laboratory* does not usually render a water surface unfit to exhibit the camphor movements.

The thickness of the oil films here investigated is of course much below the range of the forces of cohesion; and thus the tension of the oily surface may be expected to differ from that due to a complete film, and obtained by addition of the tensions of a water-oil surface and of an oil-air surface. The precise determination of the tension of oily surfaces is not an easy matter. A capillary tube is hardly available, as there would be no security that the degree of contamination within the tube was the same as outside. Better results may be obtained from the rise of liquid between two parallel plates. Two such plates of glass, separated at the corners by thin sheet metal, and pressed together near the centre, dipped into the bath. In one experiment of this kind the height of the water when clean was measured by 62. When a small quantity of oil, about sufficient to stop the camphor motions, was communicated to the surface of the water, it spread also over the surface included between the plates, and the height was depressed to 48. Further additions of oil, even in considerable quantity, only depressed the level to 38.

The effect of a small quantity of oleate of soda is much greater. By this agent the height was depressed to 24, which shows that the tension of a surface of soapy water is much less than the combined tensions of a water-oil and of an oil-air surface. According to Quincke, these latter tensions are respectively 2·1 and 3·8, giving by addition 5·9; that of a water-air surface being 8·3. When soapy water is substituted for clean, the last number certainly falls to less than half its value, and therefore much below 5·9.

V. "On the Stability of a Rotating Spheroid of Perfect Liquid." By G. H. BRYAN. Communicated by Professor G. H. DARWIN, F.R.S. Received March 12, 1890.

1. In my communication on "The Waves on a Rotating Liquid Spheroid of Finite Ellipticity,"† I stated that it did not appear possible to give a complete investigation of the criteria of stability of Maclaurin's spheroid when the liquid forming it is free from all traces of viscosity, and equilibrium is liable to be broken by a disturbance of a perfectly general character. As the problem in question appeared to be one of considerable interest, I have, since writing the above paper, put the question to the test of numerical calculation in the case of the simpler types of disturbance, and the results thus obtained have been such as to allow of extension to a perfectly general disturbance.

* In the country.

† 'Phil. Trans.,' A, 1889, p. 187.

On page 210 of my paper, I showed that, if we consider only displacements determined by the spheroidal sectorial harmonic of the second degree, the limit of eccentricity consistent with stability as obtained from my period-equations agrees with that obtained by Riemann* and Basset.† This, of course, it should do, for the type of displacement considered in both investigations is the same, viz., one in which the deformed surface becomes an ellipsoid, but does not remain one of revolution. We thus have a *necessary* condition for stability. But we do not know that it is a *sufficient* condition. In order that this may be so, it is necessary that the critical form thus obtained shall be stable for all *other* types of displacement. The object of the present paper is to show that such is, in fact, the case. Were it otherwise, the limit of eccentricity consistent with stability would have to be determined afresh. It is needless to remark that we are here exclusively considering what Poincaré calls "ordinary" stability, as distinguished from "secular" stability.

2. The symbols employed in the present paper are the same as in my former communication, and the results there proved will be here assumed. For the sake of convenience, the notation and results required for the present work are collected below, and references to the paper in question will be denoted by the letter [E].

The letters α, ζ are used as defined in [E], § 4, (11), (12), viz., if e be the eccentricity of the spheroid—

$$\alpha = \sin^{-1} e,$$
$$\zeta = \cot\alpha = (1-e^2)^{\frac{1}{2}}/e.$$

so that $e = (1+\zeta^2)^{-\frac{1}{2}}$ and ζ is the reciprocal of the quantity denoted by f in Thomson and Tait's 'Natural Philosophy' (vol. 2, § 771).

