"On the Effect of Polish on the Reflexion of Light from the Surface of Iceland Spar." By C. SPURGE, B.A., St. Catherine's College, Cambridge. Communicated by R. T. GLAZEBROOK, M.A., F.R.S. Received November 18,—Read December 16, 1886. Revised March 3, 1887.

I. *Introduction.*

The optical effect of polishing the surface of a transparent body has received the most complete investigation at the hands of Seebeck,* and till very recently† Seebeck's were almost the only experiments made on the subject. Seebeck's method consisted in observing with a Nicol the light of a lamp reflected from the surface of the body. By means of a divided circle, the angle of polarisation was measured, and it was from an alteration in this angle that a change in the state of the surface was inferred. But it has been since shown by Jamin‡ that, when *plane* polarised light is incident upon the surface of a transparent body, the reflected light is in general not plane but to a measureable degree elliptically polarised, and consequently there is no angle of incidence at which the light can be *completely* quenched by a Nicol. It follows that, as regards our present state of knowledge, Seebeck's investigation is to some extent incomplete, and also that there is some uncertainty in the determination of the angles of polarisation, which may affect our conclusions as regards the state of the surface, especially since the difference produced by polishing is according to Seebeck not very large. Both Sir David Brewster§ and M. Jamin were of the opinion that Seebeck's experiments should be repeated, and the latter promised to consider the effect of polish later on but appears never to have done so. Mr. Glazebrook kindly pointed out to me that the subject presented a suitable field for research, and, at his instance, I undertook the present investigation.

My object has been to attain *greater accuracy* than hitherto by employing for an analyser a quarter undulation plate in addition to a Nicol, so as to make the extinction of the reflected light very complete. The angle of incidence of the polarised light falling on the surface of the crystal was kept constant, in order to measure as *directly* as possible the alteration produced by change of polish. Both the azimuth of the major axis and the ratio of the axes of the elliptically polarised light were calculated. These quantities furnish

* 'Poggendorff, Annalen,' vol. 20, 1830, p. 27; vol. 21, 1831, p. 290.
† Sir J. Conroy, 'Roy. Soc. Proc.,' Feb., 1886.
‡ 'Annales de Chimie,' vol. 29, 1850, p. 263.
§ 'Edinb. Journ. Sci.,' vol. 5, 1831.

us with *two* independent tests of a change of surface, and completely determine the nature of the light, so that a knowledge of their values before and after polishing enables us to state the precise alteration produced in the reflected light, a question which has never been investigated, and is, I believe new to this paper.

II. *Apparatus.*

A series of preliminary experiments were made to discover what apparatus was best suited for the investigation. I found that, whether a Nicol or a Nicol and a quarter undulation plate were used as analyser, it was best to polarise the light before incidence. Also, observations showed me that a Nicol and a quarter wave plate were a more sensitive arrangement than a simple Nicol, supposing that in each case the light was polarised before incidence. I have therefore deemed it necessary to employ the Nicol and quarter wave plate arrangement in order to secure all the accuracy that is possible by completely quenching the reflected light.

The instrument I used was an elliptic analyser kindly lent for the purpose of these experiments by Professor Stokes.

A very full description of the instrument will be found in the 'Phil. Mag.' for 1851, but the following abbreviated account is given in order to explain the way in which it was used during the course of the experiments.

The elliptic analyser consists of a brass annulus attached to a vertical stem which fits into a hollow cylindrical foot. When the foot is placed on a table, the plane of the annulus is vertical. Within the annulus turns a brass graduated disk; and the angle through which it turns is read off by means of verniers engraved on the annulus. These verniers are therefore fixed. The disk is pierced by a central aperture on the side of which opposite the incident light is a screw thread, so that a cell containing a quarter wave plate can be screwed into the disk. In front the disk carries a hollow cylinder turned in the lathe with the disk itself. Round the cylinder turns a collar into which is screwed a tube containing the analysing Nicol. The collar carries a pair of level edged verniers by which the angle may be read off through which the Nicol has been turned. These verniers are therefore moveable. Thus the quarter wave plate moves in azimuth, carrying the Nicol along with it, and the Nicol has likewise an independent motion in azimuth. In observing, the light is extinguished by a combination of the two movements, in which case the elliptically polarised light is converted by the quarter wave plate into plane polarised, which is then quenched by the Nicol. There are two principal positions in which the light can be quenched, and, since either Nicol or quarter plate may be reversed by turning through 180°, there are four subordinate positions corresponding to

each principal position. The position of the Nicol is determined by the readings of the two moveable and that of the quarter wave plate by the readings of the two fixed verniers. Thus each principal position is determined by eight readings, and in the tables which follow, each number is the mean of eight readings.

Suppose that R, R′ are the mean readings of the fixed, r, r' the mean readings of the moveable verniers, then the quantities, which it is the object of the present investigation to determine, are tan ϖ, the ratio of the axes of the ellipse, I, the azimuth of the major axis of the ellipse, and these are given by the formulæ—

$$\cos 2\varpi = \sin (r'-r) / \sin (\mathrm{R}'-\mathrm{R}),$$

and

$$\mathrm{I} = \tfrac{1}{2}(\mathrm{R}'+\mathrm{R}).$$

These equations determine ϖ absolutely, but I will be measured from an arbitrary zero which will remain fixed so long as the quarter plate is not unscrewed from its containing tube, but which will be changed by a constant amount, if for any reason it is unscrewed and rescrewed up.

A subsidiary quantity is ρ, the retardation of the crystal plate, which may be determined by means of the equation,

$$\cos \rho = \tan (r'-r) / \tan (\mathrm{R}'-\mathrm{R}).$$

The source of light employed was an Argand burner, the rays from which were polarised by means of a Nicol before incidence.

III. *Adjustments.*

The tube of the polariser was levelled and its axis placed in a direct line with the centre of the flame. The height of a small brass table, on which the crystal was placed was adjusted so that the reflected light passed through the tube of the analyser. A number of preliminary observations were made to determine the best angle of incidence, *i.e.*, the angle at which the extinction was most rapid.

The best position of the analyser having been found, the centre of the tube of the analyser was adjusted to the same height as the centre of the tube of the polariser and the centre of the face of the crystal. The tube of the analyser was so directed that a ray of light from the centre of the flame passing along the axis of the polariser was reflected so as to enter at the centre of the tube of the analyser and leave at the centre of the tube.

As the present experiments were directed to discover a difference which at the outset was recognised as possibly small, especial care was taken to secure fixity of position in the parts of the instrument and in the position of the face of the crystal of Iceland spar.

