ficial strata are not magnetic, regions of high vertical force comparable in size with small counties, and ridge lines or loci of attraction as long and almost as clearly defined as the rivers. Their course is closely connected with the geology of the districts through which they run.

II. "The Variations occurring in certain Decapod Crustacea.—I. *Crangon vulgaris*." By W. F. R. WELDON, M.A., Fellow of St. John's College, Cambridge, and Lecturer on Invertebrate Morphology in the University. Communicated by Professor M. FOSTER, Sec. R.S. Received March 20, 1890.

It is well known that two sets of animals, belonging to the same species, but living in different places, exhibit differences from one another by which they can, in many cases, be easily distinguished. But it is at the same time equally certain that the forces determining the differences between local races of the same species do not so act as to produce the same effect upon all individuals of the same race: for I am aware of no case in which the individuals composing any race of animals—however small and isolated the area in which they live, however uniform the conditions which obtain throughout that area—have been shown to resemble one another *exactly* in any character.

Since the adjustment of a local race to the average proper to it is not complete, the question arises, whether it is not possible to determine the degree of accuracy with which this adjustment is effected, and the law which governs the occurrence of deviations from the average. The object of this paper is to give an account of certain observations made at the laboratory of the Marine Biological Association at Plymouth, in order to determine, *first*, the average length of three or four organs which admitted of accurate measurement, and, *secondly*, the frequency with which the average length and every deviation from it occurred in one or two local races of *Crangon vulgaris*.

In making this investigation, I have had the great privilege of being constantly advised and helped, in every possible way, by Mr. Galton. My ignorance of statistical methods was so great that, without Mr. Galton's constant help, given by letter at the expenditure of a very great amount of time and trouble, this paper would never have been written. I am glad to take this opportunity of expressing my gratitude for his generous conduct. I have also to thank Dr. Donald MacAlister for explaining to me many points connected with the law of error, and for helping me in various ways.

Mr. Galton has, as is well known, studied the frequency with which deviations from the average size of certain organs occur in man, in certain plants, and in moths. The result of his investigations has been to show that deviations from the average size of the organs measured by him occur in every case with the frequency indicated by the law of error. These results were, however, based on an investigation either of civilised man, or of a domesticated animal or plant: and Mr. Galton has himself pointed out that in the majority of cases studied, the effect of natural selection is probably insignificant. The similar investigations of Quetelet and others are also confined to civilised man. It has, therefore, seemed worth while to attempt an investigation of the variations in the size of certain organs which occur in a species living in a wild state, upon which natural selection and the other destructive or plastic influences from which domestic animals and civilised man are alike protected may be supposed to act with full effect.

In his recent work on heredity,* Mr. Galton predicted that selection would not have the effect of altering the law which expresses the frequency of occurrence of deviations from the average: so that he expected the frequency, with which deviations from the average size of an organ occurred, to obey the law of error in all cases, whether the animals observed were under the action of natural selection or not. The results of the observations here described are such as to fully justify Mr. Galton's prediction.

These observations relate entirely to the lengths of organs, or parts of organs. The measurements of these lengths were made either with a pair of compasses, in the case of the greater lengths, or in the case of smaller parts by means of a microscope, provided with cross-wires, and travelling on a screw of known pitch. The results are, I believe, accurate to within about 0·1 mm. The edges of the parts measured were in many cases so uneven, and the effect of the spirit in which the specimens were preserved was probably so considerable, that a greater degree of apparent accuracy in the measurements would not have implied a more reliable result.

In order to compare the organs of one individual with the corresponding parts of another individual of different size, it was evidently necessary to express the dimensions of each organ in terms of the length of the body of the individual to which it belonged. All the measurements used in this paper are therefore, expressed in terms of the total length of the body, which is taken as 1000.

Having obtained measures of the length of an organ in a sufficiently large sample of individuals, the frequency with which the various magnitudes occur may be conveniently exhibited in the fol-

* 'Natural Inheritance,' pp. 119—124.

lowing way, which is that adopted by Mr. Galton:—The values obtained are sorted and arranged in order of magnitude; then, at equal distances along a given base, ordinates are erected equal in number to the observations, one ordinate being proportional to each observed value of the organ. By joining the tops of these ordinates, a curve is obtained such as that drawn in fig. 2.

