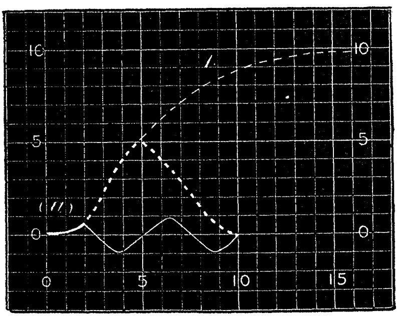
III. "On the Limiting Distance of Speech by Telephone." By William Henry Preece, F.R.S. Received February 17, 1887.

The law that determines the distance to which speaking by telephone on land lines is possible, is just the same as that which determines the number of currents which can be transmitted through a submarine cable in a second. The experimental evidence upon which this law is based was carried out in 1853 by Mr. Latimer Clark (whose assistant I then was). The experiments were made by me in the presence of Faraday; many were his own; he made them the subject of a Friday evening discourse at the Royal Institution, January 20, 1854, and they are published in his 'Researches' (vol. 3, p. 508). They received full mathematical development by Sir William Thomson in 1855 ('Roy. Soc. Proc.,' May 24), who determined the law, the accuracy of which was proved by Fleeming Jenkin and by Cromwell Varley, and the 110,000 miles of cable that now lie at the bottom of the ocean afford a constant proof in their daily working.

Hockin reduced Thomson's law to the following series:—

$$x = \mathrm{C}(1-2\{(\tfrac{3}{4})^{t/a}-(\tfrac{3}{4})^{4t/a}+(\tfrac{3}{4})^{9t/a}-(\tfrac{3}{4})^{16t/a}+\&c.\}),$$

which allows it to be expressed by the following curve (1):—



Now a is a time-constant dependent on the conditions of the circuit, invariable for the same uniform circuit but differing for different circuits. It represents the time that elapses from the instant contact is made at the sending end to the instant that the current begins to appear at the receiving end. It is given by the following equation:—

$$a = \mathrm{B}krl^2,$$

B being a constant dependent principally on the units used; k the

inductive capacity per unit length (mile or knot); r the resistance per unit length, and l the length in miles or knots.

a, therefore, limits the number of vibrations per second that can be sent through any circuit. If a be 0·196 second, as it was in the French Atlantic cable of 1869,* 2584 knots long, then it is impossible to send 5·1 currents per second through that cable; but it would be possible to send 5 or $2\frac{1}{2}$ complete reversals per second. Moreover, as the number of reversals varies inversely with the square of the length, it shows that such a cable if of 100 miles length would allow 1562 reversals to pass through it. It is necessary to remark that these expressions involve no mention of E.M.F., or of current, and therefore the number of reversals which can be produced at the end of a wire is quite independent of the impressed E.M.F., and therefore of the strength of the current. But the number of reversals is dependent upon the sensitiveness of the apparatus used to receive the currents, for if we use an instrument which will respond to a current indicated by the full line, we get two currents per second; and if we use an instrument which will respond only to the dotted line, we get only one current in two seconds, which is about the current used with delicate relays in this country. This is why such discordant results are obtained by different observers who attempt to measure the velocity of currents of electricity. It is also why the telephone is such an admirable instrument for research—for it is sensitive to the least increment or decrement of current.

Before proceeding with this inquiry it was necessary to determine very accurately the inductive capacity of overhead and underground wires. This was done with great care on very dry days in different parts of the country by means of a Thomson mirror galvanometer and a standard condenser.

The results come out as follows:—

	Capacity per mile microfarads.	Resistance per mile B. A. ohms.
No. $7\frac{1}{2}$ iron wire	0·0168	12·0
No. $12\frac{1}{2}$ copper wire	0·0124	5·7
Gutta-percha-covered wire in iron pipes	0·2500	23·0
Gutta-percha-covered wire in cables	0·2900	10·25

The capacity can be calculated from the following formula (also due to Sir William Thomson):—

$$K = \frac{l}{2 \log (4h/d)},$$

* Fleeming Jenkin, 'Electricity and Magnetism,' p. 331.

where d is the diameter of the wire and h the height above the ground. Taking h at 500 cm. and d at 0·243 cm., the capacity comes out for $12\frac{1}{2}$ wire at 0·0113 instead of 0·0124 microfarad per mile. The cause of this difference is referred to further on.

