## IV. "Some Measures of Young's Modulus for Crystals, &c." By A. Mallock. Communicated by Lord Rayleigh, Sec. R.S.

Received March 9, 1891.

The Table at the end of this communication contains the results of experiments made in October, 1889—February, 1890, on the elasticity of various bodies.

The measures relating to crystalline bodies are, I believe, new.

The method used to obtain these results was applicable to very small specimens. This was a necessary condition in the case of most crystals, because of the difficulty of getting large specimens without flaws.

In the experiments now to be described, I am dealing only with the values of Young's modulus, but by a modification of the apparatus measures can be made of the simple rigidity, which will, I hope, form the subject of a future communication.

Of course, the simple rigidity must lie between one-half and one-third of the value of Young's modulus, according to the ratio between longitudinal extension and lateral contraction (Poisson's constant) for the substance.

The apparatus used in my experiments is shown in figs. 1 and 2.

Fig. 1 shows the general arrangement of the parts, and fig. 2 is a full-sized diagram of the mirrors and knife-edges.

The lettering is the same in both figures. A, is a vertical brass

Fig. 1.

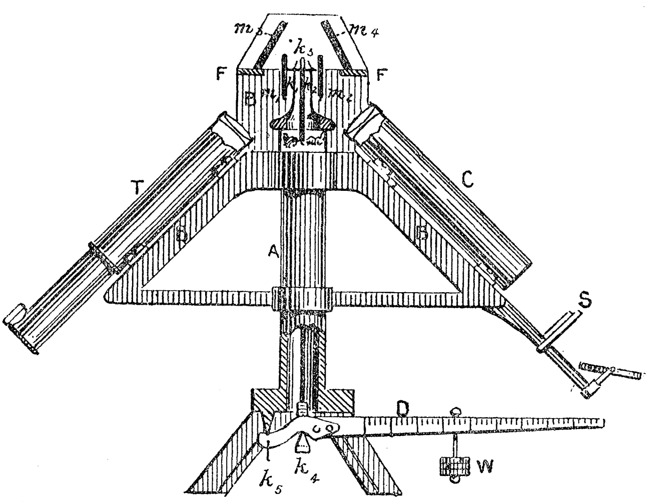
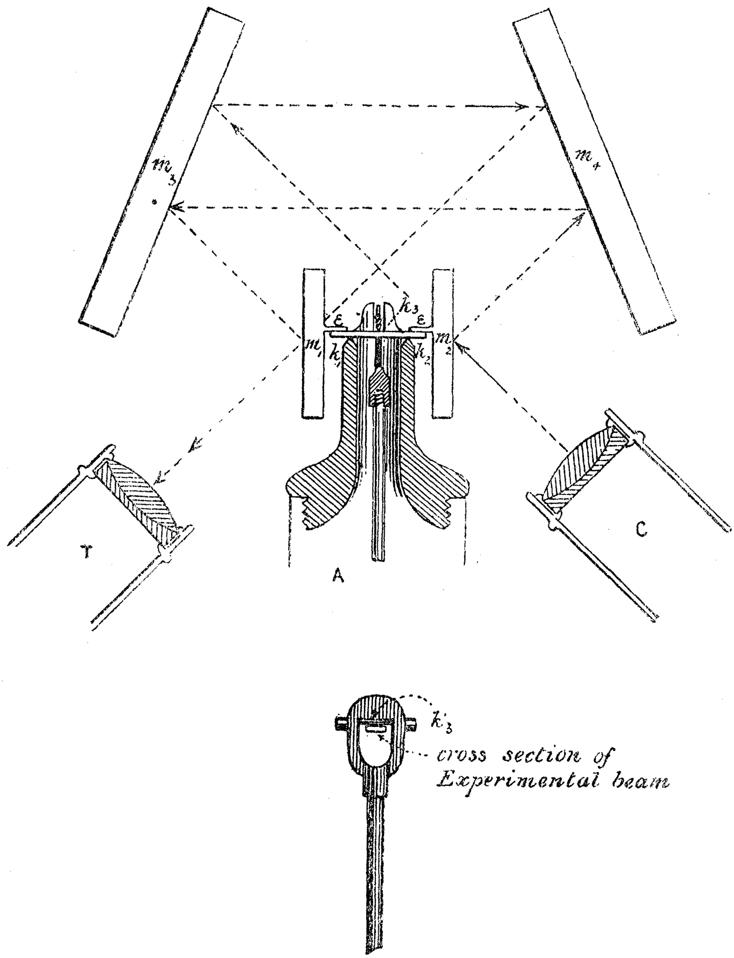


Fig. 2.




tube carrying the oblique arms B, B′. On B is mounted the telescope T, and on B′ the collimator C, having in its principal focus the glass scale S.

$K_1$ and $K_2$ are two parallel horizontal knife edges, mounted on a brass support at the upper end of A. On these knife edges the substance to be examined rests, and a third knife edge, $K_3$, parallel to the other two, and half-way between them, which is properly guided and free to move only in the vertical plane passing through its edge, presses on the substance with a force determined by the magnitude of the weight W and its position on the graduated arm D.

The fulcrum of D is the knife-edge $K_5$, and a wire passing through A connects the knife-edge $K_4$ with $K_3$.

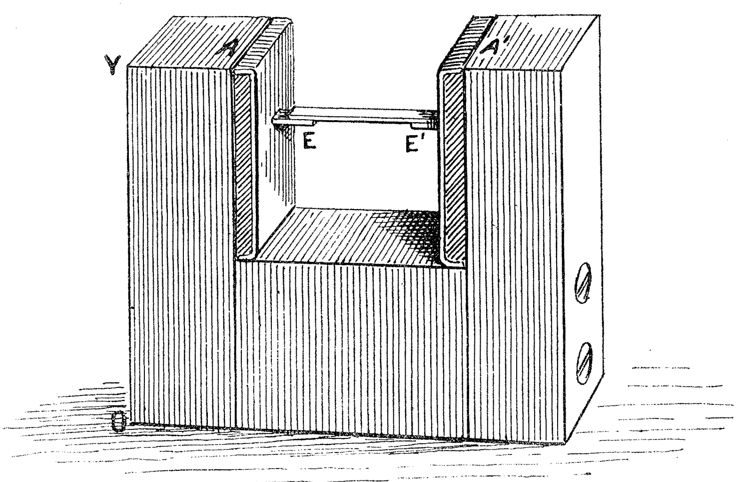
The substance to be examined is formed into a small rectangular beam, rather longer than the distance between the fixed knife-edges $K_1$, $K_2$, and to the projecting ends of the beam the mirrors $M_1$, $M_2$ are cemented. These mirrors are mounted in brass frames, and from the back of each frame a small brass tongue, EE′ (fig. 2), projects, which is the actual part to which the cement is applied.

The two other larger mirrors $M_3$, $M_4$ are inclined to one another at an angle of 45° nearly. They are fixed in a rigid brass mounting, which rests on the horizontal flat surface FF, from which two studs project, so placed that when the mounting of $M_3$, $M_4$ is in contact with both, the intersection of the planes of $M_1$ and $M_2$ is parallel to the knife-edges $K_1$, $K_2$.

The horizontal width of the mirrors $M_1$, $M_2$ is less than half that of $M_3$, $M_4$, and the telescope and collimator are so placed that their axes of collimation graze the vertical edges of $M_1$ and $M_2$.

It is necessary that the planes of $M_1$ and $M_2$ should be nearly, but not quite, parallel, and this is effected by cementing the mirrors to the experimental beam whilst the former are held in the gauge shown in fig. 3.

Fig. 3.




A spring, not shown in the figure, keeps the mirrors pressed against the plane faces A, A′ of the gauge. These are parallel in the vertical direction but inclined to one another about 2′ or 3′ horizontally, *i.e.*, the planes A, A′ intersect at this angle in a line parallel to OY.

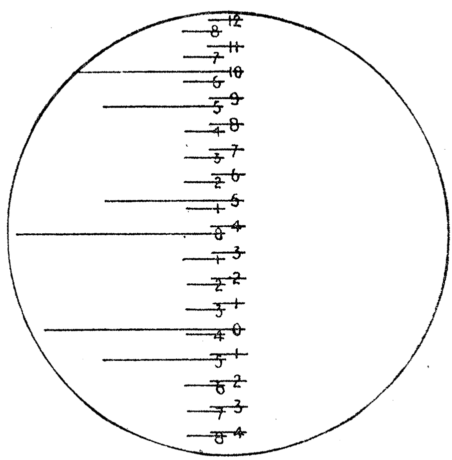
The cement is applied to E and E′ before the mirrors are placed in the gauge; when they are in position, the beam is laid with one end on E and the other on E′, and warmed. When cool, the experimental beam with the mirrors attached is removed from the gauge and laid in the proper position on $K_1$, $K_2$.

