material, and the potassium chloride formed exercises its specific influence on this reaction.

The secondary action upon potassium iodide producing iodine is practically an instantaneous one, unless the quantity of this substance is below a certain minimum. Below this the velocity observed in the mixture will be less than normal. The effect of increasing the amount of this substance to much greater than the minimum is closely analogous to that of a similar increase of any neutral salt.

The velocity is an exponential function of the temperature, as was observed in Messrs. Harcourt and Esson's investigations. As the latter increases in arithmetical progression, the former increases in geometrical progression. The rate is about doubled for a rise of 5° C. The ratio in this progression is not, however, absolutely constant, but varies a little with the temperature at which it is taken. Thus between 0° and 15° C. the rate is a little more than doubled for a rise of 5°; between 20° and 30° it is a little less than doubled.

IV. "Determination of the Viscosity of Water." By A. MALLOCK. Communicated by Lord RAYLEIGH, Sec. R.S. Received November 30, 1888.

The experiments here described, which were made during April and May of the present year (1888), to determine the constant of viscosity of water, may be of some interest on account of the newness of the method employed, and also as being on rather a larger scale than other experiments which have been made with the same object.

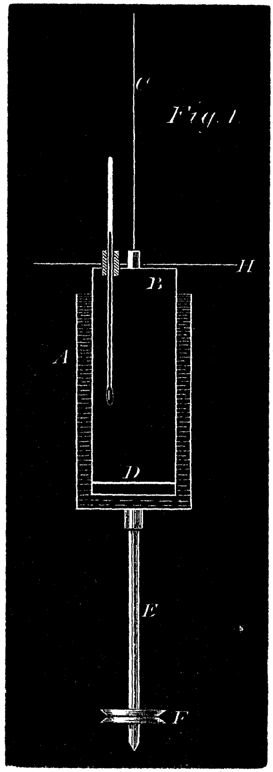
Fig. 1 gives a section of the apparatus used.

A and B are two coaxial cylinders; of these A is mounted on the vertical axis E, and can be made to rotate by a belt passing over the wheel F. B is suspended by a long fine wire C, and the annular space between A and B is filled with water or any other fluid to be experimented on.

A little way above the lower edge of B is fixed an air-tight diaphragm D, so that when the space between the two cylinders is filled with liquid air is inclosed under D, and the liquid touches B only on the cylindrical surface.

The interior of B above D is filled with water which serves the purposes of checking the torsional vibrations of B, of preventing any rapid change of temperature of the liquid in the annulus, and of holding the thermometer.

The experiments were made by driving the cylinder A at a uniform speed and recording the angle through which B is turned when it comes to rest under the action of the fluid friction on its cylindrical surface and the torsion of the suspending wire C. A was driven by



a remontoir weight in connexion with a governor, and the speed recorded electrically on a chronograph by means of a contact maker on the axis E.

The torsion of the wire C was measured on a divided circle H, attached to B.

To get the absolute value of the torsion-scale the following method was used :—

W (fig. 2) is a small weight hung at the end of a silk thread S in the neighbourhood of the torsion-wire C. H is the divided circle on B. From S a second thread, L, is taken to the circumference of H. The point of W is over a horizontal scale, and a reading of its position taken when there is no strain on L, that is, when S is hanging vertically. The weight W is then displaced by unclamping the circle H from B and winding up the thread L round its circumference, then reclamping H and allowing things to come to rest ; readings are then taken of the displacement of W and the position of H. After this the thread L is cut and the position of H read again.

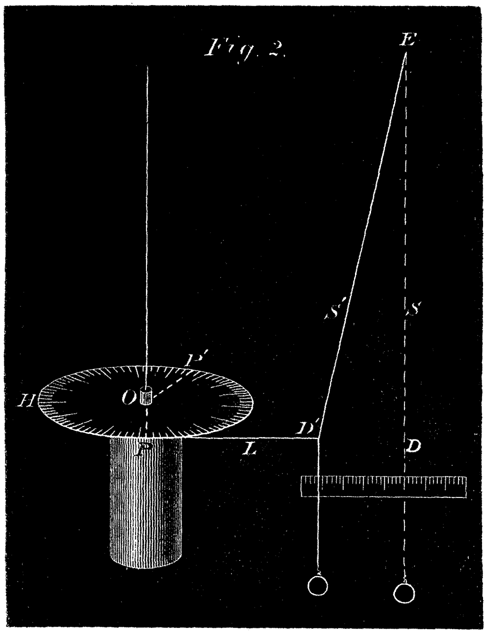


Fig. 2.

