May 9, 1889.

Professor G. G. STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The Right Hon. Baron Henry de Worms was admitted into the Society.

The following Papers were read :—

I. "On the Magnetic Rotation of the Plane of Polarisation of Light in doubly refracting Bodies." By A. W. WARD. Communicated by Professor J. J. THOMSON, F.R.S. Received April 13, 1889.

In repeating Villari's* experiment on the rotation of the plane of polarisation of light in a spinning disk of heavy glass, placed with its axis of rotation perpendicular to the lines of force in a magnetic field, it was observed that the incident plane polarised light became elliptically polarised. The elliptic polarisation was due to the centrifugal force which had the effect of stretching the glass along the radii of the disk and compressing it parallel to the axis of rotation. The strained glass in the magnetic field has, therefore, the double property of elliptically polarising plane polarised light, and at the same time rotating the plane of polarisation. The strained glass therefore acted like a crystal placed in a magnetic field, and so before Villari's experiment could be properly interpreted, it was necessary to examine how the elliptic polarisation and magnetic rotation affect each other. The following investigation is an attempt to solve this question, and its conclusions show that the apparent magnetic rotation in a doubly refractive medium is a periodic function of the length of the path of light in the medium. This result entirely accounts for the effects observed by Villari, and those observed by Lüdtge in a piece of compressed glass.

Let the axes of the doubly refracting diamagnetic medium be taken as those of x and y, and let the axis of z be the direction in which the light travels.

Let α be the inclination of the plane of vibration of the incident light to that of xz. The equation of the incident light may be written

* Villari, 'Rendiconti del Istituto Lombardo,' 9 June, 1870.

$$\left.\begin{aligned} x &= c\cos\alpha\cos(2\pi/\lambda)(vt-z) \\ y &= c\sin\alpha\cos(2\pi/\lambda)(vt-z) \end{aligned}\right\},$$

where c^2 is the intensity of the light, and the other symbols have their usual meanings.

Let β be the angular retardation in passing through the crystal. Then the equation of the emergent elliptically polarised light is

$$\left.\begin{aligned} x &= c\cos\alpha\cos(2\pi/\lambda)(vt-z) \\ y &= c\sin\alpha\cos(2\pi/\lambda)(vt-z+\lambda\beta/2\pi) \end{aligned}\right\}.$$

The inclination ω of the axis of this ellipse to the axis of x is given by

$$\tan 2\omega = \tan 2\alpha \cos\beta.$$

In this equation β is a function of λ and z, viz., $(2\pi z/\lambda)(\mu_1-\mu_2)$ where μ_1 and μ_2 are the refractive indices along the axes of x and y respectively. We may, therefore, put β equal to kz where k is a constant for the same medium and wave-length. Hence ω is a function of z, and we can find the increase in ω due to an increase dz in z. It is given by

$$2d\omega = -\cos^2 2\omega \tan 2\alpha \sin\beta \,.\, d\beta,$$

or

$$d\omega = -\tfrac{1}{4}k \sin 4\omega \tan kz \,.\, dz.$$

This equation gives us the rotation of the plane of polarisation due to the doubly refracting nature of the medium, while the light passes through a thickness dz. Let us suppose the effect of the magnetic rotation on the light traversing the element dz, may be represented by an additional rotation of these axes

$$d\omega = m\, dz,$$

where m is a constant depending on the nature of the medium and the strength of the magnetic field. Hence, when both these small effects are superposed we get

$$d\omega = m\, dz - \tfrac{1}{4}k \sin 4\omega \tan kz\; dz.$$

Let us denote by ω_1 the value of ω when m is positive, and ω_2 its value when m is negative. Then the apparent magnetic rotation is $\omega_1-\omega_2$, Ω say.

We have

$$d\Omega = 2m\, dz - \tfrac{1}{2}k \,.\, \sin 2\Omega \cos 2(\omega_1+\omega_2) \tan kz\; dz.$$

This equation is easily integrated when Ω is small. In that case

we may write 2Ω for $\sin 2\Omega$, and $\cos 4\omega_0$ for $\cos 2(\omega_1+\omega_2)$, where ω_0 is the value of ω when $m=0$. Since

$$\tan 2\omega_0 = \tan 2\alpha \cos kz,$$

we have, putting a equal to $\tan 2\alpha$,

$$\cos 4\omega_0 = \frac{1-a^2\cos^2 kz}{1+a^2\cos^2 kz}.$$

Making these substitutions the differential equation becomes

$$d\Omega = 2m\,dz - k\Omega\,\frac{1-a^2\cos^2 kz}{1+a^2\cos^2 kz}\tan kz\,.\,dz,$$

or, putting $\beta = kz$,

$$\frac{d\Omega}{d\beta} = \frac{2m}{k} - \Omega\,\frac{1-a^2\cos^2\beta}{1+a^2\cos^2\beta}\tan\beta.$$