It then became necessary to determine the speed of the current through wires of different lengths, resistances, and capacities.

A multiplex distributor—such as we are now using in the Post Office—enables this to be done with great accuracy. At each station a circle is broken up into 162 sections, and an arm in connexion with the line wire carrying a brush sweeps over these sections and makes contact. This arm can be made to rotate at any speed. If it makes three revolutions per second it will make the duration of each contact $\frac{1}{486}$th of a second. Now the two distributors are kept running in absolute synchronism, so that the brush at each station always rests simultaneously on corresponding sections. The sections at one station can be placed in contact with the battery, and those at the other station with a galvanometer or other sensitive apparatus. If the current traversed the wire instantaneously then it would appear on the same section at the same time on the galvanometer; but the current is always retarded, and the amount of retardation or the value of a can be measured and the curve of arrival and cessation drawn by watching the indications of the galvanometer on each succeeding section.

The following table summarises a large number of experiments that have been made in different parts of the country, and on different lines, to determine by observation the connexion that exists between speed of current, distance spoken through, resistance, and capacity. The maximum current, or the crest of the wave, was observed. This was equivalent to 0·003 ampère.

It will be seen that the limiting distance through which it is possible to speak varies inversely with the speed of the current, and that the speed of the current varies inversely with the product of the total resistance and the total capacity of the circuit. Hence we can say that the number of reversals that it is possible to send through any circuit varies inversely with the product of the total resistance (R) and the total capacity (K), or the limiting distance

$$S = KR \times \text{constant}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is only another form of Thomson's law for $K = lk$, and $R = lr$, and

$$\therefore S = krl^2 \times \text{constant}.$$

It is seen that when the speed of the working current was

0·001″ speaking was perfect,
0·002″ speaking was good,
0·003″ speaking was fair,
0·004″ speaking was difficult.

Table showing the Speed of Current through Wires.

Wire.	Route.	Length in miles.	Total resistance × Total capacity.	Speed of current in seconds.	Telephonic notes.
Nos. 142 and 144, Denham and Atherstone	New West Coast.. Open...... / Covered ...	90 / 2·5	2247·0	0·0007 calculated.	Gower-Bell Telephone. Speaking good.
London and Denham	Underground..........	21	2963·5	0·0009 calculated.	Ditto ditto.
Nos. 142 and 144, Denham and Stafford..........	New West Coast.. Open...... / Covered....	119 / 5	4715·7	0·0015 calculated.	Gower-Bell Telephone. Speaking fairly good.
South Wales	.... Open...... / Covered....	79 / 1·25	5680·0	0·0016 calculated.	Gower-Bell Telephones, working satisfactorily.
Newcastle Telephone system, Warkworth to Stockton P.O..........	*Viâ* Newcastle, Sunderland, and West Hartlepool.. Open .. / Covered	63·831 / 12·039	5817·6	0·0016 calculated.	Gower-Bell Telephones. 5 intermediate electro-magnets of 1000 ohms resistance, in derived circuit. Speaking satisfactory.
Nos. 142 and 144, London and Towcester.........	New West Coast.. Open...... / Covered....	50 / 21	7341·3	0·00234 calculated.	Gower-Bell Telephone. Speaking weak, but just able to hold a conversation.
Nos. 142 and 144, Denham and Nantwich	Ditto. Open...... / Covered....	146 / 7	7612·1	0·0034 calculated.	Gower-Bell Telephone. Speaking weak, but just able to carry on a conversation.