The instruments were firmly attached to a laboratory table, and before commencing the moveable parts were examined and tightly screwed up. Round the base of the table and the foot of the analyser a small quantity of melted paraffin was poured so as to form connecting links from one part of the apparatus to another. From time to time these links were examined and found to be unbroken.

To attach the crystal to the table, hard electrical cement was used. The plan finally adopted was to place the crystal on the table, and to fix it by pouring a small quantity of melted wax down the back of it.

Since the crystal was removed from the table to polish its surface, some means of restoring it to its original position were needed. The following optical method was employed.

A circular diaphragm with a central pinhole was fitted to the brass tube containing the polarising Nicol. If the pinhole were slightly eccentric, the position of the hole would change as the disk rotated in its plane. To obviate any such alteration, a radius was drawn on the diaphragm which was set so that it was horizontal and always pointed in the same direction. In front of the source of light was placed a screen having a small hole at the same height as the pinhole in the diaphragm. The position of the screen was defined by lines drawn on the table.

Thus only a single ray of light was allowed to fall on the surface of the crystal, viz., that passing through the apertures in the screen and diaphragm. As these apertures could always be replaced in the same position, the direction of this incident ray was a fixed horizontal straight line. In a similar manner a circular diaphragm with a central pinhole was fitted to the brass tube containing the analysing Nicol, and some distance in front of this tube was placed a screen with a pinhole at the same height as the pinhole in the diaphragm. The position of the screen was defined as before by lines drawn on the table. A radius was drawn on the diaphragm, and also, since the tube itself was moveable, a mark was made on it so that it could be turned into the same position.

The crystal was placed on the brass table so that its plane was vertical and passed through the centre of the circular top.

On placing the eye opposite the aperture in the screen facing the elliptic analyser, it was found that a bright dot of light was visible. Thus the horizontal incident ray already mentioned must have been reflected by the crystal surface so that it passed through the apertures in the screen and the diaphragm fitted to the analysing tube. These apertures could be replaced in the same position, and therefore the direction of the reflected ray was a fixed horizontal line. Consequently the normal to the surface of the crystal bisecting the angle between the incident and reflected rays was a fixed direction.

Supposing the crystal to have been taken down for polishing, it

could be restored to position by placing the diaphragms and screens in their proper stations, and setting the crystal so that a bright dot of light was visible to the eye in front of the last screen.

The only possible changes in position that this method allows are a displacement parallel to the table and a rotation of the face of the crystal in its own plane. The former was prevented by means of fixed marks on the table, and the latter by taking every precaution to leave the base of the crystal in contact with the table unchanged, and later on by the use of a template.

A series of experiments were made to determine whether the diaphragms and screens could be removed and replaced in exactly the same positions. For this purpose the screens and diaphragms were removed one at a time and replaced, using only the setting lines. It was found that the dot of light remained visible, while the slightest displacement of the screens caused it to disappear.

During each set of experiments the screens and diaphragms were frequently replaced to determine if the crystal remained unmoved.

The surface of the crystal was shielded during the day by a box with apertures, and completely covered at night. In taking the readings the quarter wave plate and Nicol were turned into such positions that the centre of the field was as dark as possible.

IV. *Experiments made with a Natural Face.*

Observations were now made with the light reflected from that natural face of the crystal which seemed the best. The results are given in Tables I, II. The observations made with the crystal were always consecutive, none being rejected after the first satisfactory observation had been taken.

Table I.—Observations with a Natural Face. Mean Temp. 15°·3 C.

	r'.	r.	R′.	R.	$(r'-r)$.	(R′−R).
	90·562°	4·601°	153·895°	62·475°	85·961°	91·420°
	90·585	4·565	153·781	62·286	86·020	91·495
	90·350	4·334	153·955	62·256	86·016	91·699
	90·293	4·324	154·002	62·516	85·969	91·486
	90·285	4·525	153·840	62·462	85·760	91·378
	90·454	4·594	153·852	62·314	85·860	91·538
	90·278	4·430	153·790	62·408	85·848	91·382
	90·362	4·321	153·888	62·315	86·041	91·573
	90·486	4·470	153·866	62·339	86·016	91·527
	90·045	4·525	153·833	62·215	85·520	91·618
	90·287	4·545	153·433	62·317	85·742	91·116
	90·370	4·470	154·020	62·264	85·900	91·756
	90·312	4·450	153·687	62·285	85·862	91·402
Means..	90·359	4·473	153·834	62·342	85·886	91·492

Table II.

	ϖ.	tan ϖ.	I.
	1° 53′·5	0·03303	108·185°
	1 50·8	0·03225	108·033
	1 48·1	0·03145	108·105
	1 52·5	0·03275	108·259
	2 0·3	0·03501	108·151
	1 55·3	0·03356	108·083
	1 57·5	0·03419	108·099
	1 49·	0·03172	108·101
	1 50·4	0·03212	108·102
	2 5·4	0·03648	108·024
	2 3·2	0·03585	107·875
	1 51·2	0·03236	108·142
	1 56·8	0·03399	107·986
Means..	1 54·9	0·03344	108·088

Taking from Table I the mean values of R′—R, $r'-r$, we have—

$$\cos 2\varpi = \frac{\sin(r'-r)}{\sin(R'-R)} = \frac{\sin 85{\cdot}886}{\sin 91{\cdot}492} = 0{\cdot}997762.$$

Thus $\varpi = 1°\,55'$, and we obtain for the values of the quantities which determine the nature of the polarised light,

$$\tan\varpi = 0{\cdot}03346, \qquad I = 108{\cdot}088°.$$

We have to find whether these two quantities are altered by polishing.

The subsidiary quantity ρ is given by—

$$\cos\rho = \frac{\tan(r'-r)}{\tan(R'-R)} = \frac{\tan 85{\cdot}886}{\tan 91{\cdot}492} = -0{\cdot}3621.$$

We are calculating ρ merely for the purpose of verification, and are not using it to determine the nature of the polarised light. Take, then, ρ to be the least positive angle which satisfies the last equation. Thus $\rho = 111°\,14'$. This is the mean value of ρ. To estimate the error in determining ρ, take the 12th set of observations in Table I. We have—

$$\cos\rho = \frac{\tan 85{\cdot}900}{\tan 91{\cdot}756} = -0{\cdot}42768,$$

whence $\rho = 115°\,19'$. There is thus a difference of 4° in the extreme value of ρ from the mean. The cause of this apparently large variation in the values of ρ will be considered later on.

