ing, but time does not permit its being included in the present communication.

For Examples (5) and (6) the denominator of (10) is imaginary; and the proper modification, from (7) forwards, gives for these and such cases, instead of (14), the following :—

$$u = \frac{\cos\left[t\mu^2 f'(\mu)\right] + \sin\left[t\mu^2 f'(\mu)\right]}{t^{\frac{1}{2}}\left[\mu f''(\mu) + 2f'(\mu)\right]^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

The result is easily written down for each of the two last cases [Examples (5) and (6)].

II. "On the Formation of Coreless Vortices by the Motion of a Solid through an inviscid incompressible Fluid." By Sir W. THOMSON, Knt., LL.D., F.R.S. Received February 1, 1887.

Take the simplest case: let the moving solid be a globe, and let the fluid be of infinite extent in all directions. Let its pressure be of any given value, P, at infinite distances from the globe, and let the globe be kept moving with a given constant velocity, V.

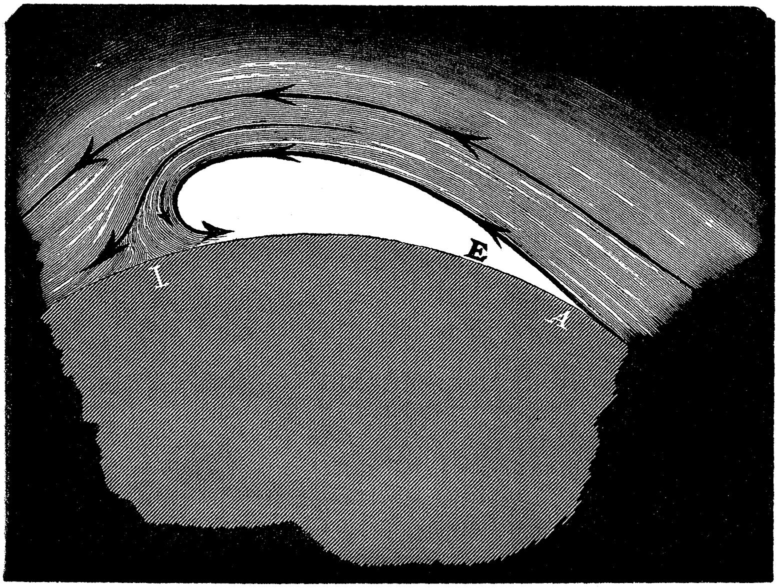
If the fluid keeps everywhere in contact with the globe, its velocity relatively to the globe at the equator (which is the place of greatest relative velocity) is $\frac{3}{2}$V. Hence, unless P $>\frac{5}{8}$V^2,* the fluid will not remain in contact with the globe.

Suppose, in the first place, P to have been $>\frac{5}{8}$V^2, and to be suddenly reduced to some constant value $<\frac{5}{8}$V^2. The fluid will be thrown off the globe at a belt of a certain breadth, and a violently disturbed motion will ensue. To describe it, it will be convenient to speak of velocities and motions *relative to the globe*. The fluid must, as indicated by the arrow-heads in fig. 1, flow partly backwards and partly forwards, at the place, I, where it impinges on the globe, after having shot off at a tangent at A. The back-flow along the belt that had been bared must bring to E some fluid; and the free surface of this fluid must collide with the surface of the fluid leaving the globe at A. It might be supposed that the result of this collision would be a "vortex sheet," which in virtue of its instability, would get drawn out and mixed up indefinitely, and be carried away by the fluid farther and farther from the globe. A definite amount of kinetic energy would be *practically annulled* in a manner which I hope to explain in an early communication to the Royal Society of Edinburgh.

But it is impossible, either in our ideal inviscid incompressible

* The density of the fluid is taken as unity.

Fig. 1.

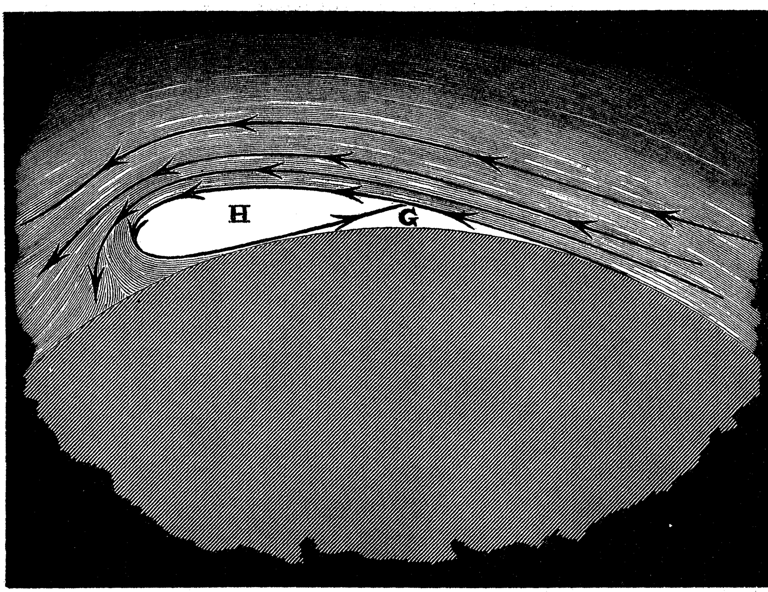


fluid, or in a real fluid such as water or air, to form a vortex sheet, that is to say an interface of finite slip by any natural action. What happens in the case at present under consideration, and in every real and imaginable case of two portions of liquid meeting one another, as for instance a drop of rain falling directly or obliquely on a horizontal surface of still water, is that continuity and the law of continuous fluid motion become established at the instant of first contact between two points, or between two lines in a class of cases of ideal symmetry to which our present subject belongs.