Table showing the Speed of Current through Wire—*continued.*

Wire.	Route.	Length in miles.	Total resistance × Total capacity.	Speed of current in seconds.	Telephonic notes.
London to Denham (with additional loop between Denham and Hanwell) ..	Underground..........	39	10221·0	0·0032 calculated.	Gower-Bell Telephone. Speaking good, but approaching the point of difficulty.
Nos. 142 and 144, Denham to Warrington	New West Coast.. Open...... / Covered....	172 / 8	10399·16	0·0038 calculated.	Only just able to speak. Impossible to carry on conversation.
Nos. 142 and 144, London to Atherstone..........	Ditto. Open...... / Covered....	90 / 23·5	12511·2	0·004 calculated.	Only just possible to speak.
New Irish Cable	Cable.... Open...... / Covered....	25 / 62·9	12587·4	0·00412 observed.	Berliner's Telephone. Only just able to talk.
Newcastle district........	.... Open...... / Covered....	30 / 30	12640·0	0·00412 calculated	Requires very good telephones and very good voices.
Copper. London to Nevin (land lines for the new Irish cable)	*Viâ* Worcester and Shrewsbury .. Open .. / Covered	251·18 / 18·5	15771·5	*0·0035 observed.	Berliner's Telephone. Some voices good.
No. 11, London Stock Exchange to Liverpool Stock Exchange (Iron wire, No. 8 gauge)......	*Viâ* London and N.W. Railway .. Open... / Covered.	199·014 / 7·243	16417·8	0·005 observed.	Telephone not tried.

N.B.—The speed is calculated from the formula $t = 32 \times 10^{-8}KR$.

* The constant for copper is 22×10^{-8}.

If we put equation (1) into this form,

$$A = krx^2, \quad \dots\dots\dots \quad (2)$$

and give to A the following values :—

Copper (overhead)	15,000
Cables and underground........	12,000
Iron (overhead)	10,000

we can find the limiting distance we can speak with any wire; for

$$x^2 = A/kr.$$

Take copper, whose constant is 15,000, and a wire whose resistance is 1^{ω} per mile, and capacity 0·0124 per mile, then—

$$x^2 = \frac{15000}{0{\cdot}0124},$$

$$x = 1100,$$

which is the limit of speaking upon such a wire.

The wire used between Paris and Brussels has a resistance of 2·4 ohms and a capacity of about 0·012 microfarad per kilometre, and as that distance is only about 200 miles the speaking must be excellent. Moreover, there is reason to believe, from the difference between observation and calculation, that the static capacity on Continental and American lines is less than that of English lines, owing to the use of earth wires on all poles in England, and therefore the distance would be greater.

Take an Atlantic cable—

$$x^2 = \frac{12000}{3 \times 0{\cdot}43},$$

$$x = 96.$$

Now I had found in 1878* that it was just possible to speak through 100 miles of such a cable—a very close agreement.

Moreover, by the law of the squares, 100 miles of an Atlantic cable ought to transmit 1562 reversals, if 2584 miles transmit $2\frac{1}{2}$, and this is probably the average number of sonorous vibrations imparted by the human voice, when hearing by telephone begins to get difficult by the loss of the higher partials and overtones.

There is another interesting consequence of Thomson's law which comes out of these experiments, and that is, whether the line be a single wire completed by the earth, or a double wire making a metallic circuit, the rate of speed between the two ends is exactly the

* 'Phil. Mag.,' April, 1878.

same, and therefore the distance we can speak through is just the same whether we use a single or double wire circuit. This is owing to the fact that though in the latter case we double the total resistance, we halve the total capacity, and therefore the product remains the same.

The difference between copper and iron is clearly due to self-induction, or to the electromagnetic inertia of the latter, and the difference between copper overground and copper underground is due to the facility that the leakage of insulators offers to the rapid discharge to earth at innumerable points, of the static charge, which in gutta-percha-covered wire can find an exit only at the ends.

It is also evident that there is no difficulty in working telephones through underground wires, even though they attain 50 miles in length, and in fact it would be better to work underground with proper copper wire from London to Brighton, than to use iron wires along the railway telegraph poles, owing to the absence of external disturbances in the former case.