V. *Determination of the Inclination of the Natural Face to the same Face Polished.*

The crystal was taken down from its position on the brass table, and, in order to ensure that the face from which the light was reflected was ground parallel to itself, the inclinations of the face to two other faces which were left untouched, were obtained before and after polishing. The measurements were made in the usual way with the spectrometer. It was found that a fair image of the slit could be obtained by reflexion at each of the faces. Three times the crystal was completely dismounted and measurements of each angle taken.

	Before polishing.		After polishing.	
	First angle.	Second angle.	First angle.	Second angle.
	105° 5′ 56″	74° 58′ 37″	105° 6′ 37″	74° 59′ 0″
	105 6 15	74 59 0	105 5 7	74 58 22
	105 7 15	74 58 7	105 5 7	74 58 50
Means..	105 6 29	74 58 35	105 5 37	74 58 44

The angles are almost unaltered and the differences are within the limits of experimental error. Lines were drawn round the sides of the crystal parallel to the edges, and after polishing remained still parallel, which was an additional confirmation. The conclusion at which we arrive is that the polished face was parallel to the natural face.

VI. *Method of Polishing the Crystal.*

The natural face which had been the subject of the experiments recorded in Tables I and II was polished. The polishing was performed by myself to ensure an exact knowledge of the treatment it received.

I am indebted to Professor Threlfall for the use of the apparatus and materials. As previous experimenters seem to have experienced a difficulty in obtaining a surface polished in the same manner, it may be well to state the exact mode of polishing the surface.

That the crystal might be polished under the same conditions of pressure, a rectangular block of lead was cemented to the crystal. In polishing especial care was taken not to press on the crystal downwards, but to exercise only lateral pressure. The crystal was first polished with emery on a plate of glass which had been rendered plane by grinding on a slate. The emery was prepared as follows:

some very fine emery was scattered over a tub of water and allowed to settle; after standing for a number of minutes, the liquid was poured off, and the sediment, which was deposited on standing for a further number of minutes, was preserved for use. Five kinds of emery were used, viz.:—

Stood	1 minute,	deposited in	5 minutes.	
„	5	„	15	„
„	15	„	30	„
„	30	„	$2\frac{1}{2}$ hours.	
„	2 hours	„	40	„

The crystal was polished about twenty minutes with each kind in a perfectly quiet room to avoid dust being deposited on the glass and causing flecks in the crystal.

Next a bed of refined pitch was prepared having its upper surface perfectly plane. Upon this the crystal was polished with rouge for about three hours. Finally, the surface was carefully cleaned by washing it in a stream of water.

VII. *Experiments made with the same Face Polished.*

The screens were carefully set in their places, and by means of these the crystal was fixed in the same position as formerly by the method described towards the end of Section III. Then the observations recorded in Table III were taken consecutively.

Table III.—Observations with a Polished Face.
Mean Temp. 14·85° C.

	r'.	r.	R′.	R.	$(r'-r)$.	(R′−R).
	91·195°	4·612°	153·714°	61·798°	86·583°	91·916°
	91·089	4·402	153·612	61·936	86·687	91·676
	90·866	4·385	153·545	61·954	86·481	91·591
	91·048	4·325	153·781	62·163	86·723	91·618
	91·162	4·194	153·229	62·095	86·968	91·134
	90·871	4·400	153·406	61·969	86·471	91·437
	91·008	4·189	153·376	62·095	86·819	91·281
	90·884	4·166	153·637	62·110	86·718	91·527
	91·020	4·191	153·465	62·244	86·829	91·221
	91·244	4·061	153·620	62·287	87·183	91·333
	91·066	4·118	153·736	62·295	86·948	91·441
	90·907	4·084	153·410	62·200	86·823	91·210
Means..	91·030	4·260	153·544	62·095	86·769	91·449

Table IV.

	ϖ.	tan ϖ.	I.
	1° 24′·9	0·02470	107·756°
	1 25·7	0·02493	107·774
	1 34·2	0·02741	107·749
	1 25·5	0·02488	107·972
	1 24·4	0·02455	107·662
	1 36·8	0·02816	107·687
	1 27·4	0·02543	107·735
	1 27·2	0·02537	107·873
	1 27·8	0·02555	107·855
	1 14·5	0·02168	107·953
	1 20·8	0·02349	108·015
	1 28·1	0·02565	107·805
Means..	1 26·5	0·02515	107·819

Taking from Table III the mean values of $R'-R$, $r'-r$, we have—

$$\cos 2\varpi = \frac{\sin 86·769}{\sin 91·449} = 0·99873.$$

Thus $\varpi = 1° \; 26·6'$, and we obtain for the values of the quantities, which we are seeking in order to determine the nature of the polarised light,

$$\tan \varpi = 0·02520, \qquad I = 107·819°.$$

Comparing these values with those formerly obtained which follow Table II, we see that the effect of polishing is to cause a small alteration of the ratio of the axes and in the inclination of the major axis of the ellipse. Thus the ratio of the axes has been changed from 0·03346 to 0·02520, while the inclination has been changed from 108·088° to 107·819°, an alteration of about 16′. These results also show that the reflected light is exceedingly nearly plane polarised.

Again, the Tables III give for the subsidiary quantity ρ,

$$\cos \rho = \frac{\tan 86·769}{\tan 91·449} = -0·44808,$$

so that

$$\rho = 116° \; 37'.$$

The value of ρ obtained before polishing was—

$$\rho = 111° \; 14'.$$

These values are not the same, but this has no bearing on the polishing, inasmuch as ρ is a constant of the instrument and is independent of the crystal.

In order to test the truth of the results we have arrived at, a Nicol's prism, the azimuth of which could be read off by a divided circle to 3′, was mounted on the side of the table opposite to that on which the elliptic analyser was, in such a position that, when the angle of incidence was the same as before, the light was reflected along the tube containing the Nicol. It was found that the light reflected from the surface could be reduced to a minimum but could never be completely eclipsed. The minimum was very small but quite perceptible. This might be due to the fact that the light was not homogeneous. But further experiments showed that, even with orange and ruby glasses there was a perceptible minimum. Next the incidence was increased and diminished in succession by 5°, and the same result obtained. We, therefore, conclude as before, that the light was slightly elliptically polarised.

VIII. *Discussion of the Determination of the Retardation of the Quarter Wave Plate.*

We have now to inquire into the discrepancy of the values of ρ, which have been found before and after polishing, viz. :—$\rho = 111° 14'$ and $\rho = 116° 37'$.

Now, cos ρ is the ratio of the tangents of two angles, one of which is very nearly 90°. Consequently, a very small error in this last angle will produce a considerable change in the value of cos ρ. We therefore come to this conclusion, that it is not a good plan to determine the instrumental constant ρ by reflexion from Iceland spar, because the light reflected is so slightly elliptically polarised. To determine more accurately the value of ρ, a steel plate was substituted for the crystal. At first, white light was used as before, but the image was found to be strongly coloured. The difference in readings for blue and red rays was several degrees. After trying a spectroscope and coloured glasses, a ruby glass was selected as the best. Then the observations recorded in Table V were made.

