filled with a uniform glow whenever the discharge passed through the primary circuit, but, when the electrostatic induction was shielded off by pieces of wet thin blotting paper connected to earth, no glow could be observed, though the wet blotting paper is not a sufficiently good conductor to shield off electromagnetic induction.

The maximum integral electromotive force round the secondary is shown to be VM/L, where V is the difference between the potentials of the coatings of the jar before discharge, L the coefficient of self-induction of the primary circuit, and M the coefficient of mutual induction between the circuits. Though in my experiments this was greater than the electromotive force requisite for a discharge through gas at the same density between terminals separated by the length of the tube, not the faintest glow could be detected. All my efforts to get a discharge through the secondary have so far been unsuccessful,* and I feel sure that the ease of getting a discharge without electrodes, say by the motion of the upper regions of the earth's atmosphere across the lines of magnetic force, has been much over-estimated. Until, however, we have got a discharge without electrodes through nothing but the gas itself, we are unable to say whether the passage of the discharge from the positive to the negative electrode which occurs in gases is a consequence of having matter in two states in the path of the discharge, or whether it is an example of a more general law, that, whenever tubes of electrostatic induction shorten in a conducting circuit, they do so in the direction of the electric displacement.

In conclusion, I have much pleasure in thanking Mr. Bartlett and Mr. Everett for the assistance they have given me in the course of this investigation.

II. "Note on the Present State of the Theory of Thin Elastic Shells." By A. E. H. Love, M.A., St. John's College, Cambridge. Communicated by Lord Rayleigh, Sec. R.S. Received January 3, 1891.

In a paper read before the Royal Society in February, 1888, and published in 'Phil. Trans.,' A, of that year, I advanced a theory of the mode of deformation that takes place when a thin shell is vibrating. The theory was founded on the form of the potential energy function, obtained by a method adapted from that of Kirchhoff for plates. It appears that, in case there are no surface-stresses on the faces of the shell, this function consists of two terms, of which one contains a certain function W_2 and the thickness $2h$ as factors, and

* Since this paper was sent in to the Royal Society, I have succeeded in getting a discharge without electrodes through a tube about 45 cm. in circumference. The discharge did not exhibit any signs of stratification.—Jan. 23, 1891.

the other contains a function W_1 and h^3 as factors. The term W_2 depends entirely on quantities σ_1, σ_2, ϖ, expressing the extension of the middle surface, while the form given for W_1 contained only quantities expressing the changes of curvature. Some previous theories proceeded as if W_1 alone occurred, and, in fact, this was the case with a paper by Lord Rayleigh in 'Proceedings of the London Mathematical Society,' vol. 13, 1882, on the "Infinitesimal Bending of Surfaces of Revolution." In the latter paper, a theory of the vibrations of bells was founded on an assumed type, viz., it was assumed that the middle surface remains unstretched. In my paper it was shown that this solution of Lord Rayleigh's fails to satisfy the boundary conditions which hold at the free edges of the bell, and further that it is, in general, impossible to satisfy these conditions, except by taking account of the extension. I, therefore, proposed to substitute for the theory of Lord Rayleigh one in which extension of the middle surface of the bell is recognised as taking place, and I did not see how to avoid the conclusion that the term W_1 must be rejected, and the term W_2 retained, for the purpose of forming the differential equations and boundary conditions that govern the motion, in other words, that the extension practically determines everything—the mode of vibration and the pitch.

Since that paper was written the subject has been investigated by Lord Rayleigh, Mr. Basset, and Professor Lamb, and the results of their work make it necessary to abandon the theory proposed. I had overlooked a circumstance which shews that my theory of extensional vibrations is incapable of giving the gravest modes of vibration of which the shell is capable, viz., the period given by Lord Rayleigh's solution, founded on the assumed type, is, in the limiting case of vanishing thickness, infinitely long in comparison with the gravest extensional period. Now it is a general dynamical theorem that the tone obtained by assuming the type cannot be graver than the gravest tone natural to the system, and it follows that the mode of deformation corresponding to the gravest tone is not included among the extensional modes. This was pointed out by Lord Rayleigh in a paper read before the Society in December, 1888, and published in the 'Proceedings.' It had still to be shown, however, that vibrations mainly dependent on the bending could take place, and the boundary conditions be satisfied. Although this has not yet been done in any particular case, the suggestion thrown out by Mr. Basset* and Professor Lamb,† probably contains the solution of the

* "On the Extension and Flexure of Cylindrical and Spherical Thin Elastic Shells," 'Phil. Trans.,' A, 1890.

† "On the Deformation of an Elastic Shell," 'London Math. Soc. Proc.,' vol. 21, 1890.

difficulty. Each of these writers has shown that, in particular statical problems relating to cylinders, the quantities expressing the extension can be very small everywhere except in the neighbourhood of an edge, and there they may increase with such rapidity as to secure the satisfaction of the boundary conditions, the total potential energy due to extension, which varies as the surface integral of hW_2 over the middle surface, being, nevertheless, negligible in comparison with that due to bending, which varies as the surface integral of h^3W_1. Mr. Basset and Professor Lamb both suggest that this may be the solution of the difficulty in the case of vibrations also, and their results point to a method of approximation which might be applied to the general case, and such that it could be verified by mathematical analysis that Lord Rayleigh's solution, founded on an assumed type, is actually a very close approximation to the state of things in any part of a vibrating bell not very close to a free edge.

It may be as well to point out what parts of the theory put forward in my paper specially require revision. (1.) On p. 500 the alteration suggested in Kirchhoff's theory is erroneous; the quantities u', v', w' are functions of α, β, and their differential coefficients must be introduced as by Kirchhoff, and afterwards neglected; this correction makes no difference to any of the results. (2.) On p. 503, Art. 4, the "products" neglected are such as occur in the equations when account is taken of the fact that the axes of reference are really not in fixed directions. If they had been retained, the part of the potential energy which is multiplied by h^3 would have contained terms depending on the extension as well as terms depending on the bending. Mr. Basset has obtained, by a different method, the form of this function for cylindrical and spherical shells, with these terms expressed. It follows that the form given for the potential energy in equation (12), p. 505, is only correct in case either (a) the shell is unextended, when its second line vanishes, or (b) the extension is the important thing, when its first line may be neglected; but it would most probably be sufficiently exact for the application of a method of approximation. (3.) The first paragraph of Art. 13, p. 521, is wrong, and so are all other paragraphs to the same effect; viz., it is incorrect to conclude that, because σ_1, σ_2, ϖ do not everywhere vanish, therefore W_1h^3 is infinitely small in comparison with W_2h. It appears, on the contrary, that the values of σ_1, σ_2, ϖ can be very small indeed everywhere except close to the edges, in such a way that the integral of W_2h, taken over the middle surface, is very small in comparison with that of W_1h^3.

The remainder of the paper must be understood as giving a theory of the extensional vibrations of the shell. Such vibrations undoubtedly can exist, but they would be difficult to excite, and the theory of them has no application to vibrating bells under ordinary conditions.