Put $\mathrm{P} = \dfrac{1-a^2\cos^2\beta}{1+a^2\cos^2\beta}\tan\beta$ for brevity, and the integral of the equation becomes

$$\Omega e^{\int \mathrm{P}d\beta} = \frac{2m}{k}\,.\int_0^\beta e^{\int \mathrm{P}d\beta}d\beta.$$

It is easily shown that

$$\int \mathrm{P}d\beta = \log\frac{1+a^2\cos^2\beta}{\cos\beta}$$

$$\therefore\ e^{\int \mathrm{P}d\beta} = \sec\beta + a^2\cos\beta.$$

Substituting we find

$$\Omega = \frac{2m}{k}\,.\,\frac{\int_0^\beta(\sec\beta+a^2\cos\beta)\,d\beta}{\sec\beta+a^2\cos\beta}$$

$$= \frac{2m}{k}\,\frac{\log_e\tan\left(\frac{1}{4}\pi+\frac{1}{2}kz\right)+a^2\sin kz}{\sec kz+a^2\cos kz}.$$

It appears from this equation that Ω is a periodic function of z, the wave-length being π/k.

If we make k very small, we may put $\log\tan\left(\frac{1}{4}\pi+\frac{1}{2}kz\right) = kz$, and the equation becomes

$$\Omega = \frac{2m}{k}\,\frac{kz+a^2kz}{1+a^2}$$

$$= 2mz,$$

an equation which gives the true magnetic rotation, as of course it should do.

If $\alpha = \pi/4$, so that a becomes infinite, the equation becomes

$$\Omega = \frac{2m}{k} \tan kz,$$

except when $kz = \pi/2$. If $kz = \pi/2$, Ω takes an indeterminate form. In this case the light is circularly polarised.

To examine this equation generally it is advisable to write it somewhat differently. Let us put

$$\Omega = \frac{2mz}{kz} \frac{\log \tan (\frac{1}{4}\pi + \frac{1}{2}kz) + a^2 \sin kz}{\sec kz + a^2 \cos kz}$$

$$= \frac{\Theta}{\beta} f(a^2, \beta)$$

where Θ is the true magnetic rotation, and β is the total retardation expressed as an angle.

Hence $$\frac{\Omega}{\Theta} = \frac{f(a,^2\beta)}{\beta}.$$

If then we trace the curve

$$y = f(a,^2x),$$

the ratio of y/x at any point gives us the ratio of the apparent to the true rotation when $\beta = x$.

With regard to the curve,

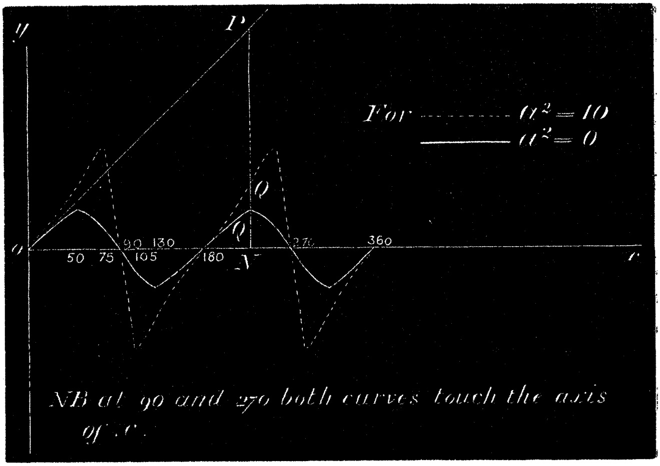
$$y = f(a,^2x),$$

it should be observed that $y = 0$ whenever x is a multiple of $\frac{1}{2}\pi$, and that $\frac{dy}{dx} = 1$ when x is an even multiple of $\frac{1}{2}\pi$, and $= 0$ when x is an odd multiple of $\frac{1}{2}\pi$. Since all along the tangent at the origin $y = x$, we may take the tangent to represent the true rotation, so that

$$\frac{y_{\text{curve}}}{y_{\text{tangent}}} = \frac{\Omega}{\Theta}.$$

Tracings of the curve when $a^2 = 0$ and 10 respectively are given below.

It appears from these curves that when $a^2 = 10$, the apparent rotation is greater than the real rotation if β is less than 80°, while if a^2 is equal to 0, Ω/Θ is always less than 1. When β is small it is easy



to see that the value of Ω/Θ is greater or less than unity, according as a is greater or less than unity. For if β is small,

$$\frac{\Omega}{\Theta} = \frac{\beta(1+a^2)}{\beta(\sec\beta + a^2\cos\beta)}$$

$$= \frac{1+a^2}{1+a^2+(\beta^2/z)(1-a^2)},$$

a fraction which is greater or less than 1 according as $1-a^2$ is negative or positive. The curve tracings also show that the apparent rotation changes sign whenever β is any multiple of a right angle.