Table V.—Observations with a Steel Plate. Mean Temp. 17° C.

r.	r'.	R′.	R.
20·468°	74·705°	89·296°	5·297°
20·515	74·661	89·082	4·885
20·529	74·334	88·917	5·097
20·495	74·117	88·978	5·286
20·692	74·685	89·160	5·305
20·277	74·135	89·034	5·190
20·520	74·282	89·214	5·635
20·527	74·767	89·235	5·400

Table VI.

ϖ.	tan ϖ.	cos ρ.	ρ.	I.
17° 39′·5	0·3184	0·14595	81° 36′	47·296°
17 43·0	0·3195	0·14063	81 55	46·983
17 52·0	0·3223	0·14797	81 30	47·007
17 57·0	0·3239	0·15006	81 22	47·132
17 46·5	0·3206	0·14814	81 29	47·232
17 50·5	0·3218	0·14768	81 30	47·112
17 52·0	0·3224	0·15355	81 10	47·424
17 38·5	0·3181	0·14999	81 22	47·317

From Table V we may find with what accuracy we can work with the elliptic analyser.

The greatest difference between the mean and a single observation is 0·003, so that the error of determination of tan ϖ is less than a percentage. The mean value of I is 47·188°, and the greatest error 0·236° or 14′. The mean value of ρ is 81° 29′, and the greatest error is somewhat less than 26′. Let d be the space retardation measured in air for wave-length λ, then—

$$\rho = 2\pi d/\lambda.$$

Thus the error in the determination of ρ expressed in wave-lengths is—

$$\delta d = \lambda\frac{\delta\rho}{360} = \lambda\frac{26}{21600} = \frac{\lambda}{831}.$$

As in the experiments tan $\varpi = 0·3209$, this is very nearly the case in which the accuracy of the analyser has been determined by Professor Stokes. The present results agree well with his limits of accuracy.

"The mean error of single observations amounted to about $\frac{1}{4}$° in the determination of the azimuth of the principal axis, about three or four thousandths in the ratio of the minor to the major axis, and a little more than a thousandth part of an undulation in the determination of ρ."*

We notice that in the mode of expressing ρ in degrees an error will be made more apparent, for the formula $\rho = 2\pi d/\lambda$ shows that the space retardation is divided by the small quantity λ. Thus, expressing the difference 5° between the two sets of observations in wave-lengths, we find that the error of determination of ρ is only about $1\frac{1}{2}$ per cent. of a wave-length.

* 'Phil. Mag.,' vol. 2, 1851.

The value of ρ which has been determined is for red light, whereas the experiments made on the crystal face were performed with white light. It was not possible to determine ρ accurately for blue rays, as a blue glass absorbed too much light. We cannot, therefore, employ the value of ρ as determined by the steel plate to accurately correct the observations made with the crystal. Nevertheless, let us see what the effect of the substitution will be.

We have
$$\cos 2\varpi = \frac{\sin (r'-r)}{\sin (\mathrm{R}'-\mathrm{R})},$$

and
$$\cos \rho = \frac{\tan (r'-r)}{\tan (\mathrm{R}'-\mathrm{R})}.$$

Since ρ is supposed known, eliminate the smaller quantity $\mathrm{R}'-\mathrm{R}$, and we find

$$\sin 2\varpi = \cos (r'-r) \sin \rho. \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{A.})$$

This formula shows that if there be any change in the ratio of the axes there must be a change in the value of $r'-r$. Looking at the Tables I, III, we see there is such a change.

Using the last written formula and the mean values of $r'-r$ from Tables I, III, we find that for a natural face

$$\tan \varpi = 0{\cdot}03552,$$

and for the polished face

$$\tan \varpi = 0{\cdot}02789.$$

Thus, even when the value of ρ for red light as determined by the steel plate is used, we find that the numerical results indicate that the *character* of the change produced by polishing remains the same.

In the case of the natural face, taking 12 observations of $r'-r$ from Table I, the greatest deviation from the mean is less than 9′, leading by formula A to a deviation from the mean of less than 5 per cent. In the case of the polished face, the greatest deviation will be, according to Table III, about 10 per cent.

The value of ρ has been determined for only one colour, and consequently we cannot calculate its value accurately for light of mean wave-length, so as to be able to compare the value of ρ obtained from the steel plate with the value of ρ obtained from observations with the crystal. Let us, however, examine in a general way what the effect of change of wave-length on the value of ρ is.

Let t be the thickness of the selenite plate, μ_x, μ'_x the ordinary and extraordinary indices for a line x of the spectrum.

Then $$\rho_x = \frac{2\pi}{\lambda_x}\{\mu_x - \mu'_x\}t,$$

and for a line y, $$\rho_y = \frac{2\pi}{\lambda_y}\{\mu_y - \mu'_y\}t.$$

Consequently, $$\frac{\rho_y}{\rho_x} = \frac{\lambda_x}{\lambda_y}\frac{\mu_y - \mu'_y}{\mu_x - \mu'_x}.$$

Now λ decreases from the lines A to E, and varies by considerably over 30 per cent. The variations in the difference $\mu - \mu'$ are very much less if any analogy holds with Iceland spar and quartz, for which the variation is less than 4 per cent.* Both causes tend to increase ρ as the wave-length diminishes. Thus, somewhere about D the retardation is 90°, and will approximate to the value derived from Table I for white light, as we approach E.

This discussion shows us that while the steel plate affords a more accurate means of determining ρ for light of given refrangibility, yet ρ is to a large extent a purely instrumental constant, such that its value has no effect on the *character* of the results, and, when we consider that white light was used in the previous experiments, the value of ρ determined by Tables I, III, would seem to be confirmed by the steel plate observations. We therefore accept the results of Tables I—IV as determining the alteration in the polarisation of the reflected light.

IX. *Statement of Results.*

The results of Tables I—IV are brought together in Table VII for the sake of future reference and comparison, so as to exhibit a synoptical view of the final result up to this stage of the work.