These experiments give directly the force which a known angular twist of the wire exerts at the known radius of the divided circle.

These experiments are recorded below, and it will be seen that the results are very fairly consistent.

The dimensions of the various parts of the apparatus are given in the computation of the viscosity constant.

In making the experiments on viscosity the velocity of the circumference of the cylinder A was made to vary from 0·5 to 50 metres per minute.

It was found that at all these speeds the force tending to turn the inner cylinder B could be represented by the sum of two terms, one varying as the velocity and the other as the square of the velocity; the latter being small compared to the former, even at the highest speed. See Diagram 1.

The cause of the square term seems to be that, owing to the action of the bottom of the revolving cylinder, a circulation is set up in the fluid in the annulus, the flow being up the side of the revolving cylinder and down the side of the stationary one, the result being that the fluid having the velocity due to a position near the outer cylinder is by this circulation continuously carried towards the inner one, thus making the variation of velocity in the neighbourhood of the

latter greater than it would otherwise be.* As far as could be observed there was no trace of eddies with axes parallel to that of the cylinders. The proportion between the two terms depends on the ratio between the length of the cylinders and the breadth of the annulus, the square term becoming smaller and smaller compared to the other as the ratio increases.

It was found that when the temperature of the fluid was altered the coefficient of the term varying as the velocity changed, but that the coefficient of the square term remained unaffected.

The value of the viscosity constant deduced from these experiments agrees closely with that obtained from the experiments of Poiseuille on the flow of liquids through capillary tubes.

I now proceed to give the method and the numerical data which were employed in the computation.

Let r_1 = radius of cylinder B . . . . = 4·636
r_2 = " " A . . . . = 5·017
h = depth of immersed surface of B . . . . . = 11·07
v = linear velocity of surface of A,
θ = torsional angle through which B is turned by the action of the water;

$$F = \kappa\,\theta = \kappa\,(Av+Bv^2) = \text{whole tangential force};$$

$$\mu = \text{coefficient of viscosity};$$

the units being the gram, centimetre, and second.

If instead of being in an annulus the water was contained between two parallel planes of infinite extent, the distortion caused by the motion of one of these planes parallel to the other would be uniform throughout the whole mass of enclosed fluid. But in the case of the liquid enclosed between two cylinders, although the distortion is uniform over each cylindrical surface in the fluid coaxial with the enclosing cylinders, yet it changes in passing from one such surface to another, increasing as the radius decreases. In fact, since the total moment transmitted by each surface is constant,† the rate of distortion necessary to produce this moment must be inversely as the area of the surface and radius of the cylinder at which it occurs; that is, the rate of distortion at radius r is proportional to $1/r^2$, hence the value of dv/dr at r is—

* Professor J. Thomson has pointed out that a circulation having a very similar origin must take place in a stream when flowing round a bend.

† [A correction has been introduced here, and in the equations (1), (2), (3). It was originally stated that the force transmitted was constant, but the error was pointed out to me by Lord Rayleigh. In consequence of this error the numerical values of μ subsequently given must be multiplied by 1·08.—January 1, 1889.]

$$\frac{dv}{dr} = \frac{\kappa A v'}{\mu} \frac{r_2}{2\pi r^2 h} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1),$$

where v' is the velocity at r_2.

Integrating between r_1 and r_2 with the conditions that when $r = r$, $v = 0$, and when $r = r_2$, $v = v'$,

$$v' = \frac{\kappa A v'}{\mu} \frac{r_2 - r_1}{2\pi r_1 h} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2),$$

whence

$$\mu = \kappa A \frac{r_2 - r_1}{2\pi r_1 h} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3).$$

The numerical value of A is that of $d\theta/dv$ at the origin of the curve in Diagram 1. The ordinate θ being the circular measure of the angle through which the cylinder B is turned by the viscosity of the water when the cylinder A has the velocity v represented by the abscissa in centimetres per second.