The functions $p_n(\zeta)$, $q_n(\zeta)$, $t_n{}^s(\zeta)$, $u_n{}^s(\zeta)$, are defined as in [E, § 5], equations (24) to (27), viz.:—

$$p_n(\zeta) = \frac{1}{2^n . n!}\left(\frac{d}{d\zeta}\right)^n (\zeta^2+1)^n = (-1)^{\frac{1}{2}n}\mathrm{P}_n(\zeta\sqrt{-1}) \quad \ldots\ldots\ldots\ldots \quad (1.)$$

$$t_n{}^s(\zeta) = (\zeta^2+1)^{\frac{1}{2}s}\left(\frac{d}{d\zeta}\right)^s p_n(\zeta) = \frac{(\zeta^2+1)^{\frac{1}{2}s}}{2^n . n!}\left(\frac{d}{d\zeta}\right)^{n+s}(\zeta^2+1)^n \quad \ldots\ldots \quad (2.),$$

$$q_n(\zeta) = p_n(\zeta)\int_\zeta^\infty \frac{d\zeta}{(\zeta^2+1)\{p_n(\zeta)\}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.),$$

$$u_n{}^s(\zeta) = t_n{}^s(\zeta)\int_\zeta^\infty \frac{d\zeta}{(\zeta^2+1)\{t_n{}^s(\zeta)\}^2} = (-1)^s\frac{n-s!}{n+s!}(\zeta^2+1)^{\frac{1}{2}s}\left(\frac{d}{d\zeta}\right)^s q_n(\zeta) \quad \ldots\ldots \quad (4.).$$

* 'Göttingen, Abhandlungen,' vol. 9 (1860), *Mathemat.*, § 9.

† 'Treatise on Hydrodynamics,' vol. 2, p. 124.

The quantities $q_n(\zeta)$ and $u_n^s(\zeta)$ are expressible in a finite form in exactly the same way as the ordinary spherical harmonics of the second kind.* We have, in fact,

$$q_n(\zeta) = (-1)^n\{p_n(\zeta)\cot^{-1}\zeta - \text{R}\} \dots\dots\dots\dots\dots\dots (5.),$$

$$u_n^s(\zeta) = (-1)^{n-s}\frac{n-s\,!}{n+s\,!}\left\{t_n^s(\zeta)\cdot\cot^{-1}\zeta - \frac{\text{R}'}{(\zeta^2+1)^{\frac{1}{2}s}}\right\} \;..\; (6.),$$

where R, R′ are known rational algebraic functions of degree $n-1$ and $n+s-1$ respectively in which *all the coefficients are positive.* For example:—

$$p_1(\zeta) = \zeta, \qquad q_1(\zeta) = -\{\zeta\cot^{-1}\zeta - 1\},$$

$$t_1^1(\zeta) = (\zeta^2+1)^{\frac{1}{2}}, \qquad u_1^1(\zeta) = +\frac{1}{1.2}\left\{(\zeta^2+1)^{\frac{1}{2}}\cot^{-1}\zeta - \frac{\zeta}{(\zeta^2+1)^{\frac{1}{2}}}\right\},$$

$$p_2(\zeta) = \tfrac{1}{2}(3\zeta^2+1), \qquad q_2(\zeta) = \tfrac{1}{2}(3\zeta^2+1)\cot^{-1}\zeta - \tfrac{3}{2}\zeta,$$

$$t_2^1(\zeta) = 3\zeta(\zeta^2+1)^{\frac{1}{2}}, \quad u_2^1(\zeta) = -\frac{1}{2.3}\left\{3\zeta(\zeta^2+1)^{\frac{1}{2}}\cot^{-1}\zeta - \frac{3\zeta^2+2}{(\zeta^2+1)^{\frac{1}{2}}}\right\},$$

$$t_2^2(\zeta) = 3(\zeta^2+1), \qquad u_2^2(\zeta) = +\frac{1}{1.2.3.4}\left\{3(\zeta^2+1)\cot^{-1}\zeta - \frac{3\zeta^3+5\zeta}{\zeta^2+1}\right\},$$

and the corresponding functions of the third, fourth, and fifth degrees can be readily written down from my table in the 'Cambridge Philosophical Proceedings' (*loc. cit.*), by introducing the necessary changes in the signs, and putting "cot $^{-1}$" in place of "coth $^{-1}$."

3. In [E, § 20] I showed that if we consider only displacements of the surface determined by a spheroidal harmonic of degree n and rank s, the condition of secular stability, which, in the present notation, is

$$p_1(\zeta)\cdot q_1(\zeta) - t_n^s(\zeta)\cdot u_n^s(\zeta) > 0 \dots\dots\dots\dots (7.),$$

is a *sufficient*, albeit not a *necessary*, condition for stability when the liquid forming the spheroid is perfect. That the left-hand member of this inequality is essentially positive when $n-s$ is odd has been proved by Poincaré,† and another proof is given below (§ 9).