On looking through the telescope T, two images of the scales S will be seen side by side. One of these images being formed by successive reflection from the four mirrors $M_2$, $M_4$, $M_3$, $M_1$, in the order named, and the other by reflection from $M_3$ and $M_4$ only.

The course followed by the two sets of rays is indicated by the arrows on the dotted lines in fig. 2.

The appearance of the scales in the field of the telescope is shown in fig. 4. If $M_1$ and $M_2$ were absolutely parallel in a horizontal direc-

FIG. 4.




tion, the scale images would at times exactly overlap one another, which would make readings difficult.

As long as the condition of approximate parallelism of the intersections of the pairs of mirrors is fulfilled, any shifting of one image of the scale past the other is due to an alteration of the angle between one pair or other of the mirrors, and to that alone, and since $M_3$, $M_4$ are in a rigid mounting, any relative motion of the images is due to an alteration in the angle between $M_1$ and $M_2$.

The experiments were made by noting the relative motion of the images when force was applied to the beam by the central knife-edge $K_3$. The course usually followed was to start with no load on $K_3$, and having noted the relative position of the scale images, to move W along D until the scales had moved relatively through one division.

The reading was then taken on D. W being then moved forwards until another scale division had been reached, the position of W on D was again read, and so on. The process was afterwards reversed and readings of the same kind taken when the load was being diminished step by step.

All these results were then plotted, and the curve drawn through the observations gave a measure of the angle between the ends of the beam in terms of the force applied to its middle point. In nearly all cases the lines drawn through the plotted observations were straight lines, within the limits of errors of observation, but, in general, the lines for each substance differed appreciably, according to whether the strains were increasing or decreasing. Some of the plotted diagrams are appended to show the kind of accuracy attained.

The actual linear motion of the central knife-edge was always very small, not in any case exceeding 0·00016 inch.

I pass now to the treatment applied to the experimental results, in order to deduce from them the values of Young's modulus.

If $l$, $b$, $t$ are the length, breadth, and thickness of a beam (originally straight),
$q$ = Young's modulus,
F = Normal force applied at its mid-length,
$\theta$ = Angle made by the tangent at each end with the tangent at its mid-length,

$$\theta = \tfrac{3}{4}\frac{Fl^2}{qbt^3} \qquad \text{and} \qquad q = \frac{3Fl^2}{4\theta bt^3}.$$

If, now, $\delta$ be the distance between the divisions of the scale S,
$n$, the number of divisions through which the images of the scales are relatively displaced, and
$f$ the focal length of the collimator,
the angle observed, $\phi = n\delta/f$.

Now $\phi = 4\theta$, because $2\theta$ is the actual alteration of angle between $M_1$ and $M_2$, and this is multiplied by two by the reflection.

Also, if R be the reading of the position of W on the arm D, and $r$ the distance between $K_4$ and $K_5$, the downward force acting on $K_3$ is

$$F = W\frac{R}{r};$$

hence

$$q = \frac{3WRfl^2}{n\delta rbt^3}.$$

In this expression, for the value of $q$, the factor $fl^2/\delta r$ is a constant, depending only on the apparatus, since $l$ is the distance between $K_1$ and $K_2$.

$1/bt^3$ is a constant for each beam, and $R/n$ is the inclination of the straight line passing through the plotted observations to the axis of $n$.

Hence putting A for $\frac{3fb^2}{\delta r}$, B for $\frac{1}{bt^3}$, and C for $W\frac{R}{n}$,

$$\text{Log } q = \log A + \log B + \log C.$$

The numerical values of quantities involved in A were as follows:—

Focal length of collimator, $f = 8{\cdot}87$ inches.
Length between knife-edges, $K_1$, $K_2$, $l = 0{\cdot}3422$ inch.
Distance between division of scale $\delta = 0{\cdot}01$.
Distance between knife-edges $K_4$, $K_5 = 1{\cdot}0$.

The values employed for $b$ and $t$ varied between 0·1 and 0·01 inch, and for W from 0·02 to 0·25 pound. The chief and indeed the only considerable source of error in these experiments is in the measurement of $t$. The measures were made with a screw micrometer reading to 0·0001 by estimation.

The average value of $t$ was between 0·03 and 0·04, so that the measurement was probably accurate to something like 1 in 400. Hence, there may be an error in $t^3$ approaching 1 per cent.

In the case of crystalline substances, beams cut from the same neighbourhood of the same crystals exhibit a constancy, in the results obtained from them, of this order, but in passing to other specimens more difference was observed.

In many substances, and notably in the case of zinc, lead, and white marble, it was found that the full deflection due to a given load was not reached until a considerable time had elapsed, and experiments with such substances would of course lead to different values for Young's modulus, according as the observations were made in rapid succession or slowly.

The behaviour of zinc in this respect was so marked, that a separate set of observations were made with that material, the results of which are shown in diagrams (10) and (11), pp. 394–395.

It will be seen on examining these diagrams that, starting with a freshly annealed piece of rolled zinc (and similar results were obtained from a beam cut from a large crystal of cast zinc), that, on the first application of the force, the bending immediately produced continues to increase for many minutes, and that, when the load is removed, the beam does not recover itself all at once, and also that a permanent set has taken place.

On the second application of the force, however, if the force is not greater than that first applied, the behaviour of the zinc is quite different. It now very rapidly assumes its maximum deflection and

drops back to its equilibrium position, on the removal of the force, still more rapidly, but little further permanent set being produced.

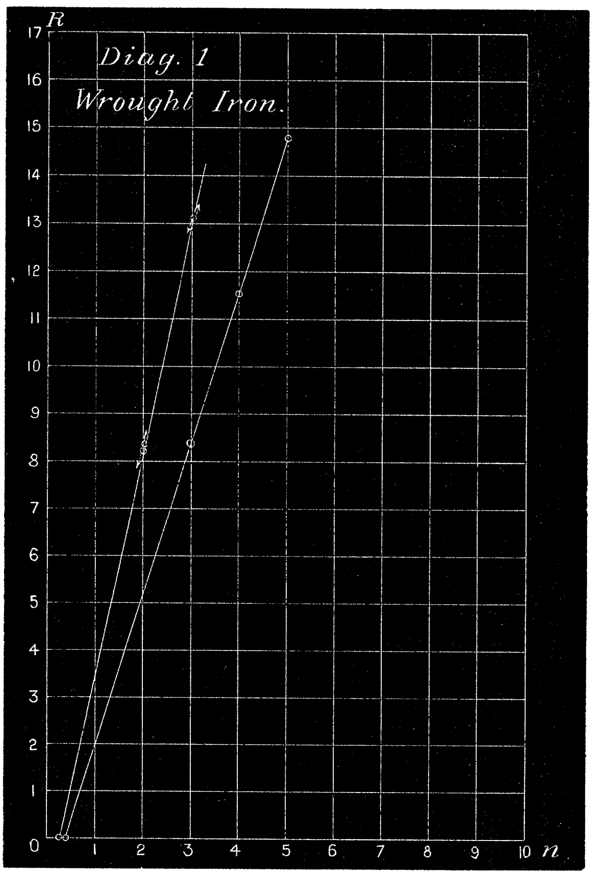
On again increasing the force so as to exceed that first applied, a further gradual extension, lasting a considerable time, takes place; and additional permanent set is found when the force is removed.

Diagram (11) shows that—

(1.) The immediate elastic bending and the permanent set are proportional in amount to the force causing them.

(2.) That the increment of extension or deflection which happens in time is something like a constant quantity.