To determine κ the following measures were made:—

In fig. 2 let

$$\begin{aligned} w &= \text{weight of W}, \\ \text{ED} &= b, \\ \text{D'D} &= x, \\ \text{POP}' &= \phi, \\ \text{OP} &= \text{R}. \end{aligned}$$

x is the displacement of w from the vertical caused by the torsion of the wire C through the angle ϕ acting at radius R.

$$\therefore \text{K} \frac{\text{R}}{r_1} \phi = w \frac{x}{b},$$

and

$$\kappa = w \frac{r_1}{\text{R}b} \frac{x}{\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4).$$

The experiments gave the following values for x and ϕ:—

	x c.m.	ϕ°.	log x/ϕ.
Experiment 1	10·51	316·6	$\bar{2}$·51109
,, 2	8·08	245·4	·51759
,, 3	9·2	276·8	·52162
,, 4	9·88	298·0	·52044
,, 5	10·8	324·0	·52287
,, 6	10·62	321·0	·51961
	Mean		·52092

Also

$$\begin{array}{rl} \log w &= 0{\cdot}81151 \\ \log r_1 &= 0{\cdot}53705 \\ \hline & 1{\cdot}34856 \\ \log \mathrm{R}b &= 3{\cdot}25583 \\ \hline & \bar{2}{\cdot}09273 \\ \log \dfrac{x}{\phi} &= \bar{2}{\cdot}52092 \\ \hline & \bar{4}{\cdot}61365 \end{array} \qquad \begin{array}{rl} \log \mathrm{R} &= 1{\cdot}06354 \\ \log b &= 2{\cdot}19229 \\ \hline & 3{\cdot}25583 \end{array}$$

Multiplying by 57·3 to convert to circular measure

$$\begin{array}{rl} \log 57{\cdot}3 &= 1{\cdot}75815 \\ \hline & \bar{2}{\cdot}37180 \end{array}$$

Whence $\kappa = 0{\cdot}02354.$

The diagrams, which were taken at random from many similar ones plotted during the course of the experiments, give A at the temperatures at 4°, 13·8°, and 48° C.

We have

$$\begin{aligned} \mathrm{A}_4 &= 0{\cdot}0582. \\ \mathrm{A}_{13\cdot8} &= 0{\cdot}0458. \\ \mathrm{A}_{48} &= 0{\cdot}023. \end{aligned}$$

Also since

$$\begin{aligned} \kappa &= 0{\cdot}02354, \\ r_2 &= 5{\cdot}017, \\ h &= 11{\cdot}07, \\ r_2 - r_1 &= 0{\cdot}381, \end{aligned}$$

we have

$$\frac{\kappa(r_2 - r_1)}{2\pi r_2 h} = 2{\cdot}606 \times 10^{-5},$$

whence

$$\begin{aligned} \mu_4 &= 15{\cdot}166 \times 10^{-7}, \\ \mu_{13\cdot8} &= 11{\cdot}93 \;\ldots\ldots, \\ \mu_{48} &= 5{\cdot}99 \;\ldots\ldots \end{aligned}$$

The results are shown in the form of a curve in Diagram 2, the ordinates being the values of μ_T and the abscissæ the temperature.

Poiseuille's results are shown by the dotted curve.

The chief interest of these experiments, beyond that attaching to an independent determination of μ by a new method, lies in the comparatively high velocities at which the viscous forces remain the principal cause of resistance.

In all other experiments on fluid friction with which I am acquainted (those on capillary tubes excepted) the term depending on the square of the velocity becomes the most important at speeds far below those used in this series.

Many experiments were made on the viscosity of fluids other than water, but as I find that the results do not differ materially from those of Poiseuille it is unnecessary to give them here.

DIAGRAM 1.