If the base-line of such a curve be divided into one hundred parts, then the proportion of individuals measured, which possess the organ from which the curve is constructed, of a size greater or less than any given magnitude, can be readily ascertained. For example, in fig. 2, which shows the distribution of lengths of the carapace in 400 female shrimps from Plymouth, the ordinate, whose length is 256, stands at grade 20°, showing that 20 per cent. of the individuals examined had the carapace longer than 256 (the body length being 1000), while in the remaining 80 per cent. the carapace was shorter than this.

A curve constructed in the manner directed is nearly always symmetrical about its middle point: and this point therefore closely approximates to the average of the whole number of observations from which the curve was constructed. The value of the middle ordinate will always be taken, in what follows, as the average value: it will, in accordance with Mr. Galton's notation, be spoken of as the Median, and denoted by the symbol M. Each curve, therefore, gives by simple inspection the average value of the organ to which it refers.

In estimating the *deviations from the average* which occur in each case, the magnitude of the average itself is evidently of no importance: and the ordinates of the curve may therefore be considered with reference to an axis passing through the point M, so that the ordinate of M becomes zero. When measured from this axis, half the ordinates of the curve are of course positive, the other half being negative.

If the frequency, with which the observed deviations from the average occur, obeys the law of error, then the curve just described should be a "curve of error," whose "probable error" is represented by the ordinates at the 25th and 75th grades. These grades are the boundaries of the first and third quarters of the base: they will, therefore, be spoken of (again in accordance with Mr. Galton's notation) as Quartiles, and will be denoted by the symbols Q_1 and Q_3 respectively. In a perfectly normal curve, Q_1 and Q_3 are of course equal in magnitude and opposite in sign. In the observed curves there was generally a slight difference between the two: and the mean value of the two is therefore adopted as the "probable error," which will be denoted by the symbol Q.

In order to determine the correspondence between the observed

curve and the curve of error, the ordinates of the observed curve will be compared with those of the curve of error at certain fixed grades.

This may be most conveniently done by considering the ordinates of the curve of error at those grades as multiples of the "probable error" of the curve.

The grades chosen, together with the ordinates of a curve of error, expressed in terms of its probable error, at those grades, are as follows:—

Table I.—Ordinates of a Curve of Error, in Terms of Q.

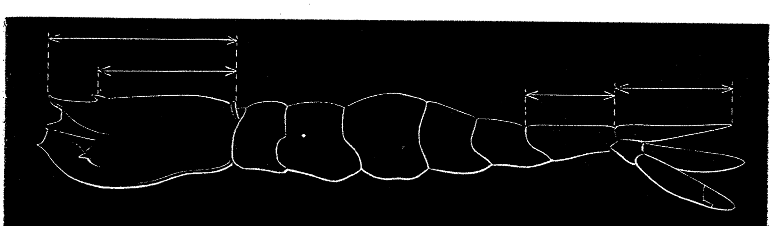
Grade.	Ordinate = Q ×	Grade.	Ordinate = Q ×
5°	+2·44	60°	−0·38
10	+1·90	70	−0·78
20	+1·25	75	−1·00
25	+1·00	80	−1·25
30	+0·78	90	−1·90
40	+0·38	95	−2·44
50	0·00		

It will be seen that, in order to compare a curve constructed from a number of observations with a curve of error, the following process is performed: the ordinates at the selected grades are determined, and the observed value of M is subtracted from each of these. The remainders, divided by $\frac{1}{2}(Q_1 - Q_3)$, should give the coefficients of Q which appear in the above table.

Such a comparison will now be made between the curve of error and the curves obtained from the measurements.

The organs measured are four: the total length of the carapace; the distance from the posterior margin of the carapace to the front of the median spine; the length of the sixth abdominal tergum; and the length of the telson. The parts are shown in the accompanying woodcut.

Fig. 1.