In [E, § 16] I showed that, in the case of the zonal harmonic displacements of even degree n, the necessary and sufficient condition for ordinary stability is

$$p_1(\zeta)\cdot q_1(\zeta) - p_n(\zeta)\cdot q_n(\zeta) + \frac{4}{n(n+1)}\{t_1^1(\zeta)\cdot u_1^1(\zeta) - p_1(\zeta)\cdot q_1(\zeta)\} > 0 \dots\dots (8.),$$

* 'Cambridge Philosophical Society Proceedings,' 1888, p. 292.

† 'Acta Mathemat.,' vol. 7, p. 326. Write R_i for $t_n^s(\zeta)$ and S_i for $(2n+1)u_n^s(\zeta)$.

and in [E, § 18] that, for a sectorial harmonic displacement, the necessary and sufficient condition is that

$$p_1(\zeta)\cdot q_1(\zeta)-t_n^n(\zeta)\cdot u_n^n(\zeta)+\frac{1}{n}\{t_1^1(\zeta)\cdot u_1^1(\zeta)-p_1(\zeta)\cdot q_1(\zeta)\}>0\,.\;(9.)\,;$$

while [E, § 20] if $n = 2$, the last condition leads to exactly the same results as Riemann's and Basset's investigations (as already mentioned), and gives for the critical form

$$1/\zeta = 3{\cdot}1414567,$$

whence $$\zeta = {\cdot}3183236, \text{ approximately} \ldots\ldots\ldots \quad (10.),$$

and the eccentricity $= \sin 72^\circ\, 20'\, 33'' = {\cdot}9528867$.

4. To prove that the spheroid is "ordinarily" stable until this critical form is reached, we only have to show that conditions (7), (8), or (9) (as the case may be) are satisfied by this value of ζ for every value of n and s. For this purpose I have calculated the numerical values of the products $p_n(\zeta)\,.\,q_n(\zeta)$, and $t_n^s(\zeta)\,.\,u_n^s(\zeta)$ for values of n up to 4, and, in the case of the sectorial harmonics ($s = n$), up to $n = 6$ inclusive, taking $\zeta = {\cdot}3183236$. The results calculated to four places of decimals are as follows, the last figure being only approximate :—

n.	s.	$t_n^s(\zeta)\,.\,u_n^s(\zeta)$.	$p_1(\zeta)\cdot q_1(\zeta)-t_n^s(\zeta)\,.\,u_n^s(\zeta)$.
1	0	$p_1(\zeta)\,.\,q_1(\zeta) = {\cdot}1904$	—
1	1 (sectorial)	·5360	− ·3456
2	0	·2153	− ·0249
2	2 (sectorial)	·3632	− ·1728
3	1	·1566	+ ·0338
3	3 (sectorial)	·2803	− ·0900
4	0	·1116	+ ·0788
4	2	·1266	+ ·0638
4	4 (sectorial)	·2303	− ·0400
5	5 (sectorial)	·1967	− ·0063
6	6 ,,	·1743	+ ·0161

From this Table it appears that the expression

$$p_1(\zeta)\cdot q_1(\zeta)-t_n^s(\zeta)\cdot u_n^s(\zeta)$$

is positive, except when $n = 2$, $s = 0$, and in the case of the first five

sectorial harmonics ($n = s$) in the Table. Thus in every case in which the exact conditions of stability have not been investigated, the *sufficient* condition for secular stability given by the inequality (7) is satisfied. It remains to apply the criteria (8) and (9) to the cases where (7) is not satisfied.

5. First take the case of $n = 2$, $s = 0$.

The fact that $p_1(\zeta) \cdot q_1(\zeta) - p_2(\zeta) \cdot q_2(\zeta)$ is negative in the above Table does not indicate that the spheroid in question is secularly unstable for this particular type of displacement. Its meaning is that the spheroid is more oblate than that form for which the angular velocity is a maximum. As pointed out in Poincaré's memoir,* the disturbed form is here also a spheroid of revolution, and there is no form of "bifurcation" when $p_1(\zeta) \cdot q_1(\zeta) - p_2(\zeta) \cdot q_2(\zeta)$ changes sign.