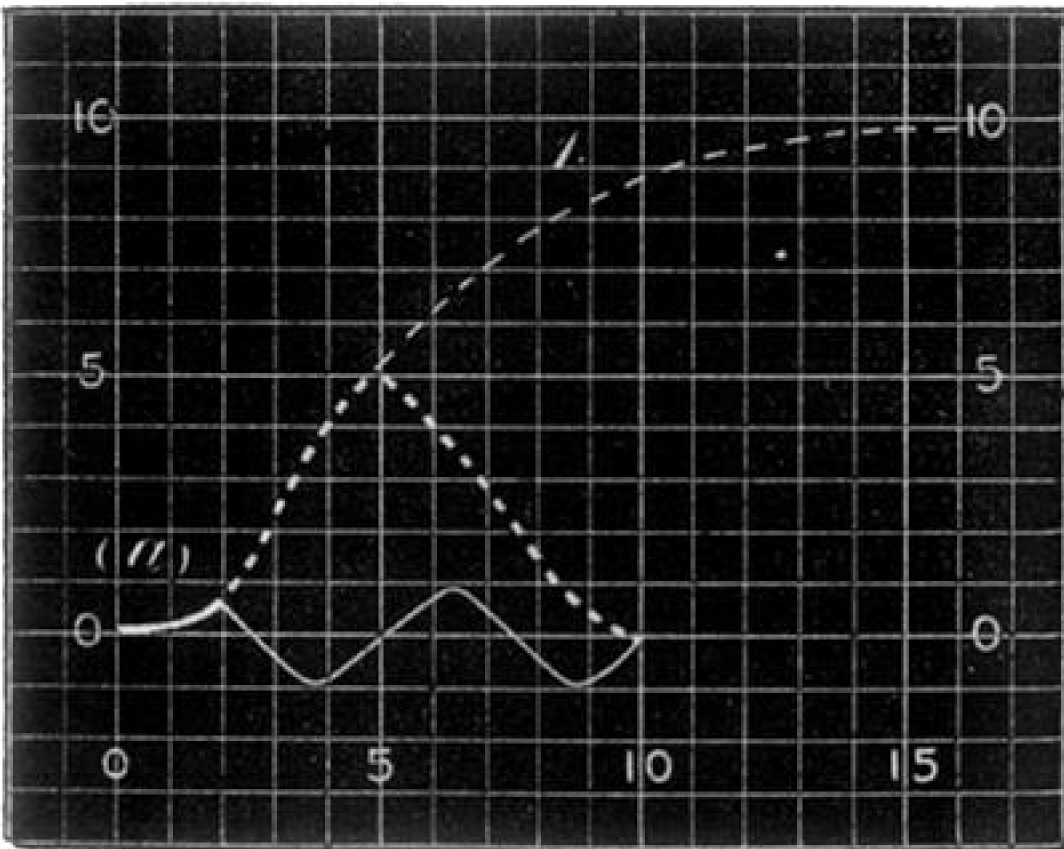
The limit of working of different insulated wires is easily obtained by equation (2), and the following table gives that information for different gutta-percha-covered wires.

No.	k.	r.	Limit of speech.
$\frac{20}{11}$	0·270 mf.	45·00 ohms.	32 miles.
$\frac{18}{7\frac{1}{2}}$	0·250 ,,	23·00 ,,	46 ,,
$\frac{16}{4}$	0·240 ,,	13·00 ,,	62 ,,
$\frac{107 \text{ lbs.}}{150 \text{ lbs.}}$	0·290 ,,	10·25 ,,	64 ,,

N.B.—The top number indicates the gauge of wire, and the lower number that of the gutta-percha.

IV. "The Etiology of Scarlet Fever." By E. KLEIN, M.D., F.R.S., Lecturer on General Anatomy and Physiology at the Medical School of St. Bartholomew's Hospital, London. Received February 23, 1887.

The investigation, the results of which I now record, was commenced at the end of December, 1885. It arose out of an inquiry into the prevalence of scarlatina in different quarters of London, under-



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