Table VII.

	tan ϖ.	I.	1st Angle.	2nd Angle.
			° ′ ″	° ′ ″
Before polishing ..	0·03345	108·088°	105 6 29	74 58 35
After polishing ...	0·02517	107·819	105 5 37	74 58 44
Difference	−0·00828	−16′ 8″	−52	+9

X. *Variation of Surface State of a Polished Crystal with the Time.*

It is a point of importance to determine if the state of the surface of a *polished* crystal is so permanent that it does not alter with the

* Rudberg, 'Poggendorff, Annalen,' vol. 14, p. 45; Glazebrook, 'Phil. Trans.,' 1879.

time, for otherwise the experiments made would not have very great value unless the time elapsed since polishing were specified. I found that preceding experimenters had made investigations on this point in regard to a few bodies, and had come to the conclusion that the surface did not alter with the time. Thus Seebeck found such a result for some glass experimented upon,* and Sir John Conroy has proved that in the case of metallic surfaces the surface state is a fairly permanent one, not being destroyed by contact with a liquid or a considerable amount of rubbing with a chamois leather.† It therefore naturally occurred to me to examine this question for Iceland spar.

For this purpose a simple analyser consisting of a Nicol and a graduated circle was set up on the side of the table opposite to the elliptic analyser, and placed in such a position that the light could be reduced to a minimum. The observations taken are recorded in Table VIII.

Table VIII.

Date.	Temp.	No. of readings.	Mean reading of 1st vernier.	Mean reading of 2nd vernier.	Mean of verniers.
Dec. 8–11.	9°·5 C.	40	21° 10′·9	201° 20′·5	111° 15′·7
Jan. 20...	9·7	60	21 11·76	201 21·6	111 16·68
	9·8	60	21 11·93	201 20·8	111 16·36

Thus there is in the period of six weeks no time variation of a *polished* surface, for the differences between the means are quite within the limits of experimental error. Also this result is confirmed by the general character of the observations made with the elliptic analyser, which often extended over some weeks, during which no change was noticed.

XI. *Variation of Surface State of a Polished Crystal with Change of Temperature.*

An attempt was made to secure greater accuracy by altering the arrangement of the parts of the elliptic analyser. For this purpose the screw fixing the vertical stem was loosened, and the circular rim rotated through two right angles about the stem as axis. The Nicol was unscrewed from its collar, and the cell containing the quarter wave plate from the disk. The quarter wave plate was now connected with the collar and the Nicol with the disk. In this mode of

* 'Poggendorff, Annalen,' vol. 20.

† 'Roy. Soc. Proc.,' vol. 31, 1881.

arrangement, therefore, the *Nicol* moves in azimuth, carrying the quarter wave plate with it, while the quarter wave plate has likewise an independent motion in azimuth.

After taking 700 readings I became convinced that the accuracy of this mode of using the analyser was less than previously. A subsequent investigation of the theory, made by tracing the surface locus of a point whose coordinates represented the intensity of the transmitted light, and the azimuths of the Nicol and quarter wave plate, seemed to confirm this conclusion.

It is well known that in the case of some bodies the application of heat considerably influences the mode in which they reflect light. Thus, in the case of flint glass, Sir David Brewster produced as great an alteration as 9° in the polarising angle by varying the temperature.* To test the alteration in the case of Iceland spar, a small tray filled with ice was placed on the top of the crystal, all the rest of the apparatus being carefully protected, especially the woodwork. I found that, leaving the quarter wave plate and Nicol untouched, no perceptible effect was produced by lowering the temperature of the crystal 8°, and also that no difference could be detected when observations were taken. I conclude that in the case of Iceland spar for moderate ranges the effect of temperature is insensible.

XII. *New Series of Observations.*

Since the change in the mode of using the elliptic analyser described in the last section had diminished the accuracy of the observations, it was considered best to revert to the original arrangement of the parts of the analyser. The instrument was therefore reversed. The screw which fixed the stem of the analyser was unloosened, and the disk rotated through two right angles about the stem as axis. The tube was set to point in the same direction as before by means of the diaphragms and screens. The quarter wave plate was unscrewed from the collar and the Nicol from the disk. The quarter wave plate was now connected with the disk and the Nicol with the collar. Thus the arrangement of the parts of the analyser was now exactly like that described in Section II.

Since the quarter wave plate has been unscrewed, we must no longer expect the values of I to be the same as in Table IV. There will be a constant difference between the preceding series of values and those which follow, because the arbitrary zero from which I is measured is now changed. For these reasons it is necessary to take a new set of observations to serve as a standard of reference.

Sir John Conroy has attempted to determine the effect of polishing a crystal of Iceland spar by observing with a simple analyser the

* 'Phil. Trans.,' 1815.

light of a lamp reflected from the surface immersed in water.* With a view to a comparison of my own work with his results, a second Argand burner was placed close to the former. The brass table was unclamped and rotated till the face of the crystal was in such a position that the incident light was reflected to the opposite side of the table to that on which the elliptic analyser was placed. The reflected light was observed with a simple analyser consisting of a Nicol, the azimuth of which could be determined by means of a divided circle and verniers to 3′.

The observations made with the elliptic and simple analysers are given in Tables IX and X respectively.

Table IX.—Second Series of Observations with the same Polished Face (Elliptic Analyser).

r'.	r.	R′.	R.	I.
136·891°	43·732°	133·320°	42·164°	87·742°
136·825	43·817	133·591	42·212	87·901
136·779	43·557	133·555	42·200	87·877
136·940	43·311	133·431	42·066	87·748
136·980	43·345	133·149	41·840	87·494
136·410	43·105	133·307	42·095	87·701
137·070	43·845	133·431	41·792	87·611
136·903	43·452	133·137	41·769	87·453
137·131	43·782	133·085	41·657	87·371
136·878	43·476	133·294	41·945	87·619
137·406	43·946	133·277	41·966	87·621
136·566	43·360	133·220	42·025	87·622
136·767	43·503	133·427	41·762	87·595
136·974	43·524	133·591	42·011	87·801

Table X.—Second Series of Observations with the same Polished Face (Simple Analyser).

	Mean of ten readings of 1st vernier.	Mean of ten readings of 2nd vernier.
	356° 5′·0	176° 13′·8
	355 57·4	176 6·1
	355 59·4	176 8·2
	356 2·9	176 12·1
	355 59·4	176 8·6
Mean ..	356 0·82	176 9·76

Mean of 100 = 266° 5·3′.

* 'Roy. Soc. Proc.,' Feb., 1886.