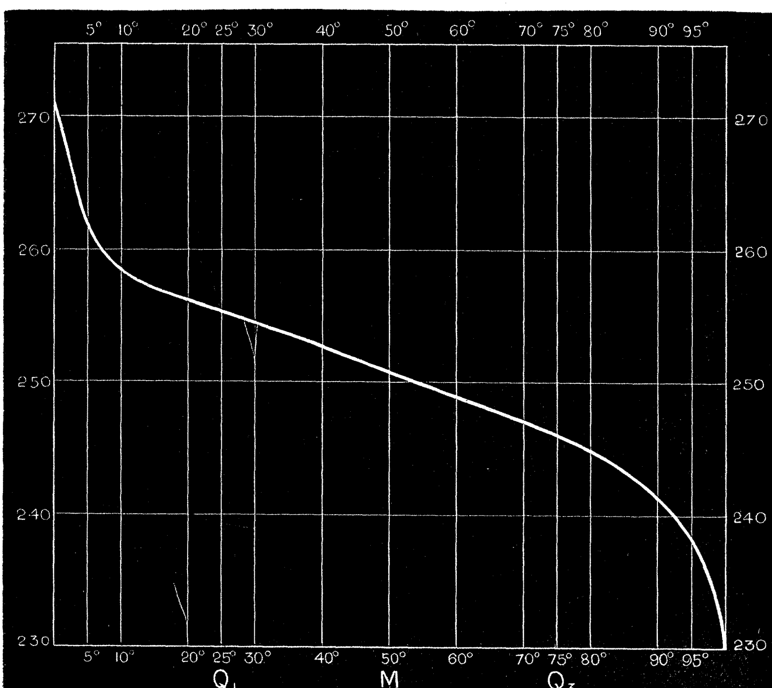


In measuring the length of the sixth tergum and of the telson, these organs were removed from the body; so that the small portion of each which projects inwards into a fold of skin, and serves as an attachment for muscles, is included in the total length.

The individuals measured were all adult females; they were collected from widely separate places. The first sample measured consisted of 400 individuals from Plymouth Sound; a second sample, containing 300 females, was obtained from Southport by Mr. W. Garstang; and a third sample, of which 300 were measured, was sent to me from Sheerness by Mr. W. H. Shrubsole.

Total Length of the Carapace.—The curve obtained by treating the total length of the carapace of the Plymouth specimens in the way described is shown in the woodcut, Fig. 2.

FIG. 2.



It will be seen that the median ordinate has a length of 250·52, which is therefore the value of M. The ordinate at 25° is 255·07 = M+4·55; that at 75° is 246·00 = M−4·52; so that the mean value of $Q = \frac{1}{2}(4{\cdot}55+4{\cdot}52) = 4{\cdot}53$.

The following table will show the relation between this curve and the normal curve of error:—

Table II.—Curve of Distribution of Carapace Lengths—Plymouth.

Grade.	Ordinate.	Ord. − M.	$\frac{\text{Ord.} - \text{M}}{\text{Q.}}$	Normal curve.
5°	261·50	+10·98	+2·42	+2·44
10	258·95	+ 8·43	+1·86	+1·90
20	256·05	+ 5·53	+1·22	+1·25
25	255·07	+ 4·55	+1·00	+1·00
30	254·10	+ 3·58	+0·79	+0·78
40	252·27	+ 1·75	+0·39	+0·38
50	250·52	0·00	0·00	0·00
60	249·09	− 1·43	−0·32	−0·38
70	247·29	− 3·23	−0·71	−0·78
75	246·00	− 4·52	−1·00	−1·00
80	244·74	− 5·78	−1·28	−1·25
90	241·39	− 9·13	−2·10	−1·90
95	238·60	−11·92	−2·63	−2·44

It will be seen therefore that the average length of the carapace in the Plymouth specimens was 250·52-thousandths of the body length, and that the frequency with which this length and the various observed deviations from it occurred was almost exactly that indicated by a curve of error whose prob. error = 4·53.

In the above table, all the steps in the determination of the coefficients of Q are indicated. To indicate all the steps in this determination in the case of each race would involve a great deal of vain repetition; and, therefore, in the following table the coefficients themselves are alone indicated. It will be understood that the entry opposite each grade in this table is found by subtracting the value of M from the observed ordinate at that grade, and dividing the remainder by $\frac{1}{2}(Q_1 - Q_3)$. The value thus obtained should be the coefficient of Q in the table of ordinates of a normal curve given on page 448. These coefficients are repeated in the last column of the following table.