[If β is greater than 90°, equal say to $n\,90°+\gamma$, where γ is less than a right angle, then if n is even,

$$\frac{\Omega}{\Theta} = \frac{f(a^2,\gamma)}{\beta};$$

and if n is odd,

$$\frac{\Omega}{\Theta} = \frac{f(a^2,\,90°-\gamma)}{\beta},$$

quantities which become very small when β is large.—May 14.]

In the case of quartz one centimetre thick, where the direction of the ray of light makes an angle of 90° with the optic axis, $\beta = 627\,\pi/2$ very nearly. Hence Ω/Θ in this case is about $\frac{1}{627}$ at best, which gives a value of Ω quite inappreciable by any known methods. To find the true magnetic rotation in quartz we must use thin sections about 0·01 mm., and a^2 should be equal to 1.

This value for Ω/Θ accounts for the fact that doubly refracting bodies do not exhibit any rotation of the plane of polarisation,* if the direction of the ray of light is inclined to the optic axis. When light goes through a crystal in the direction of the optic axis, magnetic rotation has been observed. E. Becquerel has observed rotation in tourmalin and rock crystal, while Wertheim† has detected a well-marked rotation in beryl, a feeble one in quartz, but nothing in Iceland spar.

[Lüdtge‡ determined the magnetic rotation along the optic axis of quartz and at various inclinations. The following table gives his results and the corresponding values of β:—

Inclination to optic axis	0°	1°	2°	3°	5°
Rotation (Ω)	1·1	1	0·9	0·6	0·4
Retardation (β)	0	13	52	117	325

Lüdtge adds that these figures are not to be taken as giving exact measurements. No magnetic rotation has been observed in Iceland spar even along the optic axis, and it is worthy of note that in this crystal β very rapidly increases with the inclination to the optic axis. If the length of the spar be 1 centimetre, then at an inclination of $n°$ to the optic axis for D line $\beta = 318° \times n^2$, while in quartz $\beta = 17° \times n^2$.—May 14.]

Wertheim has also shown that if a piece of heavy glass be compressed, the magnetic rotation is diminished, even when the retardation due to the doubly refracting nature of the compressed glass is much less than a wave-length.

Lüdtge's experiments on compressed glass show again how the magnetic rotation is diminished as the doubly refracting property increases. Lüdtge gives the following table, where n represents part of a wave-length retardation and d the magnetic rotation:—

n 0	0·01	0·2	0·25	0·3	0·45	0·5	0·6
d .... 5°	4·6°	4·2°	4°	3·7°	3·5°	3°	2·4°.

These results cannot be directly compared with what we should expect from a crystal, since the ends of the glass are free from strain, and a rotation is there produced, which is observed.

Villari's results are very similar to Lüdtge's. Villari, by spinning a disk of glass very rapidly, strained it, and on observing the magnetic rotation found it get less and less as the strain got greater and greater. There is, however, one noticeable difference between

* Wiedemann, 'Die Lehre von der Elektricität,' vol. 3, sec. 1097.

† Wertheim, 'Compt. Rend.,' vol. 32.

‡ Lüdtge, 'Poggendorff, Annalen,' vol. 137.

Villari's strained disk and Lüdtge's strained prism. The disk was free from strain in the middle, the prism free from strain at the ends.

I have repeated Villari's experiment at the Cavendish Laboratory, using, at Mr. Glazebrook's suggestion, an elliptic analyser to determine the magnetic rotation. With the disk spinning about 200 times a second, the magnetic rotation was reduced from 10° to 6°. This is not so great a diminution as Villari observed, but his glass may have been softer and more easily strained.

Villari thought that the effect he observed was due to the time required to magnetise the glass. That this supposition was erroneous has been clearly established by the experiments of Bichat and Blondlot, recently repeated by Dr. Lodge. In these experiments the oscillating discharge of a Leyden jar was found to rotate the plane of polarisation in time with the oscillations. Before hearing of these results I had myself attacked the problem in a somewhat similar manner. A coil of wire was wound round a piece of heavy glass, and a current alternated 250 times a second by a tuning-fork was sent through the coil. The current was measured by a dynamometer and a tangent galvanometer. The first gave the measure of the current independently of its sign, the second showed that the integral current was zero. When the current was passing it was found impossible to extinguish the light, owing to the rapid alternations of the plane of polarisation.

In conclusion, I have to express my thanks to Professor Thomson and Mr. Glazebrook for many kind suggestions and encouragement, and especially to Professor Thomson for the privilege of using the Cavendish Laboratory.