Taking the mean values of r', r, R′, R from Table IX, the elliptic analyser gives

$$\cos 2\varpi = \frac{\sin (r'-r)}{\sin (R'-R)} = \frac{\sin 93{\cdot}340}{\sin 91{\cdot}380} = 0{\cdot}998591,$$

whence $\varpi = 1° \; 31{\cdot}2'$, and we have for the values of the quantities which fully determine the nature of the reflected light

$$\tan \varpi = 0{\cdot}02625, \qquad \text{and} \qquad I = 87{\cdot}654°.$$

The previous experiments, the results of which are recorded in Table VII, give

$$\tan \varpi = 0{\cdot}02517, \qquad \text{and} \qquad I = 107{\cdot}819°.$$

The reason of the difference of the values of I is the change of the index error. In order to compare the values of I found in a subsequent part of the paper with those already obtained, we must therefore subtract 20·165°. The changes in tan ϖ may be ascribed to two causes—error in resetting the instrument so that it was not exactly in its former position, and error of experiment. It should be noted that the difference produced by the reversal of the instrument is not at all comparable with the difference produced by polishing.

In the case of the readings taken with the simple analyser, the mean reading will correspond to the azimuth of the major axis of the ellipse, determined by the elliptic analyser. Thus from Table X

$$I' = 266° \; 5{\cdot}3'.$$

XIII. *Determination of the Effect of Rotation of the Crystal Face in its own Plane.*

It was suggested that the setting of the crystal was such as to allow a rotation of the face in its own plane, which might possibly be the cause of the differences hitherto observed. A wedge of 4° 27′ angle was constructed, and upon this the crystal was placed. So delicate was the mode of setting by screens that, even with the labour of hours, I was unable to turn the crystal into such a position that the reflected ray emerged through the pinholes. I therefore took a thin sheet of paper and gently wore its surface away so as to obtain a fine wedge which was drawn under a corner of the crystal. By this means the reflected ray with some trouble was rendered visible. The observations are divided in the following table into sets of two or three. Each set was taken after the crystal had been taken off the wedge and replaced upon it in position by means of the screens. By this means the accuracy of setting can be judged.

Table XI.—Observations with Polished Face.

r'.	Mean r'.	r.	Mean r.	R′.	Mean R′.	R.	Mean R.
137·129°		43·050°		133·041°		41·621°	
137·404		42·727		133·269		41·319	
137·357		43·080		133·421		41·536	
	137·297°		42·952°		133·244°		41·492°
137·307		43·104		133·331		41·556	
137·075		42·875		133·267		41·332	
137·242		43·062		133·135		41·474	
	137·208		43·014		133·244		41·454
137·031		43·289		133·121		41·617	
137·401		43·097		133·556		41·745	
137·336		43·009		133·455		41·479	
	137·256		43·132		133·377		41·613
137·155		42·890		133·035		41·366	
137·382		42·984		133·000		41·231	
	137·268		42·937		133·017		41·298
137·346		43·040		133·441		41·621	
137·255		43·091		133·299		41·812	
137·425		42·921		133·319		41·452	
	137·342		43·017		133·353		41·628
137·106		43·400		133·485		41·869	
137·047		43·257		133·474		41·910	
136·869		43·147		133·333		42·054	
137·014		43·165		133·525		42·127	
137·089		43·347		133·562		41·712	
137·155		43·124		133·250		41·549	
137·264		43·055		133·381		41·886	
	137·078		43·214		133·430		41·872
137·667		42·809		133·612		42·048	
136·954		42·938		133·489		41·665	
137·020		42·821		133·366		41·834	
136·992		42·977		133·419		41·697	
137·064		43·104		133·228		41·681	
136·730		42·859		133·124		41·704	
137·045		42·731		133·266		41·424	
137·179		42·895		133·216		41·719	
137·006		43·075		133·217		41·636	
	137·073		42·912		133·326		41·712

The mean of the 960 observations of Table XI gives—

$$r' = 137{\cdot}168^\circ, \qquad r = 43{\cdot}031^\circ,$$
$$\mathrm{R}' = 133{\cdot}321^\circ, \qquad \mathrm{R} = 41{\cdot}656^\circ.$$

Thus $$\mathrm{I} = 87{\cdot}488^\circ.$$

But Table IX gives for the mean value of I—

$$\mathrm{I} = 87{\cdot}654^\circ,$$

a difference from the above of 0·166.

Table VII shows us that the effect of polishing is to decrease I from 108·088° to 107·819°, a diminution of 0·269°. Since then a rotation of 4° 27′ produces an increase of 0·166; to produce an effect equal to that of polishing by rotation, the crystal face would have to be rotated in its own plane 8°. No such error of setting was possible.

Again,

$$r'-r = 94{\cdot}137°, \qquad \text{and} \qquad R'-R = 91{\cdot}665°.$$

The last reading is of an altogether different magnitude to what has hitherto occurred.

A template was constructed and used in future observations to test the accuracy of setting. It was found that the setting by the screens was always correct on afterwards being examined by the template, and when it was out of adjustment by the screens the template did not fit. These three grounds seem to be more than sufficient for the rejection of the hypothesis of a rotation of the crystal face in its own plane.

It may be of some interest to calculate the value of the change in the ratio of the axes produced by a rotation of 4° 27′. We therefore calculate it—

Thus $$\cos 2\varpi = \frac{\sin 94{\cdot}137}{\sin 91{\cdot}665},$$

whence $$\varpi = 1°\ 53{\cdot}6',$$

and $$\tan \varpi = 0{\cdot}03305.$$

The value before rotation was

$$\tan \varpi = 0{\cdot}02655.$$

XIV. *Determination of the Effect of Repolishing the same Face of the Crystal. Investigation of the Errors of Means of a Series of Observations with an Elliptic or Simple Analyser. Comparison of the Relative Accuracy of the Elliptic and Simple Analysers.*

Much of the value of the observations on polishing must depend on the fact whether the crystal can be polished in the same manner, that is, polished and repolished so as to obtain the same results. The crystal was, therefore, submitted to the same process of polishing as has been described already in Section VI. The observations with the same face thus repolished are recorded in Table XII.