The method, however, described in this paper is not well adapted to the investigation of these phenomena, as the state of strain in the beam on which they depend varies from + through 0 to − on

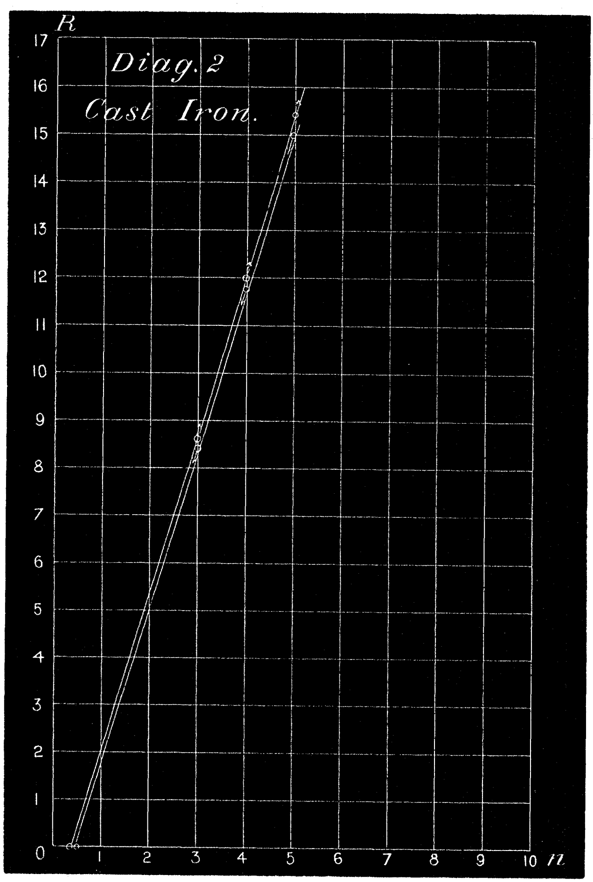


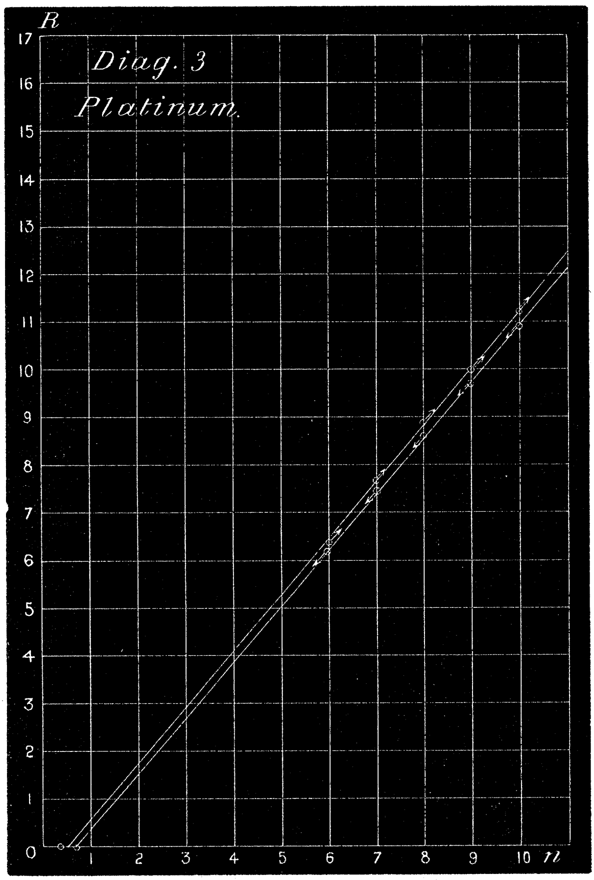
opposite sides of the neutral axis, and one cannot be sure that the very slightly strained material of the central parts acts in the same way as that near the upper and lower boundaries.

In nearly all the substances experimented on, it was found that work was done in bending and unbending the beams, *i.e.*, for a given deflection the load was always less when the latter was being diminished than when it was being increased. This effect was generally more apparent in metals than in hard crystals.

On reference to the Table (p. 398), it will be seen that only ten of the non-metallic substances examined at all approach steel in stiffness.

I regret that I have not hitherto been able to get a specimen of diamond of suitable form for measurement; but I hope to be able to give Young's modulus for this and some other crystals in a supplementary table.




R
Diag. 3
Platinum.
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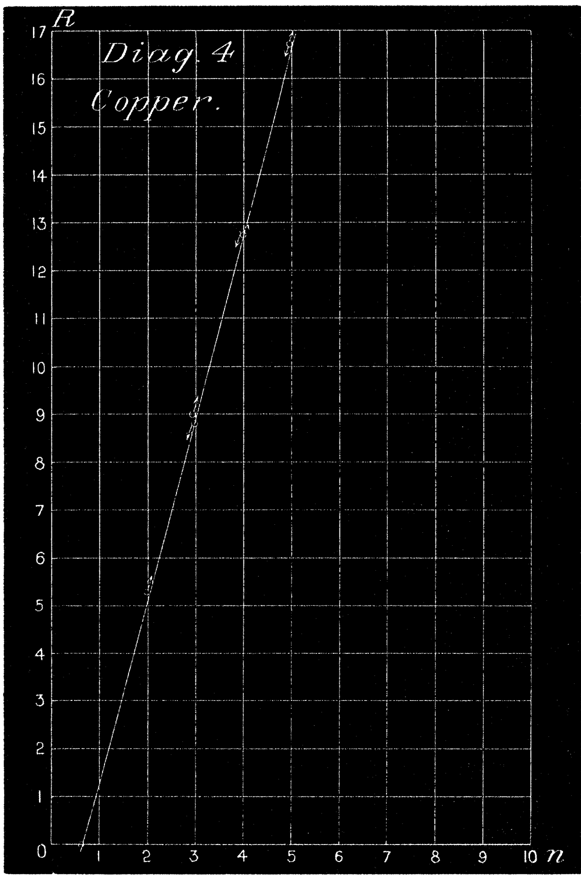
n