The condition of "ordinary" stability, from inequality (8) is

$$p_1(\zeta) \cdot q_1(\zeta) - p_2(\zeta) \cdot q_2(\zeta) + \tfrac{2}{3}\{t_1^1(\zeta) \cdot u_1^1(\zeta) - p_1(\zeta) \cdot q_1(\zeta)\} > 0.$$

For the particular value of ζ considered, the left-hand member of this inequality is

$$= -{\cdot}0249 + \tfrac{2}{3}({\cdot}3456) = -{\cdot}0249 + {\cdot}2304 = {\cdot}2055,$$

and is positive; therefore (8) is satisfied.

Even in the extreme case when the spheroid becomes flattened out indefinitely, so that ζ approaches the limit zero, we find

$$p_1(\zeta) \cdot q_1(\zeta) - p_2(\zeta) \cdot q_2(\zeta) + \tfrac{2}{3}\{t_1^1(\zeta) \cdot u_1^1(\zeta) - p_1(\zeta) \cdot q_1(\zeta)\}$$
$$= 0 - \frac{1}{4}\frac{\pi}{2} + \frac{2}{3}\left\{\frac{1}{2}\frac{\pi}{2} - 0\right\} = -\frac{\pi}{8} + \frac{\pi}{6} = \frac{\pi}{24},$$

and is positive. This accords with Sir William Thomson's result that Maclaurin's spheroid is essentially stable, however oblate, if it is supposed constrained to remain spheroidal.

6. Next consider the sectorial harmonics. As the displacement corresponding to $n = s = 1$ is a mere shifting of the mass as a whole, and we are dealing with the critical value of ζ for displacements determined by the harmonic of degree and rank 2, there are only three cases to consider. Now since

$$t_1^1(\zeta) \cdot u_1^1(\zeta) - p_1(\zeta) \cdot q_1(\zeta) = {\cdot}3456,$$

we find

$$p_1(\zeta) \cdot q_1(\zeta) - t_3^3(\zeta) \cdot u_3^3(\zeta) + \tfrac{1}{3}\{t_1^1(\zeta) \cdot u_1^1(\zeta) - p_1(\zeta) \cdot q_1(\zeta)\}$$
$$= -{\cdot}0900 + {\cdot}1152 = +{\cdot}0252;$$

* 'Acta Mathemat.,' vol. 7, p. 329.

$$p_1(\zeta)\,.\,q_1(\zeta)-t_4^4(\zeta)\,.\,u_4^4(\zeta)+\tfrac{1}{4}\{t_1^1(\zeta)\,.\,u_1^1(\zeta)-p_1(\zeta)\,.\,q_1(\zeta)\}$$
$$= -{\cdot}0400+{\cdot}0864 = +{\cdot}0464\,;$$

$$p_1(\zeta)\,.\,q_1(\zeta)-t_5^5(\zeta)\,.\,u_5^5(\zeta)+\tfrac{1}{5}\{t_1^1(\zeta)\,.\,u_1^1(\zeta)-p_1(\zeta)\,.\,q_1(\zeta)\}$$
$$= -{\cdot}0063+{\cdot}0691 = +{\cdot}0628.$$

The values of these expressions are all positive; therefore condition (9) is satisfied in each case, and the spheroid is "ordinarily" stable for the corresponding types of displacement.

It is therefore stable for *all* types of displacement considered in the foregoing table, except that for which it is, by hypothesis, "critical."

7. On examining the values of $t_n^s(\zeta)\,.\,u_n^s(\zeta)$ given in the Table, it appears probable that as we proceed to harmonics of higher degrees this product diminishes in value, and that condition (7) is satisfied universally in all the cases not considered above. That such is actually the case we now proceed to demonstrate. The results are a slight extension of those obtained in § 10 of Poincaré's paper, the method here employed being very similar.