An inevitable result of the separation of the liquid from the solid, whether our supposed globe or any other figure perfectly symmetrical round an axis and moving exactly in the line of the axis, is that two circles of the freed liquid surface come into contact and initiate in an instant the enclosure of two rings of vacuum (G and H in fig. 2, which, however, may be enormously far from like the true configuration).

The "circulation" (line-integral of tangential component velocity round any endless curve encircling the ring, as a ring on a ring, or one of two rings linked together) is determinate for each of these vacuum-rings, and remains constant for ever after: unless it divides

FIG. 2.



itself into two or more, or the two first formed unite into one, against which accidents there is no security.

It is conceivably possible* that a coreless ring vortex, with irrotational circulation round its hollow, shall be left oscillating in the neighbourhood of the equator of the globe; *provided* $(\frac{5}{8}V^2-P)/P$ *be not too great.* If the material of the globe be viscously elastic, the vortex settles to a steady position round the equator, in a shape perfectly symmetrical on the two sides of the equatoreal plane; and the whole motion goes on steadily henceforth for ever.

If $(\frac{5}{8}V^2-P)/P$ exceed a certain limit, I suppose coreless vortices will be successively formed and shed off behind the globe in its motion through the fluid, incessantly.

* If this conceivable possibility be impossible for a globe, it is certainly possible for some classes of prolate figures of revolution.

FIG. 1.

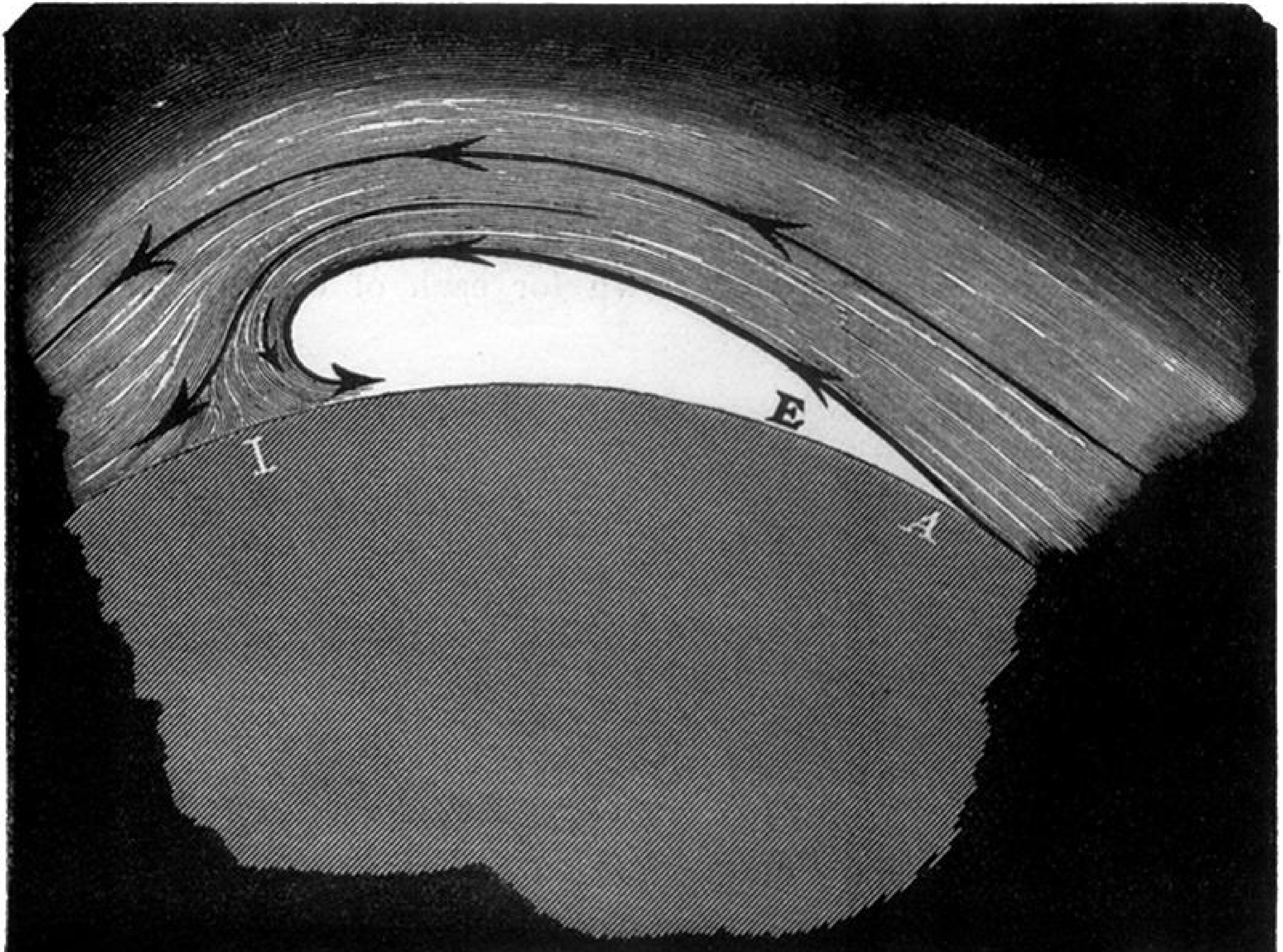


Fig. 2.