Diag. 4
Copper.
R
0
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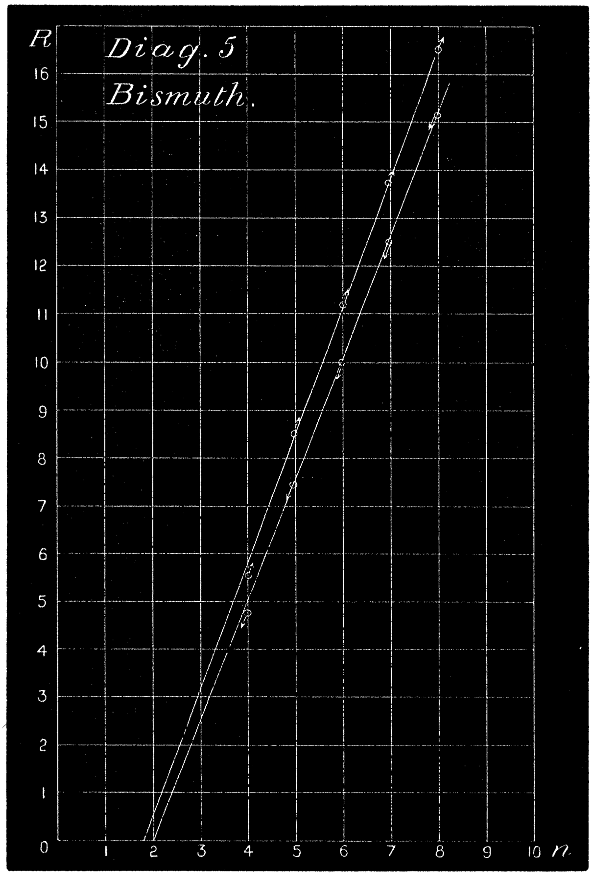
Diag. 5
Bismuth.
R
16
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14
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0
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4
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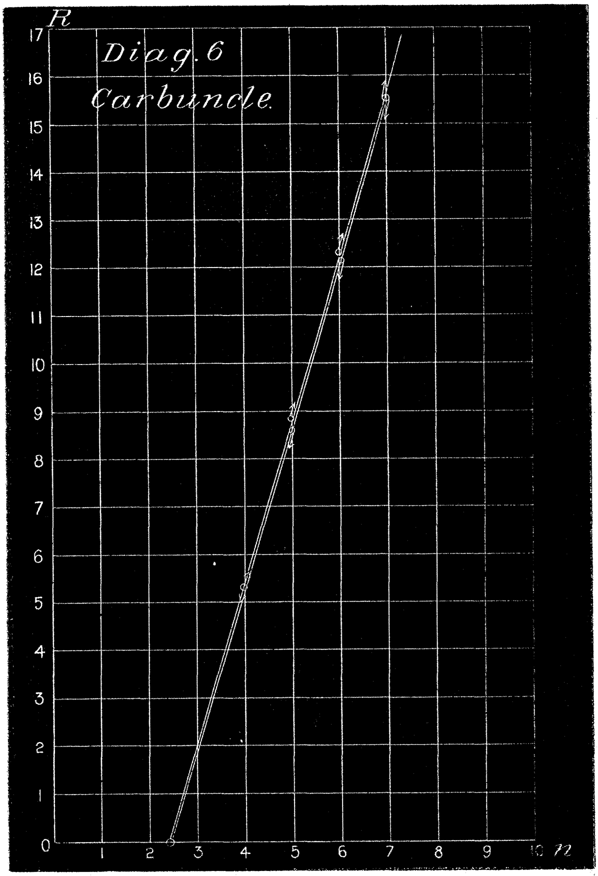
Diag. 6
Carbuncle.
R
n
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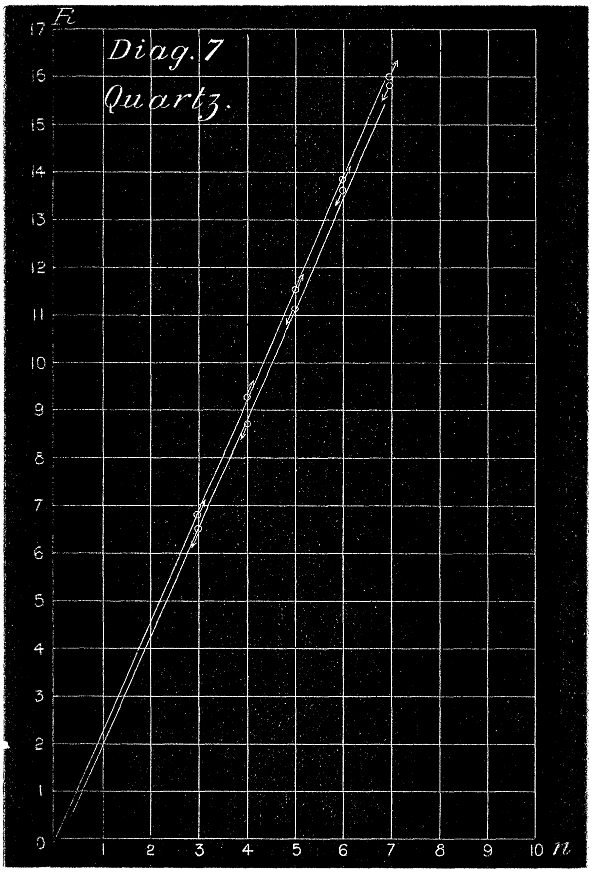
Diag. 7
Quartz.
R
n
0
1
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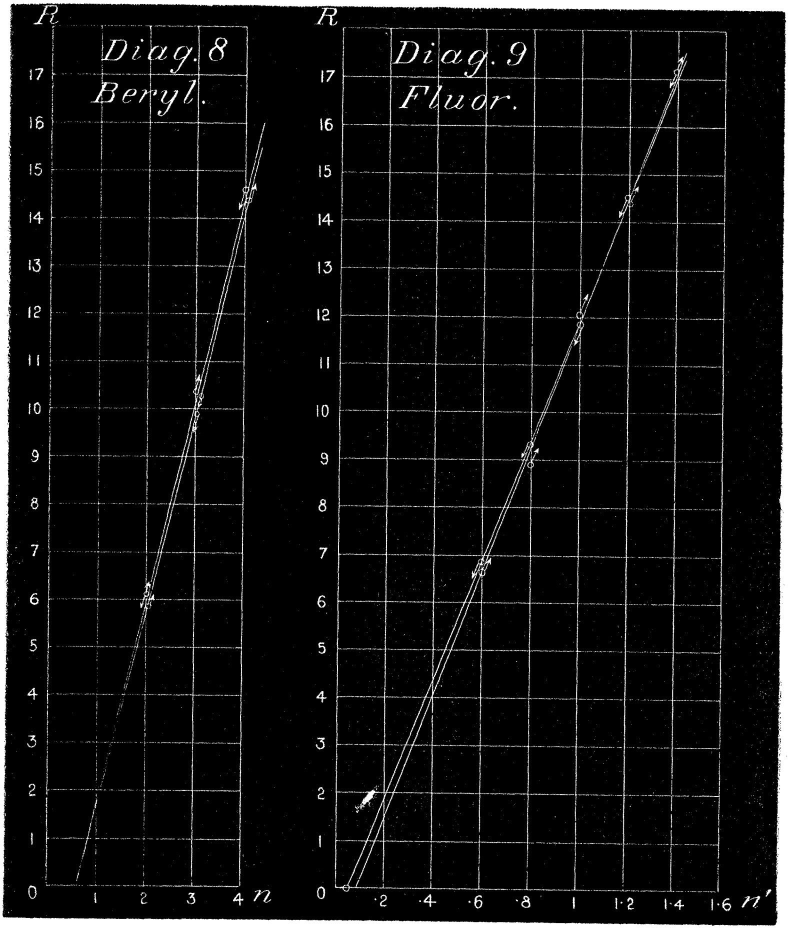
15
16
17



R
Diag. 8
Beryl.
17
16
15
14
13
12
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3
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1
0
1
2
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4 n
R
Diag. 9
Fluor.
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12
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·8
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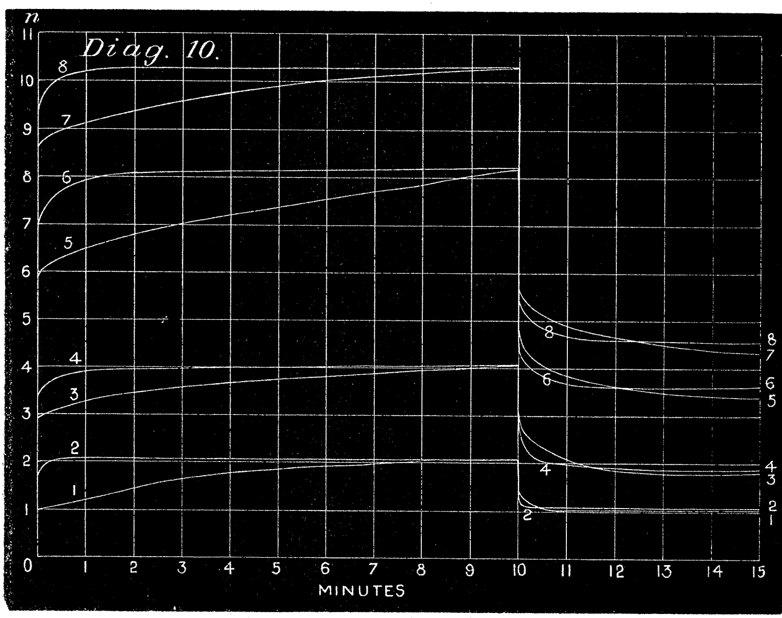
1·2
1·4
1·6 n'


Diag. 10.
n
MINUTES
0
1
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5
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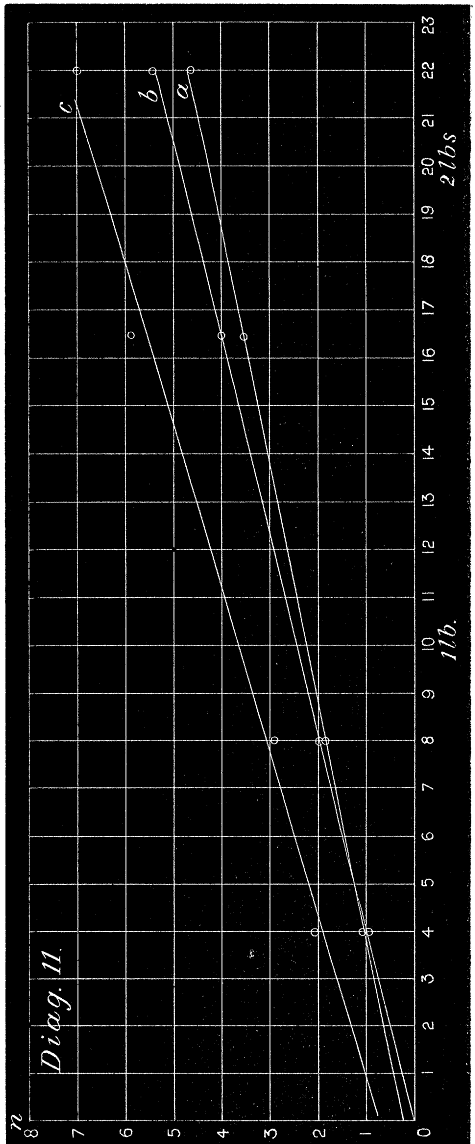
14
15


Diag. 11.
n
0 1 2 3 4 5 6 7 8
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
1lb.
2lbs
a
b
c

*Explanation of the Diagrams.*

Diagrams (1) to (9) are examples of the diagrams used in determining the ratio R/$n$.