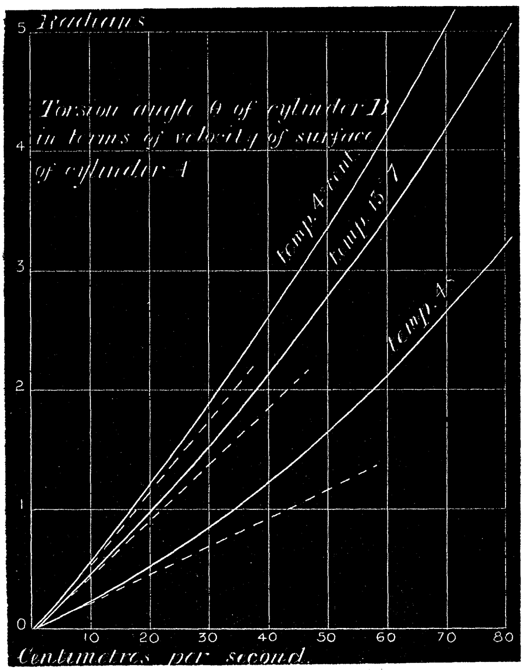
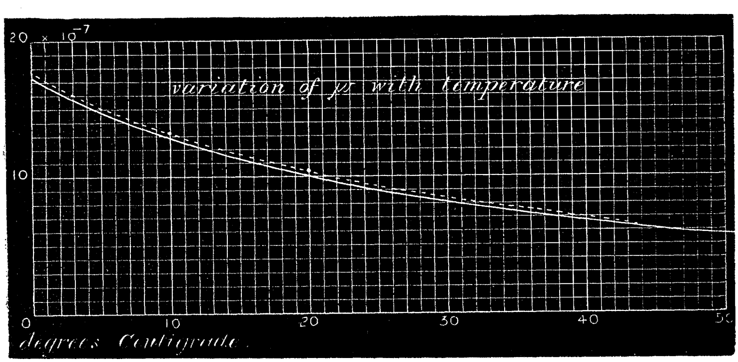


DIAGRAM 2.



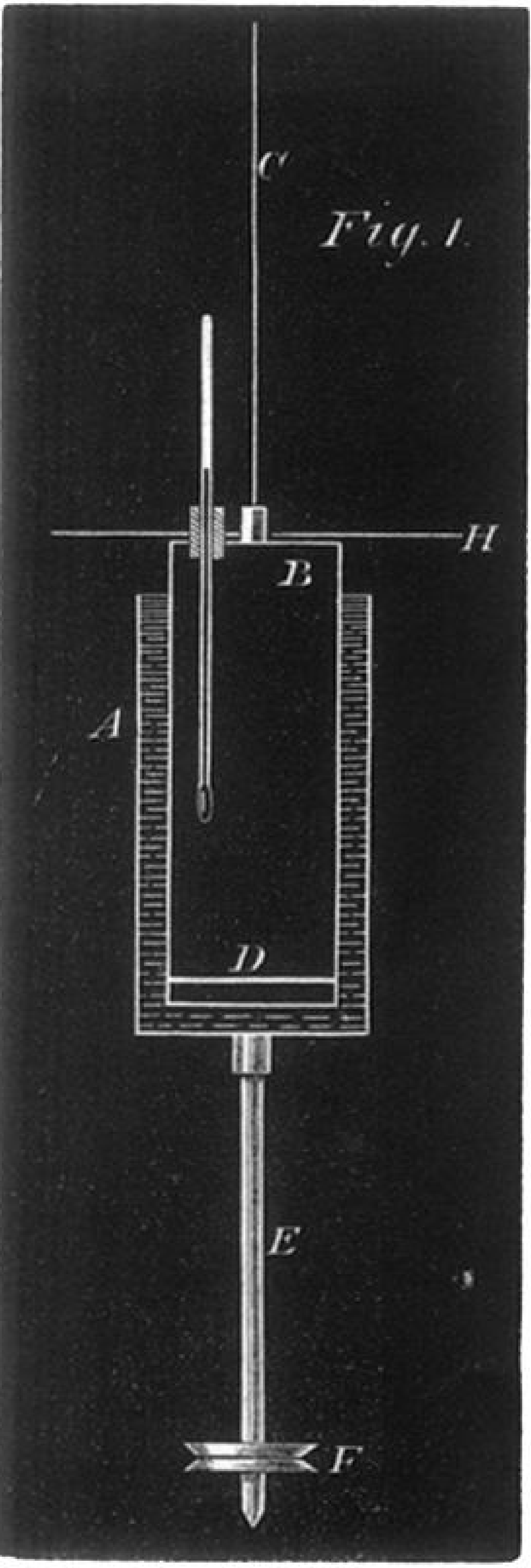


Fig. 1.

