that the circuit which is best according to the rules given by these equations is seven times as good as the best previously found.

I have then shown that the mirror must be of such a size as to have a moment of inertia one-third of that of the active bar. In the particular case considered, where the active bar consists of two pieces, one antimony and one bismuth, $5 \times 1 \times \frac{1}{4}$ mm., at a mean distance of 1 mm. apart, the diameter of the mirror should be $2\frac{3}{4}$ mm. This size both theoretically should, and practically does, enable one with certainty to observe a deflection of $\frac{1}{4}$ mm. on a scale 1 metre distant.

General considerations show that the antimony-bismuth bars cannot have too small a sectional area, but that the length when already short is only involved in a secondary manner.

It is shown that the heat in the circuit is equalised mainly by conduction, which is thirty times as effective as the Peltier action.

It is found necessary to screen the antimony and bismuth from the magnetic field by letting them swing in a hole in a piece of soft iron buried in the brass work.

I have shown that the instrument imagined in the preliminary note would be so much more than dead beat that it would not be possible to use it advantageously, but on making a corresponding calculation for the best circuit, now found, using conditions which have been proved by practice to work well together, a difference of temperature of one ten-millionth of a degree centigrade is by no means beyond the power of observation.

The figures given by an actual comparison between the newest instrument and one of the original pattern is very favourable to the former.

In conclusion, I have explained the peculiar action of the rotating pile, and have shown that it is different from that figured in Noad's 'Electricity and Magnetism.'

II. "On Hamilton's Numbers. Part II." By J. J. Sylvester, D.C.L., F.R.S., Savilian Professor of Geometry in the University of Oxford, and James Hammond, M.A., Cantab. Received March 9, 1888.

(Abstract.)

§ 4. *Continuation, to an infinite number of terms, of the Asymptotic Development for Hypothenusal Numbers.*

In the third section of this paper ('Phil. Trans.,' A., vol. 178, p. 311) it was stated, on what is now seen to be insufficient evidence, that the asymptotic development of $p-q$, the half of any hypothenusal

number, could be expressed as a series of powers of $q-r$, the half of its antecedent, in which the indices followed the sequence $2, \frac{3}{2}, 1, \frac{3}{4}, \frac{5}{8}, \frac{1}{2}, \ldots$

It was there shown that, when quantities of an order of magnitude inferior to that of $(q-r)^{\frac{3}{4}}$ are neglected,

$$p-q = (q-r)^2 + \tfrac{4}{3}(q-r)^{\frac{3}{2}} + \tfrac{11}{18}(q-r) + \tfrac{10}{81}(q-r)\ ;$$

but, on attempting to carry this development further, it was found that, though the next term came out $\frac{88}{1215}(q-r)^{\frac{5}{8}}$, there was an infinite series of terms interposed between this one and $(q-r)^{\frac{1}{2}}$.

In the present section it will be proved that between $(q-r)^{\frac{3}{4}}$ and $(q-r)^{\frac{1}{2}}$ there lies an infinite series of terms whose indices are—

$$\tfrac{5}{8},\ \tfrac{9}{16},\ \tfrac{17}{32},\ \tfrac{33}{64},\ \tfrac{65}{128},\ \ldots$$

and whose coefficients form a geometrical series of which the first term is $\frac{88}{1215}$ and the common ratio $\frac{2}{3}$.

We shall assume the law of the indices (which, it may be remarked, is identical with that given in the introduction to this paper as originally printed in the 'Proceedings') and write—

$$\begin{aligned} p-q = {} & (q-r)^2 + \tfrac{4}{3}(q-r)^{\frac{3}{2}} + \tfrac{11}{18}(q-r) + \tfrac{10}{81}(q-r)^{\frac{3}{4}} \\ & + \frac{2^3}{3^3}\mathrm{A}(q-r)^{\frac{5}{8}} + \frac{2^4}{3^4}\mathrm{B}(q-r)^{\frac{9}{16}} + \frac{2^5}{3^5}\mathrm{C}(q-r)^{\frac{17}{32}} \\ & + \frac{2^6}{3^6}\mathrm{D}(q-r)^{\frac{33}{64}} + \frac{2^7}{3^7}\mathrm{E}(q-r)^{\frac{65}{128}} + \text{\&c., } ad\ inf. \\ & + \Theta^* \quad \ldots\ldots\ldots\ldots \quad (1.) \end{aligned}$$

The law of the coefficients will then be established by proving that—

$$\mathrm{A} = \mathrm{B} = \mathrm{C} = \mathrm{D} = \mathrm{E} = \ldots = \tfrac{11}{45}.$$

If there were any terms of an order superior to that of $(q-r)^{\frac{1}{2}}$, whose indices did not obey the assumed law, any such term would make its presence felt in the course of the work; for, in the process we shall employ, the coefficient of each term has to be determined before that of any subsequent term can be found. It was in this way that the existence of terms between $(q-r)^{\frac{5}{8}}$ and $(q-r)^{\frac{1}{2}}$ was made manifest in the unsuccessful attempt to calculate the coefficient of $(q-r)^{\frac{1}{2}}$.

It thus appears that the assumed law of the indices is the true one.

It will be remembered that $p, q, r, \ldots$ are the halves of the

* In the text above Θ represents some unknown function, the asymptotic value of whose ratio to $(q-r)^{\frac{1}{2}}$ is finite.

sharpened Hamiltonian Numbers E_{n+1}, E_n, E_{n-1}, ... and that consequently the relation—

$$E_{n+1} = 1 + \frac{E_n(E_n-1)}{1 \,.\, 2} - \frac{E_{n+1}(E_{n-1}-1)(E_{n-1}-2)}{1 \,.\, 2 \,.\, 3} + \ldots .$$

may be written in the form—

$$p = \tfrac{1}{2} + \frac{q(2q-1)}{2} - \frac{r(2r-1)(2r-2)}{2 \,.\, 3} + \frac{s(2s-1)(2s-2)(2s-3)}{2 \,.\, 3 \,.\, 4}$$
$$- \frac{t(2t-1)(2t-2)(2t-3)(2t-4)}{2 \,.\, 3 \,.\, 4 \,.\, 5}$$
$$+ \frac{u(2u-1)(2u-2)(2u-3)(2u-4)(2u-5)}{2 \,.\, 3 \,.\, 4 \,.\, 5 \,.\, 6}$$
$$- \ldots \ldots \ldots \qquad (2.)$$

The comparison of this value of p with that given by (1) furnishes us with an equation which, after several reductions have been made in which special attention must be paid to the order of the quantities under consideration, ultimately leads to the determination of the values of A, B, C, ... in succession.

III. "Hydraulic Problems on the Cross-sections of Pipes and Channels." By Henry Hennessy, F.R.S., Professor of Applied Mathematics and Mechanism in the Royal College of Science for Ireland. Received March 14, 1888.

In that division of hydromechanics which is devoted to the investigation of the flow of liquids through pipes and open channels, the resistance due to the friction of the contained liquid against the sides of the pipes or channels has led to expressions for the velocity as a function of the dimensions and shape of the cross-section commonly designated as the hydraulic mean depth.

This quantity is defined as the quotient of the area of the cross-section of the liquid by that part of its perimeter in contact with the pipe or channel. In a full pipe this perimeter is identical with that of the pipe's cross-section, and in practice this is generally a circle.

It is also proved from the Calculus of Variations that a circle is the closed curve which, under a given perimeter, has the largest area, and by the same processes of analysis a segment of a circle appears to be that which includes the greatest area between its arc and its chord.

If we call the hydraulic mean depth of a pipe or channel bounded by a curved outline u, its definition gives the condition