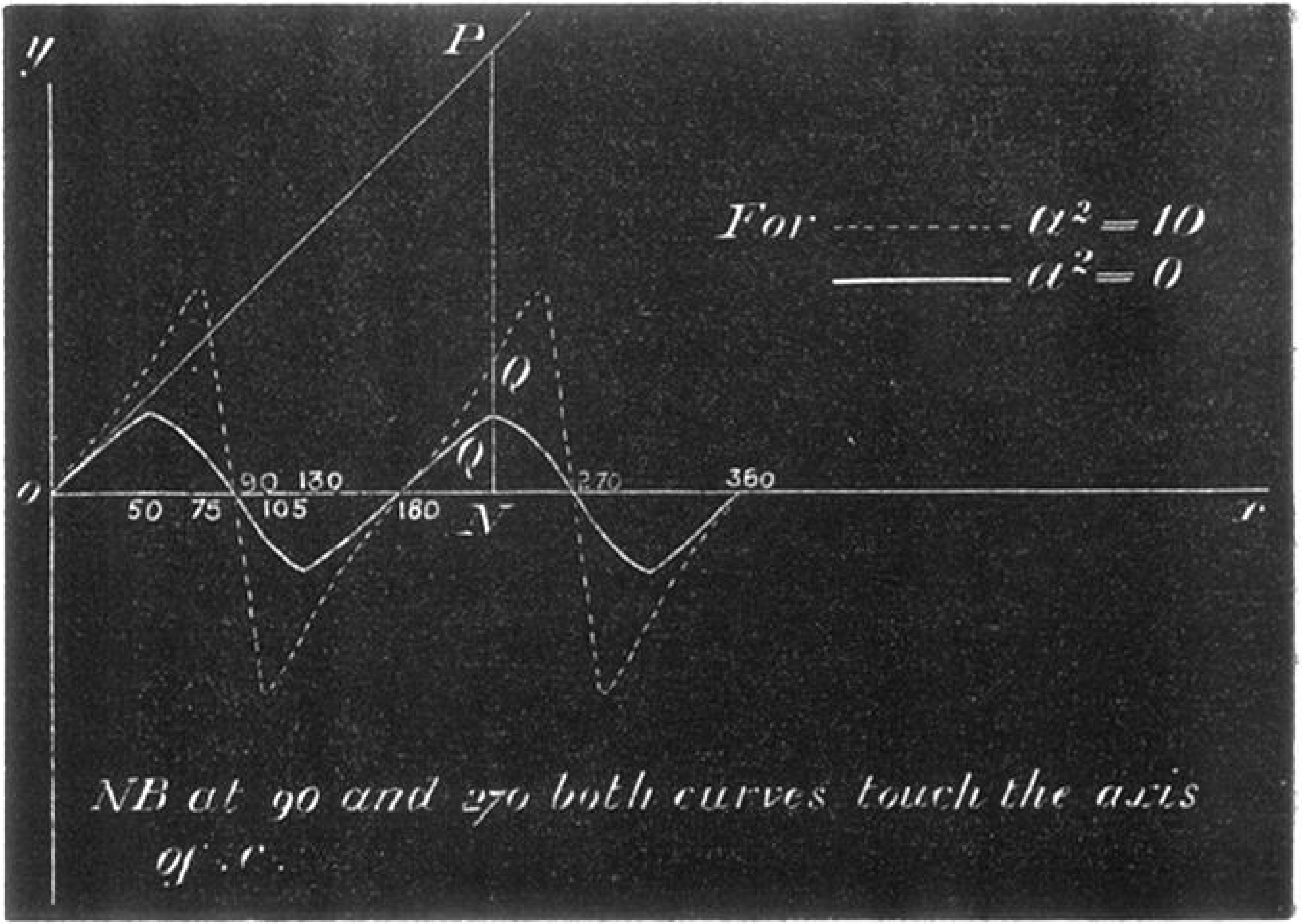
II. "Revision of the Atomic Weight of Gold." By J. W. Mallet, F.R.S., Professor of Chemistry in the University of Virginia. Received April 15, 1889.

(Abstract.)

After noticing and giving the results of the earlier determinations of the atomic weight of gold, and the recent researches of Krüss and of Thorpe and Laurie, the author reports upon experiments of his own in the same direction, which have occupied much of his time and labour for the last three or four years.

The difficulties connected with the accurate determination of the atomic weight of this metal are remarked upon, and the general principles are reviewed which ought to be observed in all investigations of this kind.

The means and methods of weighing used are stated, and the pre-


y
P
For $a^2 = 10$
$a^2 = 0$
Q
Q
o
90
130
270
360
50
75
105
180
N
x
NB at 90 and 270 both curves touch the axis of x.