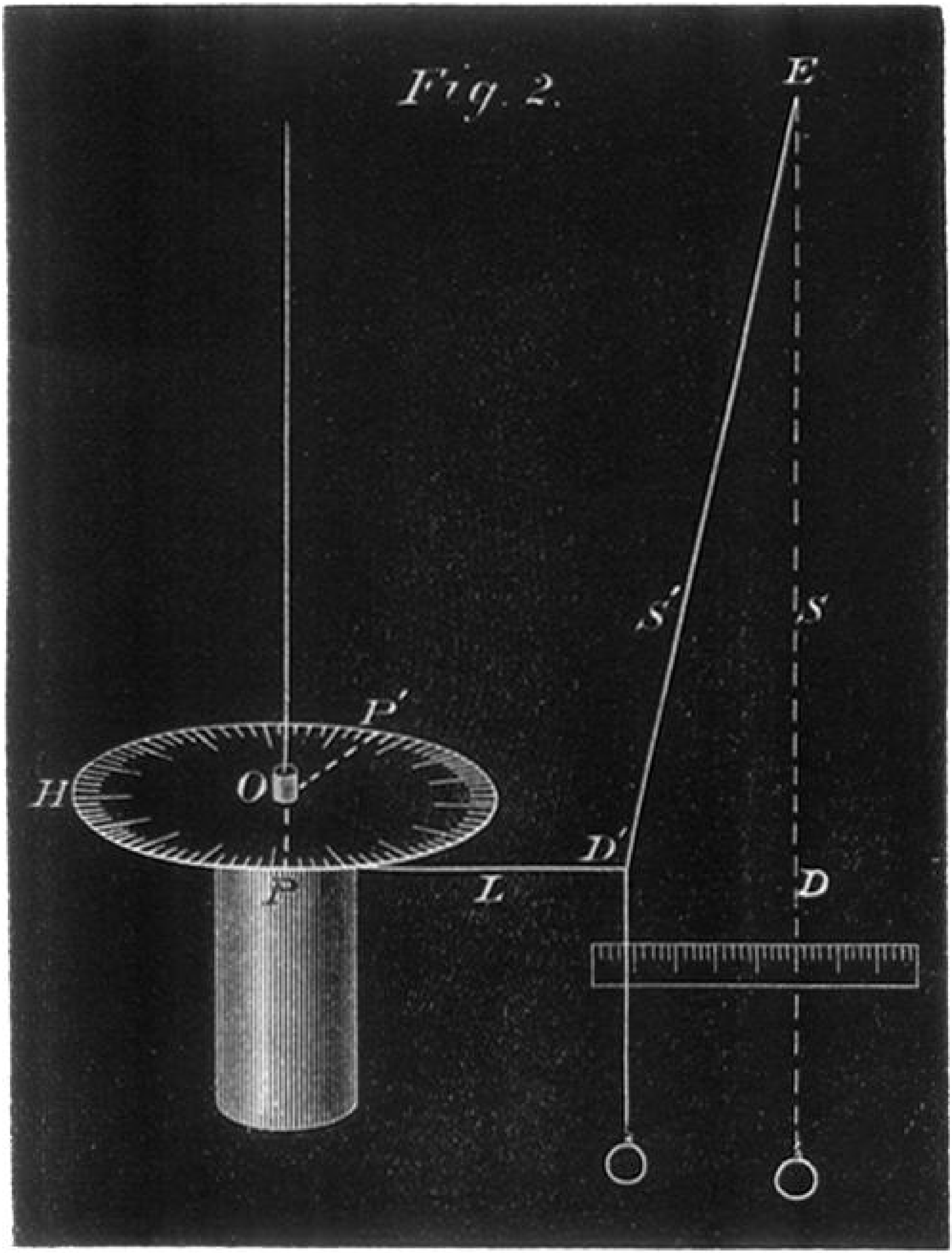