Table III.—Ordinates of the Curves of Deviation of Carapace Lengths, each in Terms of its own Q.

$$\text{Each entry} = \frac{\text{observed ordinate} - \text{M}}{\text{Q}}.$$

Grade.	Plymouth. 400 specimens.	Southport. 300 specimens.	Sheerness. 300 specimens.	Normal curve.
5°	+2·42	+2·86	+3·34	+2·44
10	+1·86	+2·11	+2·29	+1·90
20	+1·22	+1·29	+1·35	+1·25
25	+1·00	+0·97	+1·02	+1·00
30	+0·79	+0·70	+0·76	+0·78
40	+0·39	+0·34	+0·35	+0·38
50	0·00	0·00	0·00	0·00
60	−0·32	−0·37	−0·35	−0·38
70	−0·71	−0·80	−0·74	−0·78
75	−1·00	−1·03	−0·99	−1·00
80	−1·28	−1·27	−1·28	−1·25
90	−2·10	−2·05	−1·97	−1·90
95	−2·63	−2·68	−2·41	−2·44

The table shows that in all the races the coefficients of Q agree fairly well with those indicated by the normal curve. When these coefficients and the values of M and Q in each case are known, it is evident therefore that the whole curve is known.

The values of M and Q are as follows:—

Plymouth .... M = 250·05 ; Q = 4·53
Southport .... M = 248·50 ; Q = 3·17
Sheerness .... M = 247·51 ; Q = 3·05

It thus appears that not only does the average size of the carapace differ in different local varieties, but the range of deviation from that average differs also. Nevertheless, the frequency with which the observed deviations from the average occur is in all the three observed cases expressed by a curve of error.

Since the variations observed in adult individuals depend not only on the variability of the individuals themselves (which is possibly nearly alike in all races), but also upon the selective action of the surrounding conditions—an action which must vary in intensity in different places—the result here obtained is precisely that which might be anticipated, and it is precisely that predicted by Mr. Galton.

The same features are presented by the curves derived from the remaining sets of measurements. The following tables give the data for constructing curves of deviation of each organ.

Table IV.

Grade.	Post-spinous portion of carapace.			Length tergum VI.*		Length telson.*		Normal curve.
	Plymouth. M=178·03. Q= 3·58.	Southport. M=179·50. Q= 3·01.	Sheerness. M=179·63. Q= 2·80.	Plymouth. M=145·46. Q= 3·24.	Southport. M=141·72. Q= 3·02.	Plymouth. M=192·44. Q= 4·56.	Southport. M=195·45. Q= 3·52.	
5°	+2·35	+2·76	+2·95	+2·60	+2·32	+2·40	+2·95	+2·44
10	+1·79	+2·01	+2·27	+2·04	+1·87	+1·78	+2·27	+1·90
20	+1·23	+1·16	+1·37	+1·24	+1·12	+1·22	+1·37	+1·25
25	+0·98	+0·97	+1·08	+0·95	+0·88	+0·97	+1·08	+1·00
30	+0·75	+0·76	+0·85	+0·66	+0·69	+0·73	+0·85	+0·78
40	+0·34	+0·39	+0·41	+0·31	+0·32	+0·35	+0·41	+0·38
50	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
60	−0·29	−0·37	−0·37	−0·35	−0·39	−0·35	−0·37	−0·38
70	−0·75	−0·79	−0·73	−0·78	−0·86	−0·80	−0·73	−0·78
75	−1·02	−1·03	−0·91	−1·05	−1·12	−1·02	−0·91	−1·00
80	−1·35	−1·50	−1·18	−1·30	−1·35	−1·23	−1·18	−1·25
90	−2·11	−1·80	−1·88	−1·84	−1·85	−1·89	−1·88	−1·90
95	−2·58	−2·30	−2·48	−2·38	−2·30	−2·39	−2·48	−2·44

* The lengths of tergum VI and of the telson were not determined in the Sheerness specimens.