Table XII.—Observations with a Repolished Face (Elliptic Analyser).

	r.	Mean r′.	r.	Mean r.	R′	Mean R′.	R.	Mean R.	Mean I.
No. 1......	136·586°		43·256°		133·879°		42·035°		
	136·846		43·276		133·265		41·680		
	136·554		43·177		133·652		42·061		
	136·794		43·090		133·478		41·915		
	136·884		43·145		133·372		41·941		
	136·619		43·370		133·578		41·779		
	136·455		43·325		133·480		42·051		
	136·607		43·535		133·476		41·875		
	136·941		43·291		133·269		41·979		
	136·872		43·496		133·525		42·247		
	136·846		43·244		133·559		41·876		
	136·841		43·267		133·485		41·977		
		136·737°		43·289°		133·501°		41·951°	87·726°
No. 2......	137·080		43·196		133·352		41·699		
	136·705		43·305		133·223		42·051		
	136·865		43·511		133·456		42·335		
	136·770		43·227		133·359		41·832		
	136·731		43·064		133·510		41·794		
	136·975		43·257		133·693		41·892		
	136·819		43·126		133·277		41·949		
	136·796		43·176		133·049		41·854		
	136·890		43·290		133·402		41·835		
	136·594		43·338		133·542		42·036		
	136·621		43·577		133·491		42·165		
	136·604		43·670		133·221		41·989		
		136·787		43·311		133·381		41·952	87·666
No. 3......	136·589		43·562		133·311		42·361		
	136·722		43·399		133·780		42·044		
	136·705		42·967		133·637		42·105		
	136·657		43·402		133·457		42·165		
	136·722		43·121		133·575		42·306		
	136·585		43·446		133·536		41·857		
	136·850		43·300		133·259		41·937		
	137·091		43·435		133·326		41·719		
	136·740		43·329		133·485		42·062		
	136·824		43·234		133·410		42·064		
	136·774		43·306		133·437		41·972		
	136·672		43·416		133·327		41·875		
		136·744		43·326		133·461		42·039	87·750
No. 4......	136·772		43·640		133·290		41·894		
	136·929		43·174		133·542		41·936		
	136·732		43·289		133·414		41·937		
	136·645		43·557		133·565		41·900		
Means for 1280 readings.		136·758		43·320		133·449		41·975	87·712

The observations in Table XII made with the elliptic analyser are divided into sets of 12, for our object is not only to determine what is the change produced by repolishing, but also to discover the error of a mean of 12 sets of observations with a view to fixing the accuracy of working with the elliptic analyser, and also the errors of the means of the sets of observations recorded in Tables I—IV. For convenience of comparison the means are exhibited together in Table XIII.

Table XIII.—The Means of Observations with a Repolished Face (Elliptic Analyser).

	$r'-r$.	R′−R.	sin $(r'-r)$.	sin (R′−R).	cos 2ϖ.	ϖ.	tan ϖ.
No. 1..	93·448	91·550	0·998190	0·999634	0·998555	1° 32′·41	0·02689
No. 2..	93·476	91·429	0·998160	0·999689	0·998471	1 35·07	0·02766
No. 3..	93·418	91·422	0·998221	0·999692	0·998529	1 33·25	0·02713
Mean..	93·447	91·467	0·998190	0·999672	0·998518	1 33·57	0·02723

We now proceed to consider the accuracy of the sets of observations with the elliptic analyser. Table XIII shows that the greatest deviation of a single set of 12 observations from the mean is less than 1·6 per cent. for the ratio of the axes. The best set differs from the mean by under $\frac{1}{4}$ per cent. Again, Table XII shows that the greatest deviation from the mean for I is 0·046° or under 3′, whereas the least deviation from the mean is under 1′. We take, then, the upper limits as the errors of observations. Let us now consider the effect produced by repolishing. Before repolishing, the calculation following Table IX shows that—

$$\tan\varpi = 0·02655.$$

After repolishing, Table XIII shows that—

$$\tan\varpi = 0·02723.$$

The difference between the two values of $\tan\varpi$ is $2\frac{1}{2}$ per cent., which may be covered by the limits of errors of observation.

Again, before repolishing—

$$I = 87·654.°$$

After repolishing, Table XII shows that—

$$I = 87·712°.$$

There is thus a difference of 0·058° or 3·48′, which is covered by the limits of errors of observation.

Again, Table VII shows that the effect of the first polishing was to *diminish* I from 108·088° to 107·819°, a *diminution* of 0·269°; but the effect of the second polishing has been to *increase* I from 87·654° to 87·712°, an *increase* of 0·058°. Thus the effect of the polishings has not been cumulative so as to cause the original deviation of polishing to increase still further on second polishing. On the contrary, I has approached by a small quantity the value it would have for a natural face. The inference may be drawn that the first polishing produced *all* the effect that polishing can bring about, and that the second polishing differs from the first by a quantity which there is some ground for regarding as an error of experiment. But if this is so, it may be thought that the values of tan ϖ, the second independent test of a change in the reflected light, should also alter in the same manner and sense as those of I have done.

Now, Table VII shows that the effect of polishing is to *diminish* tan ϖ from 0·03345 to 0·02517, a diminution of 0·00828. Table XIII and the calculation following Table IX show that the effect of re-polishing is to cause tan ϖ to *increase* from 0·02655 to 0·02723, an *increase* of 0·00068. Thus tan ϖ changes sympathetically with I.

In Table XIV are contained the results of the observations made with the simple analyser, and we have first to investigate the limits of accuracy. In regard to single observations there can be no doubt of the correctness of Sir John Conroy's observation that "in order to obtain accurate results with observations of this kind, it is necessary to make a large number of observations, and to take their mean."* Thus, in Table XIV the means of tens of readings are recorded, and for purposes of comparison the means of fifties are tabulated.

Table XIV.—Observations with a Repolished Face (Simple Analyser).

	Means of ten readings of 1st vernier.	Means of 50.	Means of ten readings of 2nd vernier.	Means of 50.	Means of 100.
No. 1......	355° 52·2′		176° 1·2′		
	355 50·3		175 59·1		
	355 57·7		176 5·6		
	355 49·9		175 59·3		
	355 51·6		176 0·7		
		355° 52·3′		176° 1·2′	265° 56·8′
No. 2......	355 59·5		176 6·1		
	355 53·6		176 1·6		
	355 58·3		176 7·8		
	355 54·9		176 4·6		
	355 50·5		175 59·6		
		355 55·3		176 3·9	265 59·6

* 'Roy. Soc. Proc.,' Feb., 1886.

Table XIV—*continued.*

	Means of ten readings of 1st vernier.	Means of 50.	Means of ten readings of 2nd vernier.	Means of 50.	Means of 100.
No. 3......	355° 49·5′		175° 58·6′		
	355 56·8		176 5·6		
	355 46·0		175 54·9		
	355 51·7		176 0·1		
	355 51·5		176 0·6		
		355° 51·1		176° 0·0′	265° 55·5
No. 4......	355 53·8		176 3·3		
	355 51·7		176 0·5		
	355 49·7		175 58·7		
	355 50·6		175 59·4		
	355 46·6		175 55·8		
		355 50·5		175 59·5	265 55·0
No. 5......	355 50·8		176 0·3		
	355 54·9		176 4·4		
	355 51·8		176 0·8		
	356 2·4		176 10·8		
	355 54·5		176 4·0		
		355 54·9		176 4·06	265 59·5
				Mean of 500 =	265 57·28

The results in Table XIV show a greatest error of 2·32′ from the mean for a 100 readings, and of about 4′ for 10. Before polishing the value of I as given by Table X is—

$$I' = 266^\circ\ 5{\cdot}3'.$$

After polishing, Table XIV gives—

$$I' = 265^\circ\ 57{\cdot}28'.$$

There is thus a difference of 8′. This is covered by the larger limit above, but not by the lower. This is not surprising when the errors and fluctuations of single observations are taken into account, and also the number of observations.