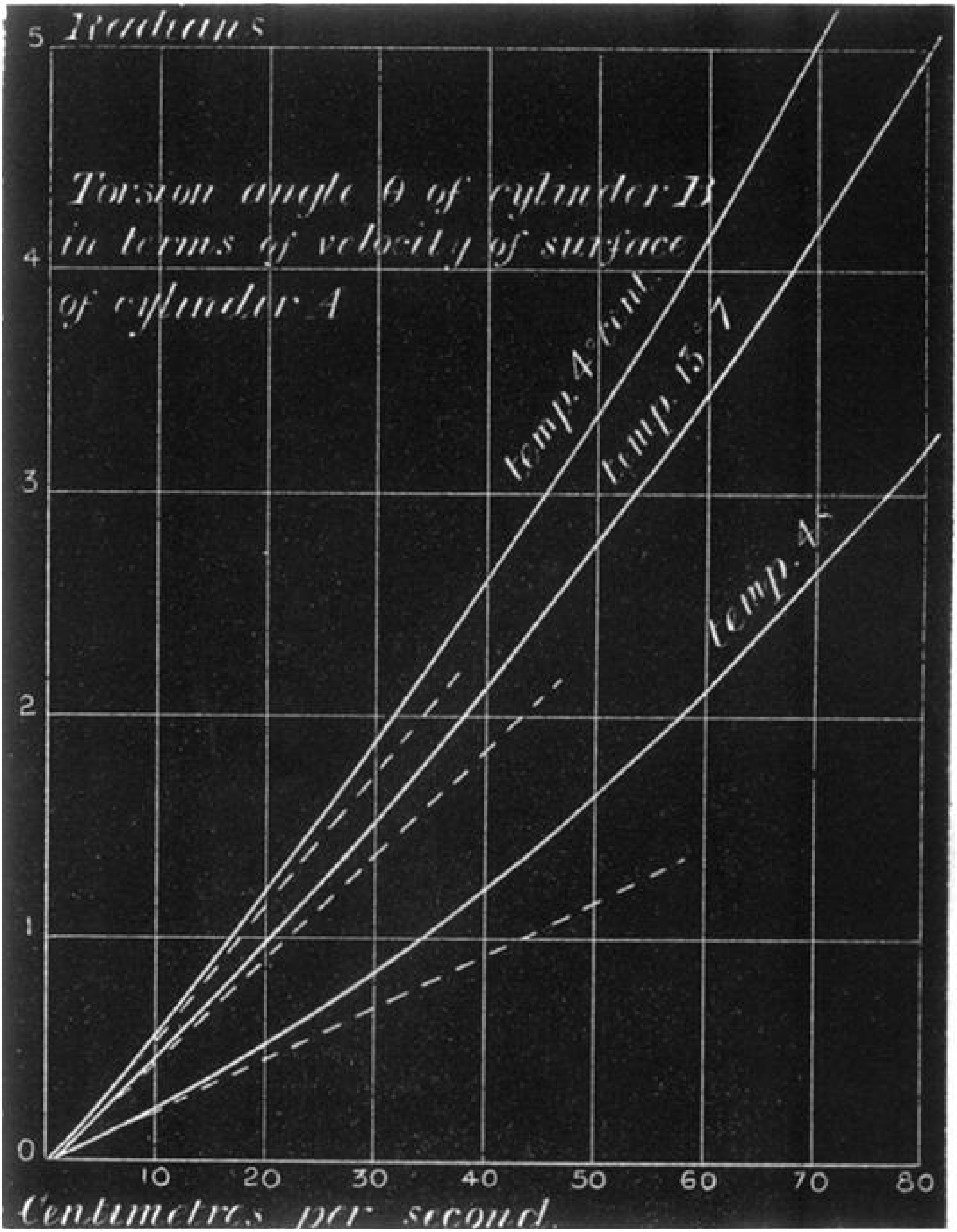