8. Consider the expression—

$$t_m^r(\zeta_0)\,.\,u_m^r(\zeta_0)-t_n^s(\zeta_0)\,.\,u_n^s(\zeta_0),$$

and let us examine under what circumstances it is essentially positive. From formula (4) we have

$$t_m^r(\zeta_0)\,.\,u_m^r(\zeta_0)-t_n^s(\zeta_0)\,.\,u_n^s(\zeta_0)$$
$$= \{t_m^r(\zeta_0)\}^2\int_{\zeta_0}^{\infty}\frac{d\zeta}{(\zeta^2+1)\{t_m^r(\zeta)\}^2}-\{t_n^s(\zeta_0)\}^2\int_{\zeta_0}^{\infty}\frac{d\zeta}{(\zeta^2+1)\{t_n^s(\zeta)\}^2}$$
$$= \int_{\zeta_0}^{\infty}\frac{d\zeta}{\zeta^2+1}\left\{\left(\frac{t_m^r(\zeta_0)}{t_m^r(\zeta)}\right)^2-\left(\frac{t_n^s(\zeta_0)}{t_n^s(\zeta)}\right)^2\right\}.$$

This will be essentially positive if the quantity to be integrated is always positive, that is, if for all values of ζ lying between ζ_0 and ∞,

$$\left(\frac{t_m^r(\zeta_0)}{t_m^r(\zeta)}\right)^2-\left(\frac{t_n^s(\zeta_0)}{t_n^s(\zeta)}\right)^2 > 0,$$

or

$$\frac{t_n^s(\zeta)}{t_m^r(\zeta)} > \frac{t_n^s(\zeta_0)}{t_m^r(\zeta_0)},$$

where $\zeta > \zeta_0$,

which will be the case if $t_n^s(\zeta)/t_m^r(\zeta)$ increases with ζ.

9. The result proved by Poincaré, and assumed in the preceding investigations, namely, that if $n-s$ be *odd*—

$$p_1(\zeta) \cdot q_1(\zeta) - t_n^s(\zeta) \cdot u_n^s(\zeta)$$

is essentially positive, follows at once. For, as just proved, this will be the case if $t_n^s(\zeta)/p_1(\zeta)$, that is, $t_n^s(\zeta)/\zeta$, increases with ζ. Now, from formula (2) it is evident that, $n-s$ being odd, $t_n^s(\zeta)$ is divisible by ζ, and the quotient will be $(\zeta^2+1)^{\frac{1}{2}s} \times$ a rational algebraic function of ζ^2 in which all the terms are positive. This quotient evidently increases with ζ, which proves the result.

10. Let us now revert to the original question, but suppose in addition that both $m-r$ and $n-s$ are even. We have just shown that

$$t_m^r(\zeta_0) \cdot u_m^r(\zeta_0) > t_n^s(\zeta_0) \cdot u_n^s(\zeta_0),$$

provided that $t_n^s(\zeta)/t_m^r(\zeta)$ increases with ζ.

This will be the case if

$$\frac{d}{d\zeta} \frac{t_n^s(\zeta)}{t_m^r(\zeta)} > 0,$$

or

$$t_m^r(\zeta) \frac{dt_n^s(\zeta)}{d\zeta} - t_n^s(\zeta) \frac{dt_m^r(\zeta)}{d\zeta} > 0;$$

or multiplying by ζ^2+1,

$$(\zeta^2+1) t_m^r(\zeta) \frac{dt_n^s(\zeta)}{d\zeta} - (\zeta^2+1) t_n^s(\zeta) \frac{dt_m^r(\zeta)}{d\zeta} > 0.$$

Since $m-r$ and $n-s$ are both even, it readily appears that $dt_n^s(\zeta)/d\zeta$ and $dt_m^r(\zeta)/d\zeta$ vanish when $\zeta = 0$.

Hence the left-hand side of the last inequality will be positive when $\zeta > 0$ if it increases with ζ; this condition gives on again differentiating—

$$t_m^r(\zeta) \frac{d}{d\zeta} \left\{ (\zeta^2+1) \frac{dt_n^s(\zeta)}{d\zeta} \right\} - t_n^s(\zeta) \frac{d}{d\zeta} \left\{ (\zeta^2+1) \frac{dt_m^r(\zeta)}{d\zeta} \right\} > 0.$$