The ordinate is the scale-reading on the arm D (fig. 1), and the abscissa the corresponding scale-reading of the images of the glass scale seen in the field of the telescope.

That the line through the observations does not in general point to the origin is due to the fact that the mirrors were not quite parallel in a vertical direction at the beginning of the experiment; in fact, according to the position in which the spring holding the mirrors against the faces of the gauge was placed while they were being cemented to the beam, variations of rather more than a minute of arc were produced between their planes. The abscissa reading when the ordinate = 0 has of course to be subtracted from $n$ in getting the true ratio R/$n$.

The spots indicate individual observations, and are marked with arrows whose directions show whether the load was being increased or diminished.

Diagram (1). $t$ = 0·0302 inch, $b$ = 0·1100 „ } wrought iron.
(*a*) W = 0·2581 lb. (*b*) W = 0·3710 lb.

Diagram (2). $t$ = 0·0322 inch, $b$ = 0·0918 „, W = 0·2511 lb. } cast iron.

Diagram (3). $t$ = 0·0208 inch, $b$ = 0·1075 „, W = 0·0671 lb. } platinum.

Diagram (4). $t$ = 0·0461 inch, $b$ = 0·0955 „, W = 0·1381 lb. } copper.

Diagram (5). $t$ = 0·0473 inch, $b$ = 0·1190 „, W = 0·0671 lb. } bismuth.

Diagram (6). $t$ = 0·0304 inch, $b$ = 0·0757 „, W = 0·0671 lb. } carbuncle

Diagram (7). $t$ = 0·0369 inch, $b$ = 0·0911 „, W = 0·0671 lb. } quartz.

Diagram (8). $t = 0{\cdot}0415$ inch, $b = 0{\cdot}0877$ ,, $W = 0{\cdot}1381$ lb. } beryl.

Diagram (9). $t = 0{\cdot}0292$ inch, $b = 0{\cdot}1065$ ,, $W = 0{\cdot}0217$ lb. } fluor.

In diagram (9) the abscissa is plotted to a different scale, since, owing to the extreme brittleness of fluor spar, a displacement of only about 1·5 of the ordinary division of the scale could be safely use when the thickness of the specimen was about 0·03 inch, and many beams of this substance were broken in attempting to produce greater flexures.

In this experiment the greatest departure of the centre of the beam from its unstrained position is about 0·00001 inch.

Diagram (10) shows some of the properties of zinc. The curves numbered 1 to 8 are really one continuous experiment. A light weight was allowed to act on a beam of sheet zinc, and the deflections caused by it were noted every thirty seconds for ten minutes. These deflections are shown by the first part of curve 1. The weight was then removed, and the recovery of the beam was observed for five minutes. This forms the second part of curve 1.

The experiment was then repeated with the same weight, and the results are shown in curve 2. Additional weight being applied, the same course of procedure gave curves 3 and 4, and in like manner by still further additions of load curves 5, 6 and 7, 8 were obtained.

The dimensions of the beam were :—

$$t = 0{\cdot}0373 \text{ inch}$$
$$b = 0{\cdot}1558 \text{ ,,}$$

| | | | |
|---|---|---|---|
| For curves, | 1, 2 | weight | = 0·4026 lbs. |
| ,, | 3, 4 | ,, | = 0·8052 ,, |
| ,, | 5, 6 | ,, | = 1·657 ,, |
| ,, | 7, 8 | ,, | = 2·110 ,, |

Diagram (11) gives (*a*) the permanent set, (*b*) the immediate elastic deflection, (*c*) the deflection at the end of ten minutes. These are taken from diagram (10).

Table of Values of Young's Modulus.

| Substance. | Young's modulus, lbs. per square inch. | Young's modulus. C.G.S. |
|---|---|---|
| Steel | 33·5 × $10^6$ | 2·311 × $10^{12}$ |
| Wrought iron | 27·0 | 1·863 |
| Platinum | 25·42 | 1·754 |
| Cast iron (soft grey) | 23·31 | 1·608 |
| Copper | 17·65 | 1·218 |
| Brass | 16·38 | 1·130 |
| Cobalt | 12·89 | 8·895 × $10^{11}$ |
| Aluminium | 8·73 | 6·025 |
| Bismuth* | 4·16 | 2·87 |
| Lead | 2·71 | 1·87 |
| Zinc† | 1·4 to 0·89 | 9·7 to 6·1 × $10^{10}$ |
| Carbuncle‡ | 34·83 × $10^6$ | 2·430 × $10^{12}$ |
| Carbuncle‡ (another specimen | 34·38 | 2·372 |
| Beryl‡ | 30·9 | 2·076 |
| Tourmaline§ (*a*) | 18·76 | 1·294 |
| Smoky topaz‡ (*a*) | 17·5 | 1·207 |
| Fluor‖ | 17·39 | 1·200 |
| Fluor¶ | 17·18 | 1·185 |
| Yellow topaz‡ | 16·38 | 1·130 |
| Yellow topaz‡ | 13·79 | 9·515 × $10^{11}$ |
| Yellow topaz‡ (*b*) | 12·75 | 8·80 |
| Tourmaline§ (*b*) | 11·79 | 8·135 |
| Quartz** | 10·82 | 7·46 |
| Hard white glass | 10·09 | 6·96 |
| Agate | 9·25 | 6·381 |
| White "Arkansas" stone†† | 8·45 | 5·83 |
| Selenite‡‡ | 7·98 | 5·505 |
| "Extra dense" flint glass | 7·48 | 5·165 |
| Bluish marble | 4·64 | 3·20 |
| White marble | 1·6 | 1·1 |

* Cast bismuth. The beam cut parallel to a natural crystalline cleavage of the metal.

† The greatest value is that obtained from observations taken in rapid succession.

‡ Relation of the faces of the beam to the crystallographic axes not known. The specimens marked (*b*) are cut at right angles to those marked (*a*).

§ A very black opaque crystal from the Ural; (*a*) cut parallel to the side of the prism, (*b*) normal to the sides.

‖ Parallel to diagonal of the cubic crystal.

¶ Parallel to face of cube.

** Parallel to sides of prism.

†† A very close-grained oilstone.

‡‡ Parallel to the principal cleavage.

FIG. 1.

