a difficult analytical process, that when the ellipsoid is moderately elongated, instability sets in by a furrowing of the ellipsoid along a line which lies in a plane perpendicular to the longest axis. It is, however, extremely remarkable that the furrow is not symmetrical with respect to the two ends, and there thus appears to be a tendency to form a dumb-bell with unequal bulbs.

M. Poincaré's work seemed so important that, although the figures above referred to were already drawn a year ago, this paper was kept back in order that an endeavour might be made to apply the principles enounced by him, concerning the stability of such systems. The attempt, which proved abortive on account of the imperfection of approximation of spherical harmonic analysis, is given in the appendix to the paper, because, notwithstanding its failure, it presents features of interest.

The calculations in this paper being made by means of spherical harmonic analysis, it is necessary to consider whether this approximate method has not been pushed too far in the computation of figures of equilibrium which depart considerably from spheres. A rough criterion of the applicability of the analysis is derived from a comparison between the two values of the ellipticity of an isolated revolutional ellipsoid of equilibrium as derived from the rigorous formula and from spherical harmonic analysis. As judged by this criterion, which is necessarily in some respects too severe, the figures drawn appear to present a fair approximation to accuracy.

Since, as above stated, the rigorous method of discussing the stability of the system fails, certain considerations are adduced which bear on the conditions under which there is a form of equilibrium consisting of two fluid masses in close proximity, and it appears that there cannot be such a form, unless the smaller of the two masses exceeds about one-thirtieth of the larger. It seemed therefore worth while to find to what results the analysis would lead when two masses, one of which is 27 times as great as the other, are brought close together. As judged by this criterion the computed result must be very far from the truth, but as the criterion is too severe, it seemed worth while to give the figure. The smaller mass is found to be

* Thomson and Tait, 'Natural Philosophy,' (1883), §778 (i). He also remarks elsewhere that by thinning a Jacobian ellipsoid in the middle, we shall get a figure of the same moment of momentum and less kinetic energy.

† 'Acta Math.,' 7, 3 and 4, 1885.

deeply furrowed in a plane parallel to the axis of rotation, so as to be shaped like a dumb-bell, and although this result can only be taken to represent the truth very roughly, yet it cannot be entirely explained by the imperfection of the analytical method employed. It appears then as if the smaller body were on the point of separating into two masses, in the same sort of way that the Jacobian ellipsoid may be traced through the dumb-bell shape until it becomes two masses.

M. Poincaré has commented in his paper on the possibility of the application of his results, so as to throw light on the genesis of a satellite according to the nebular hypothesis, and this investigation was undertaken with such an expectation. He remarks, however, that the conditions for the separation from a mass, which is strongly concentrated at its centre, are necessarily very different from those which he has treated mathematically.

However, both his investigation and the considerations adduced here seem to show that, when a portion of the central body becomes detached through increasing angular velocity, the portion should bear a far larger ratio to the remainder than is observed in our satellites, as compared with their planets; and it is hardly probable that the heterogeneity of the central body can make so great a difference in the results as would be necessary, if we are to make an application of these ideas.

It seems then at present necessary to suppose that after the birth of a satellite, if it takes place at all in this way, a series of changes occur which are still quite unknown.

VII. "The Influence of Stress and Strain on the Physical Properties of Matter. Part I. Elasticity—*continued.* The Velocity of Sound in Metals, and a Comparison of their Moduli of Torsional and Longitudinal Elasticities as determined by Statical and Kinetical Methods." By HERBERT TOMLINSON, B.A. Communicated by Professor W. GRYLLS ADAMS, M.A., F.R.S. Received April 29, 1887.

(Abstract.)

The principal object of the investigation was to ascertain whether the values of the moduli of torsional and longitudinal elasticities, as determined by statical methods, would be the same as when determined by kinetical methods, provided the deformations produced were very small.

The method of determining the modulus of longitudinal elasticity statically has been already described.* This method was applied with

* 'Phil. Trans.,' 1883, Part I.