That the deviations from the normal value of the coefficients of Q shown in the foregoing are accidents due to the small number of observations upon which the curves are based is shown by the following table, in which each entry is the mean of the corresponding entries of all the preceding tables :—

Grade.	Mean of observed coefficients of Q.	Normal.	Grade.	Mean of observed coefficients of Q.	Normal.
5°	+2·55	+2·44	60°	−0·36	−0·38
10	+1·99	+1·90	70	−0·78	−0·78
20	+1·24	+1·25	75	−1·03	−1·00
25	+0·97	+1·00	80	−1·31	−1·25
30	+0·74	+0·78	90	−1·97	−1·90
40	+0·35	+0·38	95	−2·49	−2·44
50	0·00	0·00			

Results similar to the above have been obtained from measurements of a larger series of organs, and parts of organs, in *Pandalus annulicornis* (two races) and *Palæmon serratus* (one race); but, at present, not more than 100 individuals of each race have been measured, and the curves of distribution of the magnitudes of the various organs are therefore more irregular than those given for the shrimp. In these cases, however, there is no constant deviation in any direction from the normal curve. There seems, therefore, no reason to doubt that an extended series of measurements will show that the variations of these animals obey the law of error as closely as do those of *Crangon*. I hope shortly to collect such a series of measurements.

It seems, therefore, that Mr. Galton's prediction is fully justified; and that (1) the variations in size of the organs measured occur with the frequency indicated by the law of error; and (2) the "probable error" of the same organ is different in different races of the same species.

I have attempted to apply to the organs measured the test of *correlation* given by Mr. Galton ('Roy. Soc. Proc.,' vol. 45, No. 274, pp. 135 *et seq.*); and the result seems to show that the degree of correlation between two organs is constant in all the races examined. Mr. Galton has, in a letter to myself, predicted this result. A result of this kind is, however, so important to the general theory of heredity, that I prefer to postpone a discussion of it until a larger body of evidence has been collected.

FIG. 1.

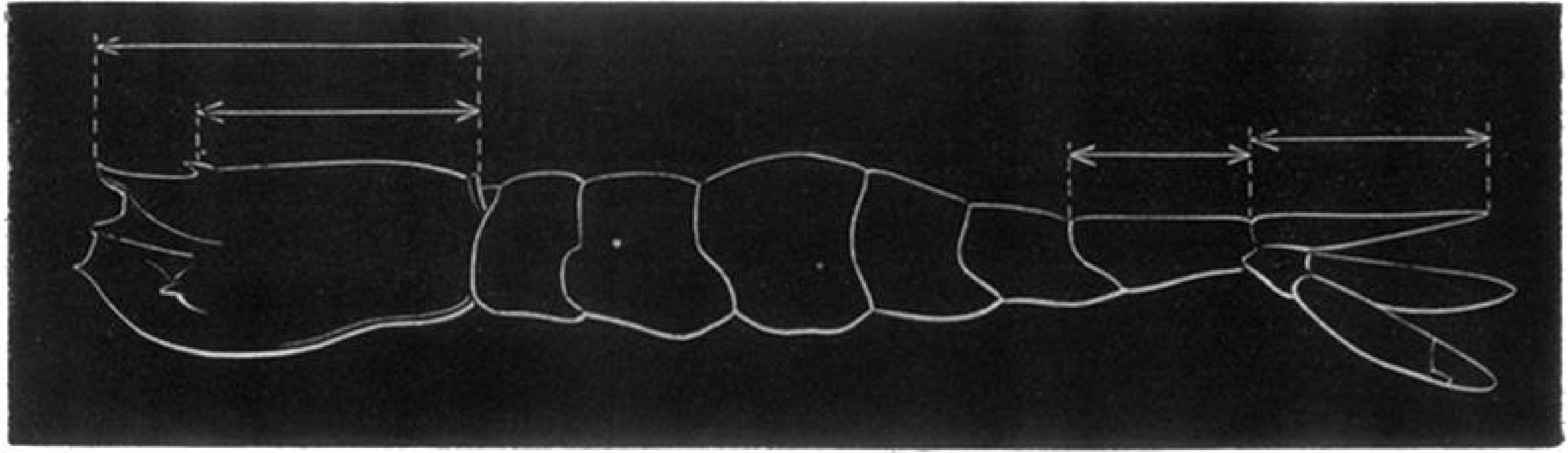


FIG. 2.