The parallelism of the polished and repolished faces was tested by means of the lines drawn round the sides. As these lines gave no perceptible inclination, it did not seem worth while to measure the angles with a spectrometer, since the differences caused by repolishing are so small.

Thus, we come to the conclusion that the repolishing produced little, if any, alteration in the nature of the reflected light, and that this alteration is very small compared with the change produced by

polishing a natural face. Also, this inference is supported by two data as regards the elliptic analyser, and by the results obtained with the simple analyser.

Sir John Conroy* has come to a different conclusion. "It did not seem worth while to make any further experiments with artificial surfaces, as it seemed certain the results would be untrustworthy."

Conroy's experiments were made with the surface immersed in water to secure a greater rotation. I am not sure that the greater rotation would compensate for the loss of light which must ensue when reflexion takes place at the surface of two media of nearly the same density and refractive index.

Another cause of error has been indicated by Conroy. There was some uncertainty whether the face was polished parallel to itself. Though it would not altogether account for the large changes observed by Conroy, yet I found in the case of a crystal, with which I performed a first set of experiments for two months, that a perceptible alteration of the inclination of the face was sufficient to reduce the elliptically polarised light to plane polarised. Thus as regards my own experiments such a change of inclination would have been quite fatal, and I abandoned the crystal.

There is another point which is worthy of attention. An extreme ill-luck seems to have befallen those who have had their crystals polished by others. Thus Seebeck† found that in the case of a glass polished by an artist there was a difference of 28·5′, but that he himself could polish it so that the difference was only 2·9′. I think most likely this may be the probable cause of the large alterations observed by Sir J. Conroy.

XV. *Experiments with a Cleavage Face parallel to the Repolished Face.*

To make the whole series of experiments as conclusive as possible, I now attempted to make a series of experiments with a natural face as at the beginning of the paper. Unfortunately the crystal did not cleave very readily. In splitting the base of the crystal came into two or three pieces, which were fixed on to it again as accurately as possible in their former position. As the sides of the crystal also came off, the inclination of the face to the repolished face could not be determined. There was some doubt as to whether the face of the crystal was in the same absolute position. The surface was not very good, being somewhat broken, and as the reflexion took place from a part of the crystal nearer an edge, it had to be placed so much on one side of the brass table that the template could not be used. Then the observations recorded in Table XV were taken.

* 'Roy. Soc. Proc.,' Feb., 1886.

† 'Poggendorff, Annalen,' vol. 20.

Table XV.—Second Series of Observations with a Natural Face.

	r'.	r.	R′.	R.
	137·020°	42·589°	134·259°	42·682°
	136·935	42·804	133·974	42·539
	137·105	43·015	133·832	42·464
	136·819	43·032	133·994	42·581
	137·046	42·496	134·302	42·510
	136·884	42·744	134·325	42·652
	136·940	42·990	133·781	42·654
	137·216	42·674	134·091	42·556
	136·900	42·971	133·921	42·710
	136·781	42·767	134·317	42·630
	136·961	42·791	134·337	42·636
	137·082	43·030	134·285	42·515
Means..	136·974	42·825	134·122	42·594

Thus we find that $I = 88{\cdot}358°$,

and $\tan \varpi = 0{\cdot}03368.$

We have now to compare these results with those recorded in Table VII, taken some fifteen months previously. Originally we had—

$$\tan \varpi = 0{\cdot}03345,$$

whereas the present value is—

$$\tan \varpi = 0{\cdot}03368.$$

The agreement is very close, and, as might be expected from a comparison of Table VII with the results of Table IX, the latter number is somewhat larger than the former on account of the resetting of the instrument. Again, the first series of observations show that the effect of polishing was according to Table VII to change I from 108·088° to 107·819°, a decrease of 0·269°, whereas the second series of experiments show that the result of repolishing has been according to Tables XII and XV to change I from 88·358° to 87·712°, a decrease of 0·646°. This result is correct as regards sense, but not so satisfactory as regards magnitude. It must, however, be remembered that the breaking up of the base of the crystal would most probably have an influence on the value of I.

We are now in a position to sum up the results of this investigation. The process of polishing a natural face of a crystal of Iceland spar with emery and rouge does most certainly alter the state of the surface. This alteration is evinced by a change both in the ratio of

the axes and in the azimuth of the major axis of the elliptically polarised light, and was observed in the case of two different crystals which were made the subject of experiment.

Since the light reflected from the surface of the crystal is exceedingly nearly plane polarised, the absolute value of the change in the ratio of the axes is small; but the relative change is considerable, for tan ϖ is altered by polishing from 0·0334 to 0·0252. Also the change in the azimuth of the major axis is not very large.

As regards the disturbing causes, it is found that temperature and time do not cause any very perceptible alterations in the surface state of a polished crystal.

The experiments prove a result unnoticed by Seebeck, that an emery-rouge polished surface gives perfectly concordant results on repolishing, and in this respect is quite as satisfactory as the chalk-polished surface recommended by him. This conclusion is supported both by the elliptic and simple analysers. And in general the results of the paper tend to confirm the views of Seebeck rather than those of Sir J. Conroy, for Seebeck in his paper prefers polished surfaces because of the liability of the natural surface to tarnish.

In conclusion, my best thanks are due to Mr. Glazebrook for his advice, and to Professor J. J. Thomson for placing at my disposal a room and apparatus in the Cavendish Laboratory.

"Further Experiments on the Distribution of Micro-organisms in Air (by Hesse's Method)." By PERCY F. FRANKLAND, Ph.D., B.Sc., F.C.S., and T. G. HART, A.R.S.M. Communicated by Prof. FRANKLAND, D.C.L., F.R.S. Received November 22,—Read December 9, 1886.

[PLATE 3.]

In a previous communication entitled "The Distribution of Micro-organisms in Air," a number of experiments have been recorded by one of us on the relative abundance of microbes in the air of various places and of the same place at different times. The numerical determination of the aërial micro-organisms in these experiments was made by means of Hesse's apparatus, the method of using which was there fully described. Since the publication of the above experiments we have been extending our investigations by means of this method, and the results which we have obtained form the subject of the present communication.

In addition to the determination of the number of micro-organisms