Now $t_m^r(\zeta)$ and $t_n^s(\zeta)$ satisfy the differential equations

$$\frac{d}{d\zeta} \left\{ (\zeta^2+1) \frac{dt_n^s(\zeta)}{d\zeta} \right\} = \left\{ n(n+1) - \frac{s^2}{\zeta^2+1} \right\} t_n^s(\zeta),$$

$$\frac{d}{d\zeta} \left\{ (\zeta^2+1) \frac{dt_m^r(\zeta)}{d\zeta} \right\} = \left\{ m(m+1) - \frac{r^2}{\zeta^2+1} \right\} t_m^r(\zeta);$$

therefore we get $\left\{ n(n+1) - m(m+1) - \frac{s^2-r^2}{\zeta^2+1} \right\} t_m^r(\zeta) . t_n^s(\zeta) > 0$, or, since $t_n^s(\zeta)$ and $t_m^r(\zeta)$ are essentially positive,

$$(n-m)(n+m+1)-\frac{s^2-r^2}{\zeta^2+1} > 0 \quad \ldots\ldots\ldots\ldots (11.).$$

Writing ζ for ζ_0, we see that inequality (11) is a sufficient condition that the expression—

$$t_m^r(\zeta) \cdot u_m^r(\zeta) - t_n^s(\zeta) \cdot u_n^s(\zeta)$$

may be positive in the case of $m-r$ and $n-s$ both even, provided that (11) is satisfied for all values of ζ between 0 and ∞.

11. We have to consider two cases:—

I. Suppose $n = m$. Then condition (11) will be satisfied, for all values of ζ, provided that $r > s$. Therefore $t_n^r(\zeta) \cdot u_n^r(\zeta)$ is always $> t_n^s(\zeta) \cdot u_n^s(\zeta)$ whatever be the value of ζ, provided that $r > s$. In other words, for given values of n, ζ, the product $t_n^s(\zeta) \cdot u_n^s(\zeta)$ increases as s increases, and is greatest when $s = n$ (corresponding to the sectorial harmonics).

II. Suppose $n-s = m-r$ and, therefore, $n-m = s-r$. Condition (11) may be written—

$$(n-m)\left\{n+m+1-\frac{s+r}{\zeta^2+1}\right\} > 0.$$

The first factor is positive provided that $n > m$. The second is necessarily positive, for r, s are not greater respectively than m, n; therefore $n+m+1 > s+r$, and therefore, *a fortiori*,

$$n+m+1 > \frac{s+r}{\zeta^2+1}$$

for all values of ζ. Hence, putting $s = n-2k$, and therefore $r = m-2k$, we have—

$$t_m^{m-2k}(\zeta) \cdot u_m^{m-2k}(\zeta) > t_n^{n-2k}(\zeta) \cdot u_n^{n-2k}(\zeta),$$

provided that m is $<n$. Therefore the product $t_n^{n-2k}(\zeta) \cdot u_n^{n-2k}(\zeta)$ decreases for any given value whatever of ζ as n increases. In particular, $t_n^n(\zeta) \cdot u_n^n(\zeta)$ decreases as the number n is increased.

12. Let us now apply these results to the spheroid under consideration. From the results of Case II,

$$t_5^3(\zeta) \cdot u_5^3(\zeta) < t_4^2(\zeta) \cdot u_4^2(\zeta),$$

$$t_5^1(\zeta) \cdot u_5^1(\zeta) < t_4^0(\zeta) \cdot u_4^0(\zeta) \; (i.e., \, p_4(\zeta) \cdot q_4(\zeta));$$

and from the Table, $t_4^2(\zeta) \cdot u_4^2(\zeta)$ and $p_4(\zeta) \cdot q_4(\zeta)$ are each less than $p_1(\zeta) \cdot q_1(\zeta)$ for this particular value of ζ. Therefore, *a fortiori*,

$$\left.\begin{array}{l} p_1(\zeta) \cdot q_1(\zeta) - t_5^3(\zeta) \cdot u_5^3(\zeta) > 0 \\ \text{and} \quad p_1(\zeta) \cdot q_1(\zeta) - t_5^1(\zeta) \cdot u_5^1(\zeta) > 0 \end{array}\right\} \text{where } \zeta = \cdot 3183\ldots,$$

and the spheroid is stable for harmonic displacements of the degree 5.