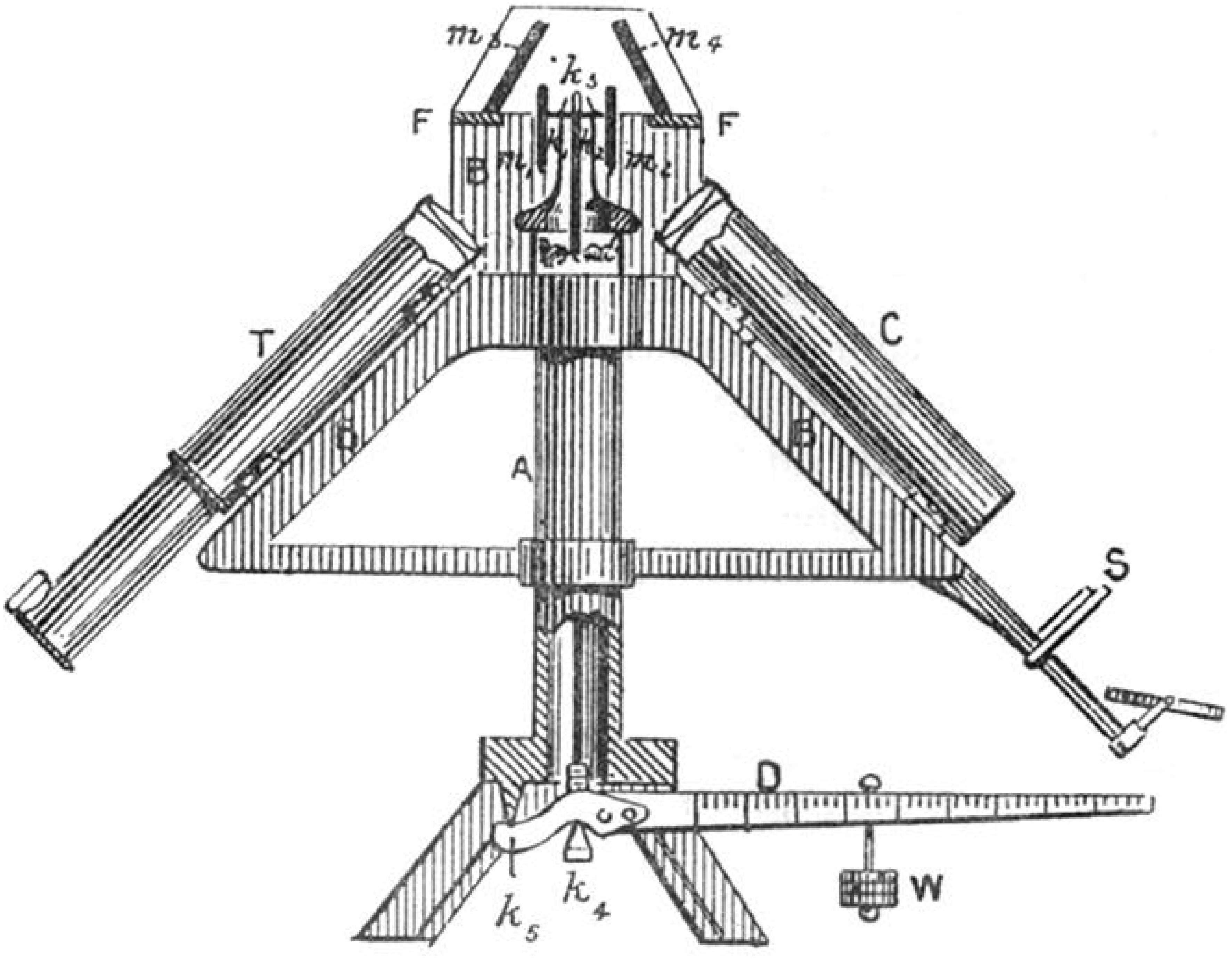
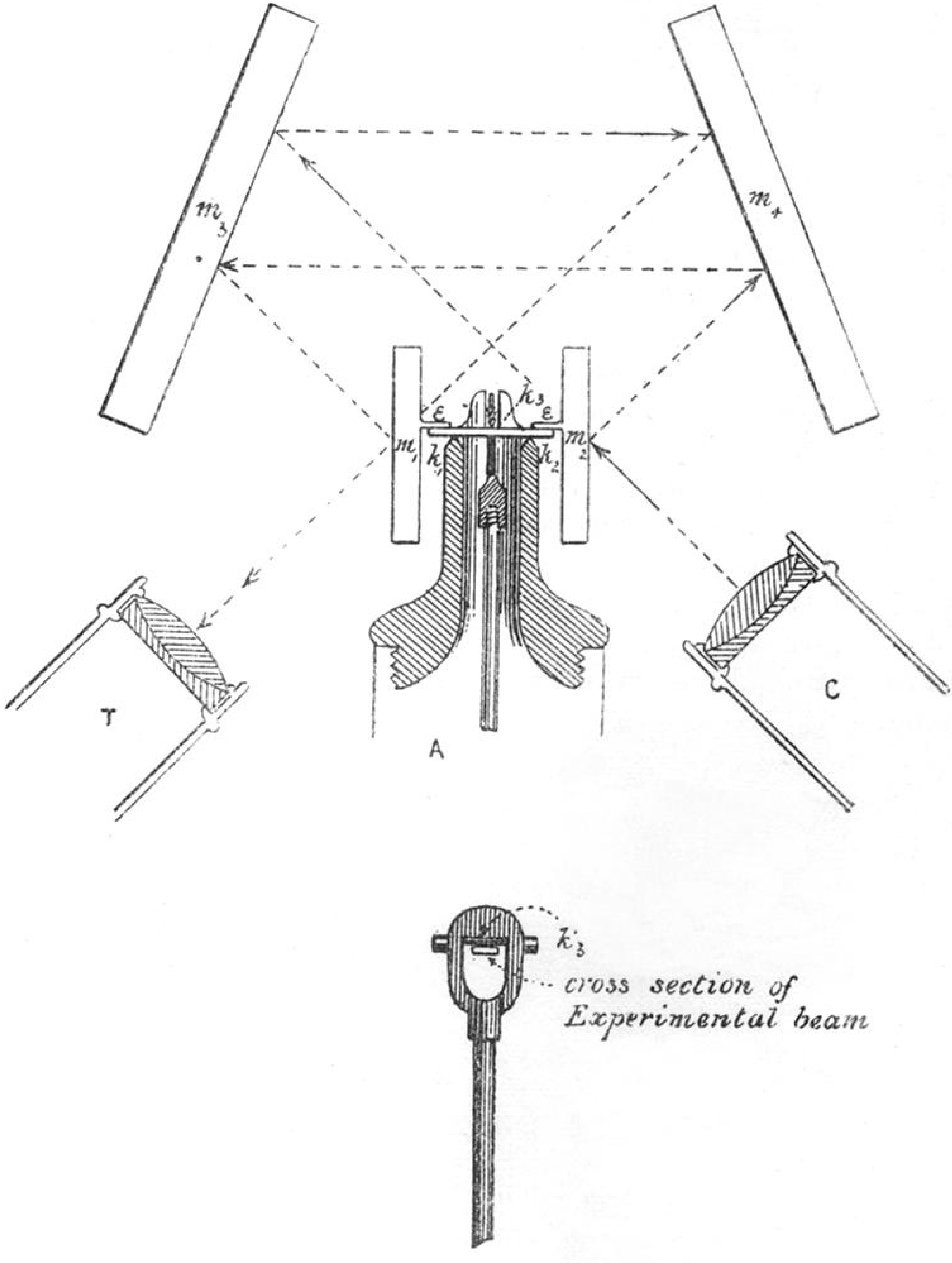
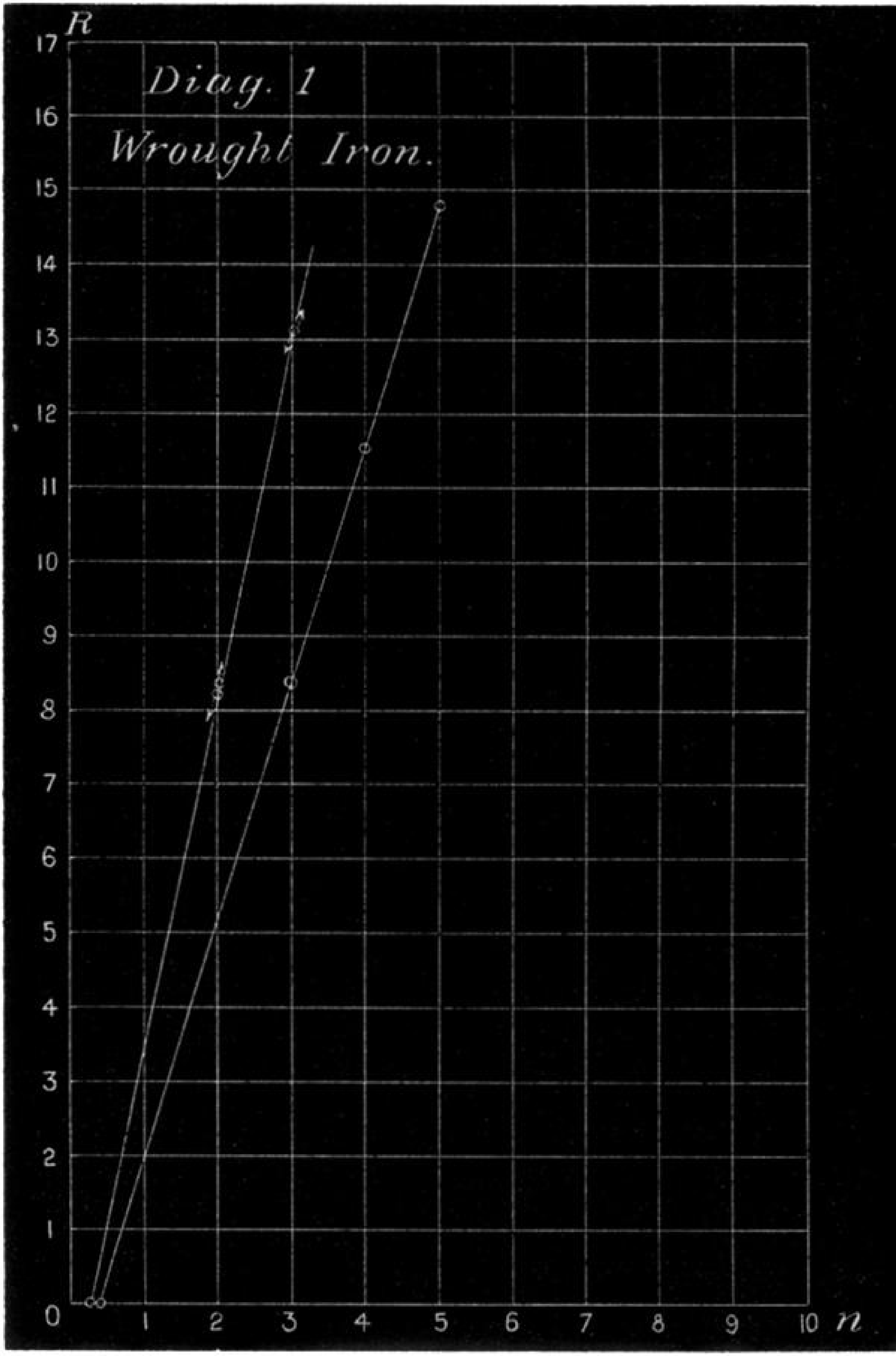