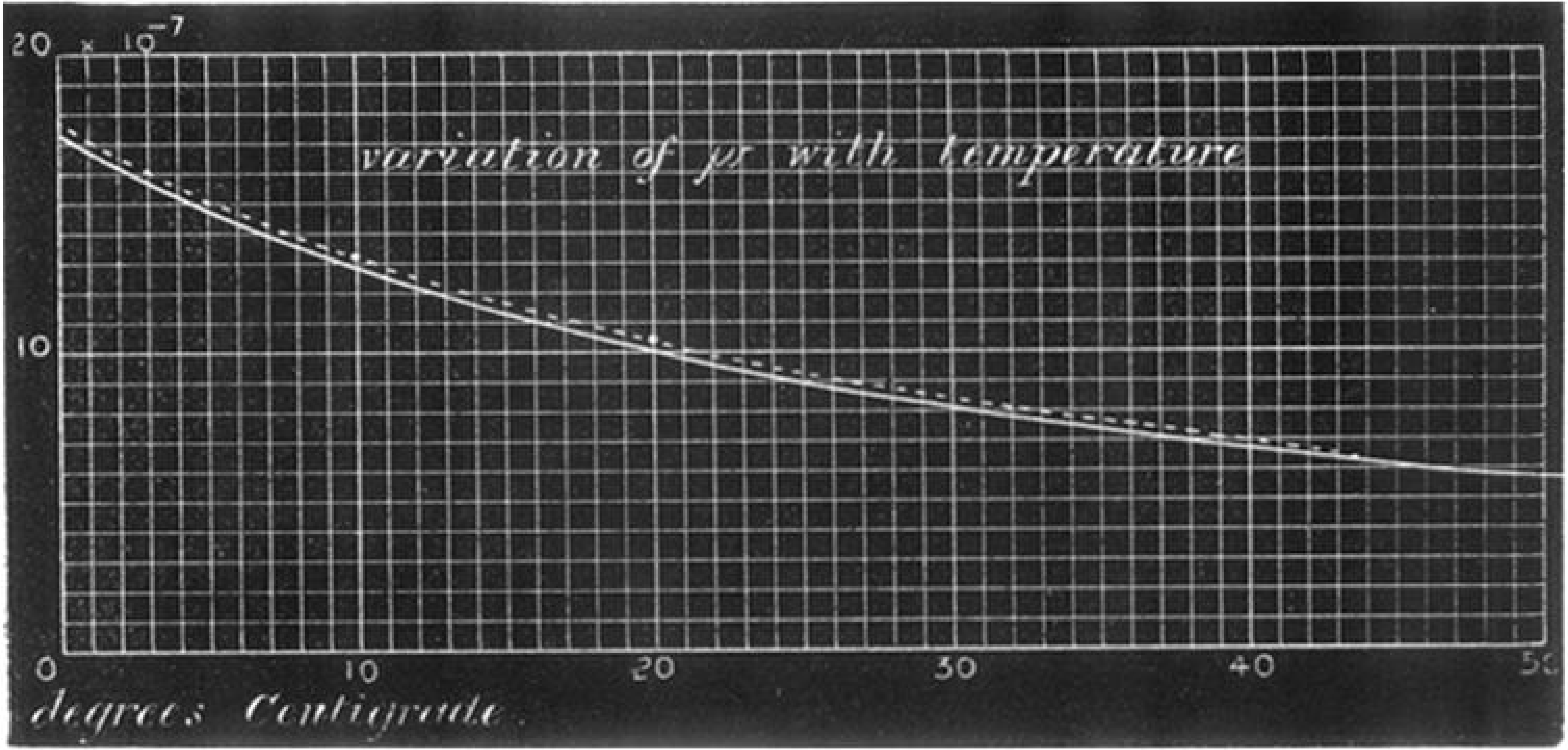
Fig. 2.

DIAGRAM 1.



Radians
5
4
3
2
1
0
Torsion angle θ of cylinder B
in terms of velocity of surface
of cylinder A
temp. 4° cent.
temp. 13°.7
temp. 4°
10
20
30
40
50
60
70
80
Centimetres per second.

DIAGRAM 2.


20 × 10⁻⁷
variation of μ with temperature
10
0
10
20
30
40
50
degrees Centigrade.