From the results of Case II we also have, if n be greater than 6,

$$t_n^n(\zeta) \,.\, u_n^n(\zeta) \;<\; t_6^6(\zeta) \,.\, u_6^6(\zeta),$$

and from the Table

$$t_6^6(\zeta) \,.\, u_6^6(\zeta) \;<\; p_1(\zeta) \,.\, q_1(\zeta)\,;$$

therefore, *a fortiori*,

$$t_n^n(\zeta) \,.\, u_n^n(\zeta) < p_1(\zeta) \,.\, q_1(\zeta), \text{ if } n > 6.$$

Moreover, by Case I,

$$t_n^s(\zeta) \,.\, u_n^s(\zeta) \;<\; t_n^n(\zeta) \,.\, u_n^n(\zeta)\,;$$

therefore, *a fortiori*,

$$t_n^s(\zeta) \,.\, u_n^s(\zeta) \;<\; p_1(\zeta) \,.\, q_1(\zeta),$$

or
$$p_1(\zeta) \,.\, q_1(\zeta) - t_n^s(\zeta) \,.\, u_n^s(\zeta) \;>\; 0,$$

where $\zeta = \cdot 3183\ldots$, and n is equal to or greater than 6.

Thus the sufficient condition of secular stability is satisfied for all types of displacement, with the exceptions already considered in which the "ordinary" conditions of stability have been proved to hold good. Hence the results of the present paper prove conclusively that *Maclaurin's spheroid, if formed of perfectly inviscid liquid, will be absolutely stable if its eccentricity be less than* 0·9528867. *If the eccentricity exceed this limit, the spheroidal form will become unstable, and the liquid will assume the form of an ellipsoid.*

13. The state of steady motion which then ensues is intermediate between the forms known as Jacobi's and Dedekind's ellipsoids. The "spin" of the liquid will be everywhere constant and equal, say, to ω, and the form of the liquid free surface will be an ellipsoid, whose principal axes rotate about the least axis with angular velocity $\frac{1}{2}\omega$. That this is initially the case is in accordance with the results of [E, §§ 14, 18], supposing that the roots of the period-equation become complex, for their *real* part will indicate that the disturbance travels round with angular velocity $\frac{1}{2}\omega$. It is unnecessary to discuss this point at greater length here.

It is also to be noted that the results of the present paper quite preclude the possibility, under ordinary circumstances, of Maclaurin's spheroid ever passing into the form of one or more rings of rotating liquid. This might probably take place if we imagined the liquid surface constrained to remain a figure of revolution. But such hypothetical circumstances are devoid of interest, and, since it appears from the results of the present analysis that, when we consider displace-

ments determined by harmonics of any even degree (n), the "coefficient of stability" for the displacement symmetrical about the axis is the *last* to change sign, it is clear that hardly any less general constraint would suffice to produce such a result.

VI. "A Determination of "v," the Ratio of the Electromagnetic Unit of Electricity to the Electrostatic Unit." By J. J. Thomson, M.A., F.R.S., Cavendish Professor of Experimental Physics, Cambridge, and G. F. C. Searle, B.A., Peterhouse, Demonstrator in the Cavendish Laboratory, Cambridge. Received March 12, 1890.

(Abstract.)

The experiments made by one of us in 1883 having given a value for "v" considerably smaller than those found in several recent researches on this subject, it was thought desirable to repeat the experiments. The method used in 1883 was to find both the electrostatic and the electromagnetic measures of the capacity of a condenser, the electrostatic measure being calculated from the dimensions of the condenser, and the electromagnetic measure by determining a resistance which would produce the same effect as that produced by repeated charging of the condenser when placed in one arm of a Wheatstone's bridge. In the experiments in 1883 the condenser used in determining the electromagnetic measure was not the same as that for which the electrostatic capacity had been calculated, but one without a guard ring, the equality of the capacity of this condenser and the guard ring condenser being tested by the method given in Maxwell's 'Electricity and Magnetism,' vol. 1, p. 324.

In repeating the experiments we adopted at first the same method as before, using, however, a key of different design for testing the equality of the condensers by Maxwell's method. We got very consistent results, practically identical with those obtained in 1883. We may mention here, since it has been suggested that the capacity of the leads might explain the low value of "v" obtained previously, that the leads are allowed for by the way the comparison between the two condensers is made, for the same leads are used in the determination of the electromagnetic measure of the capacity of the auxiliary condenser and in the comparison of the capacity of this condenser with the one with the guard ring, and the capacity of the auxiliary condenser is adjusted until its capacity, plus that of the leads, equals the capacity of the guard ring condenser; and in the electromagnetic measurements it is the capacity of the auxiliary condenser, plus that of its leads, which is found.