FIG. 2.



m3
m4
ε
k3
ε
m1
k1
k2
m2
T
A
C
k3
cross section of
Experimental beam



Diag. 1
Wrought Iron.
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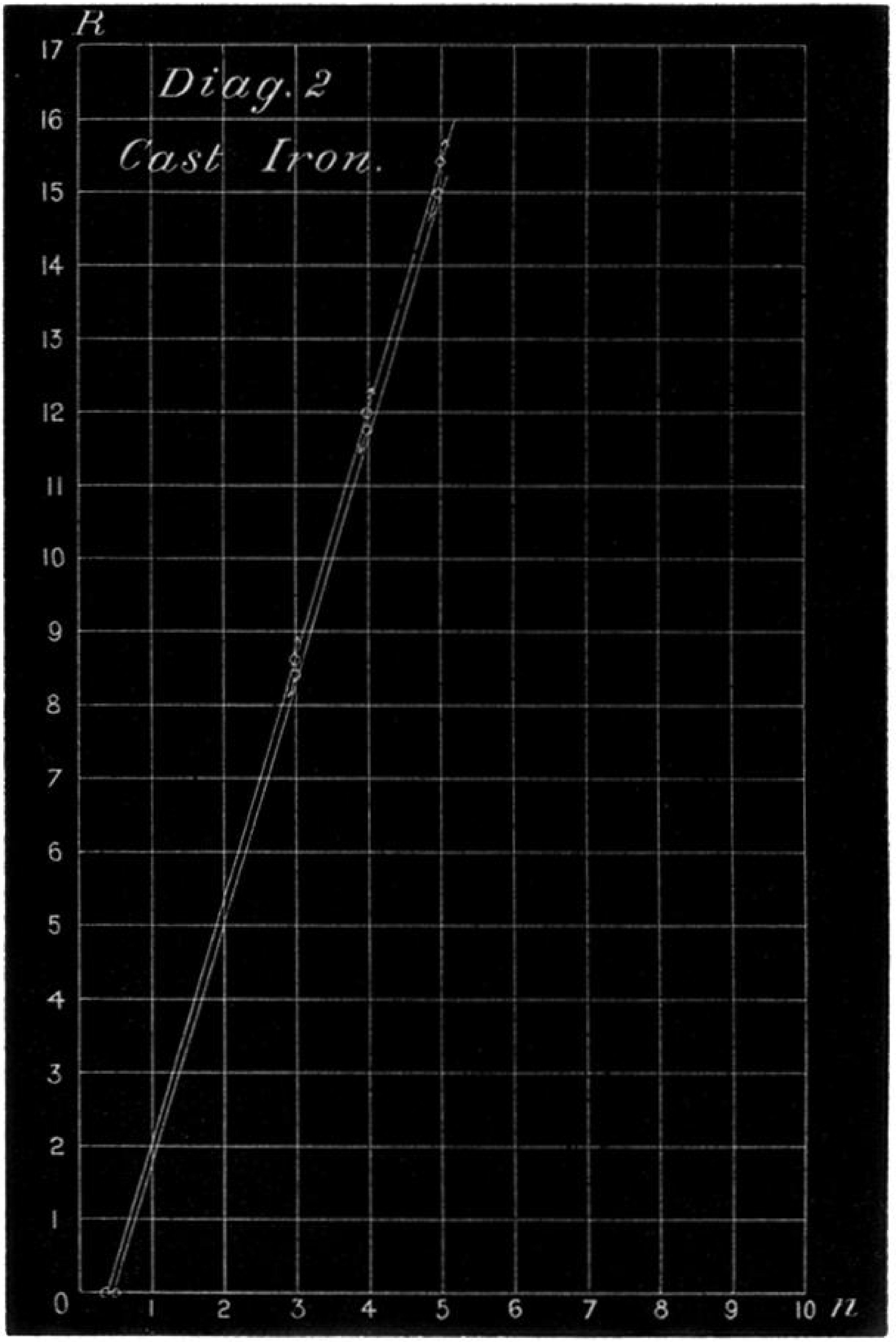

R
Diag. 2
Cast Iron.
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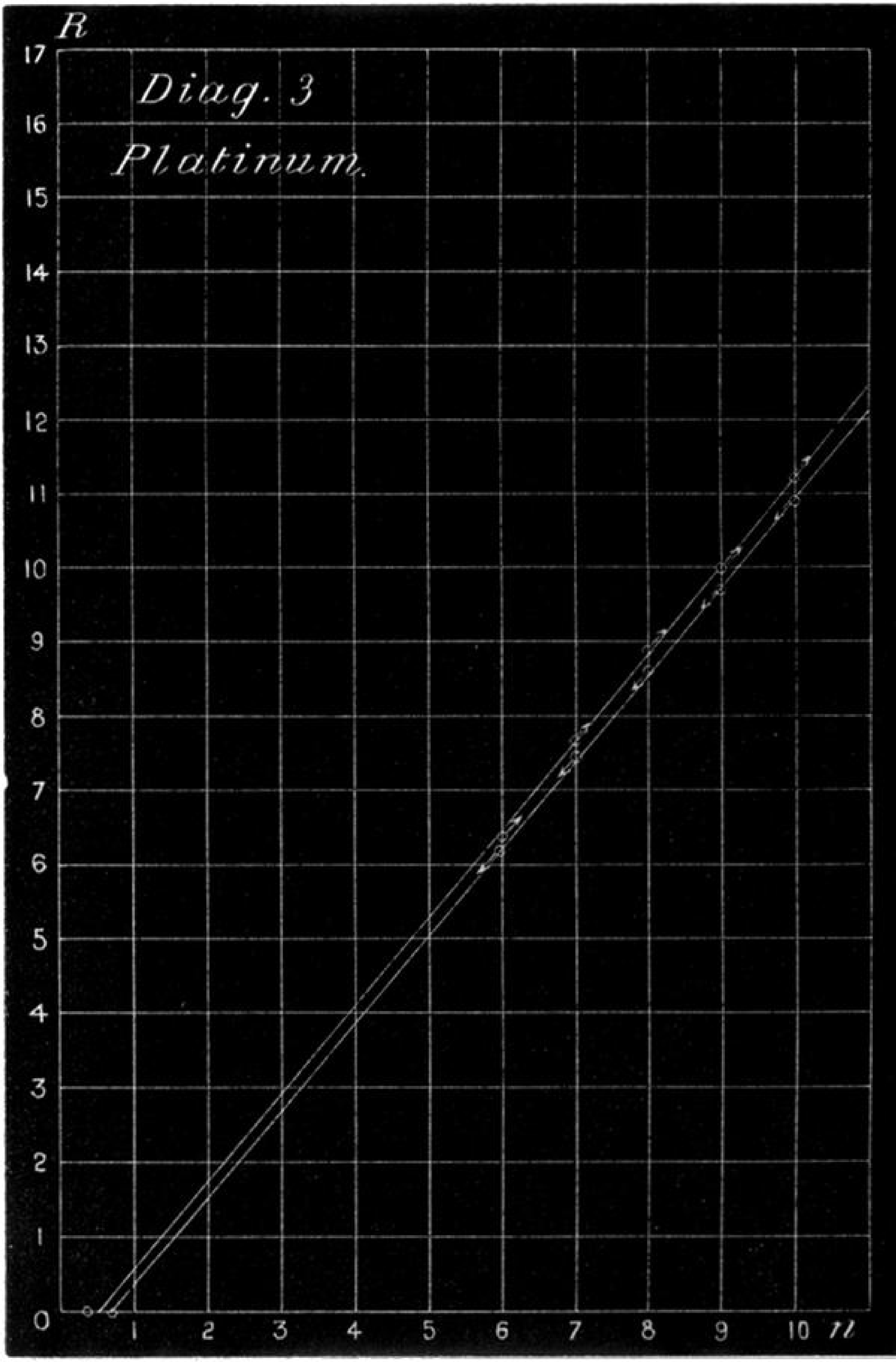
Diag. 3
Platinum.
R
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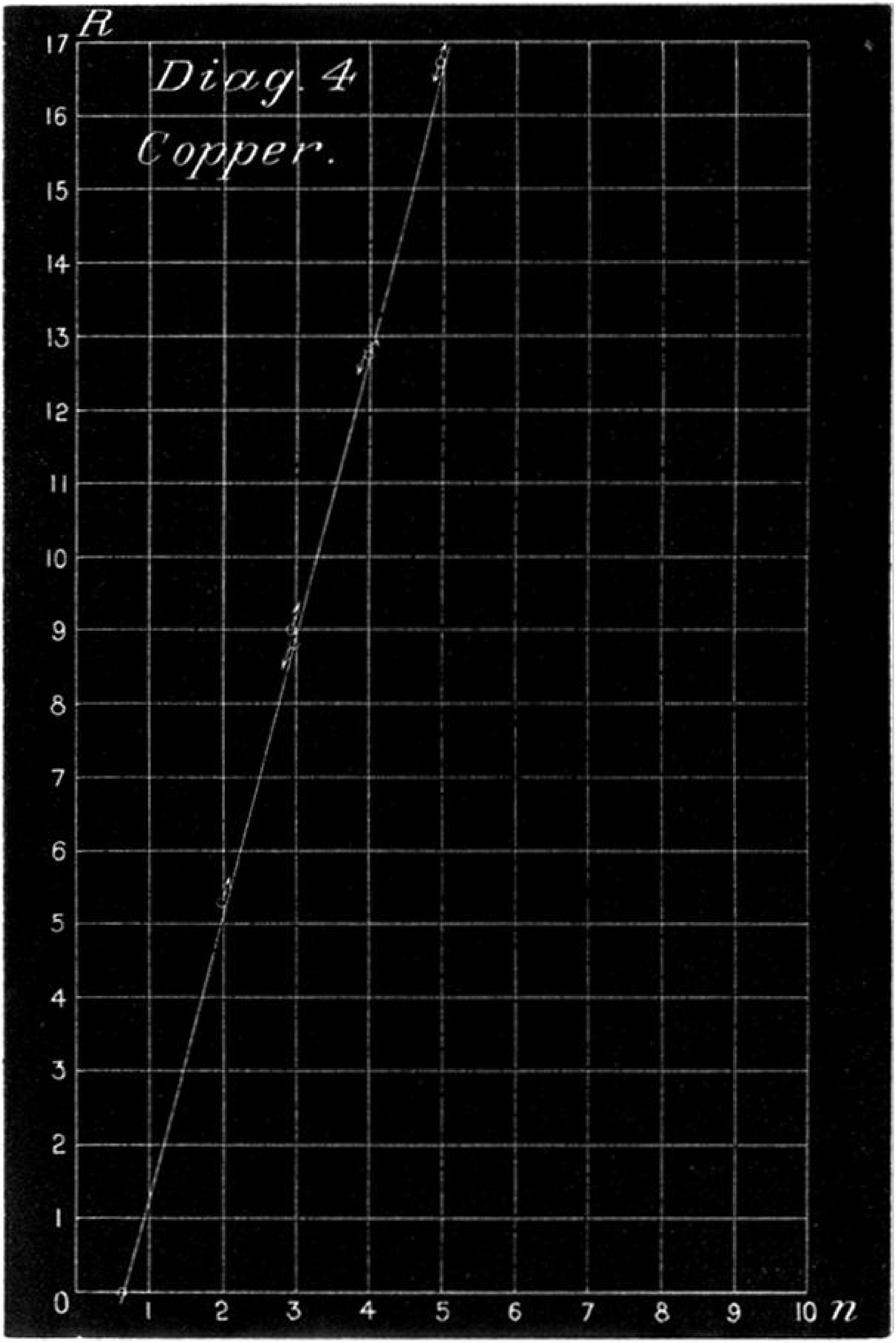
Diag. 4
Copper.
R
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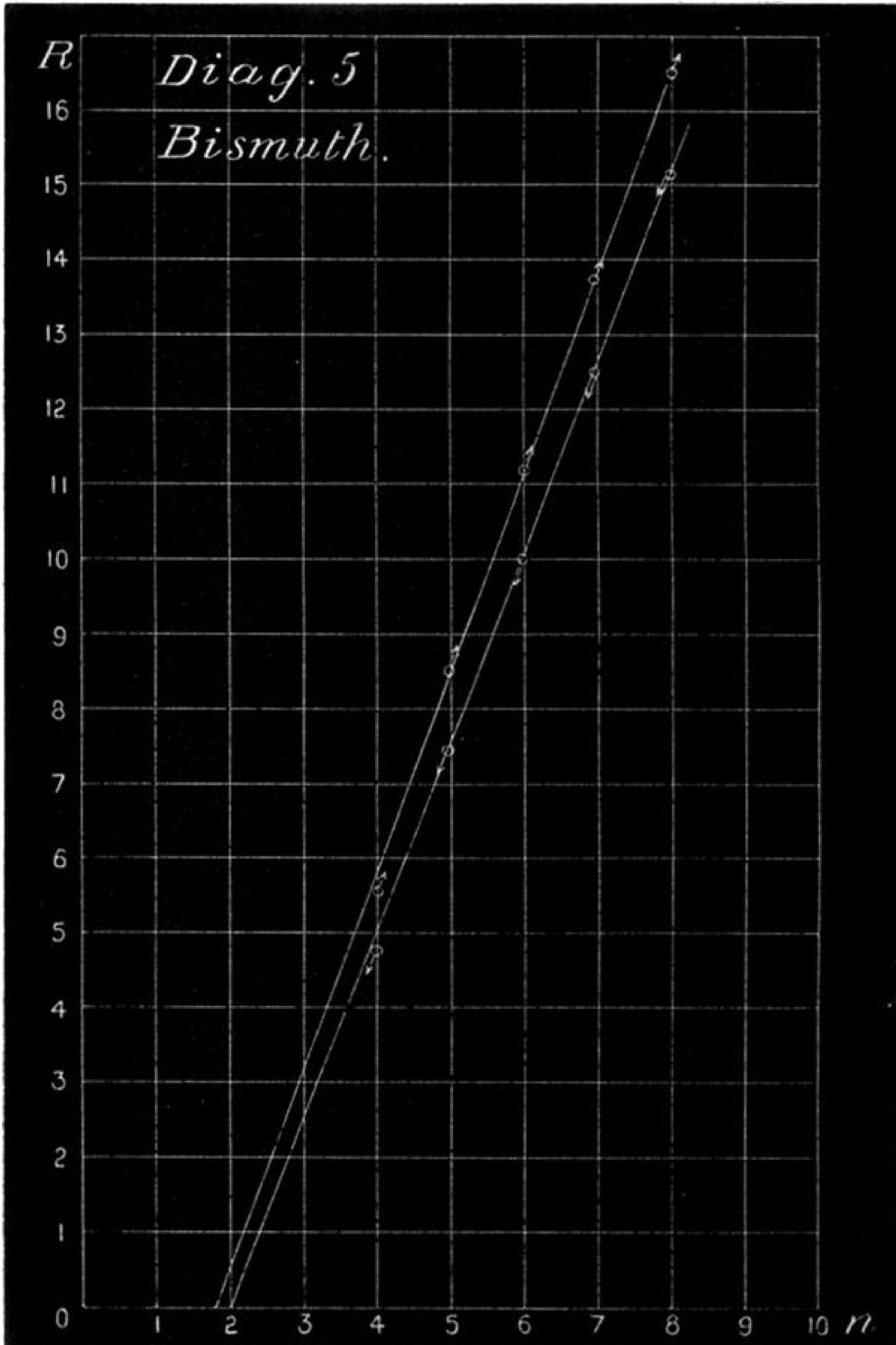
Diag. 5
Bismuth.
R
16
15
14
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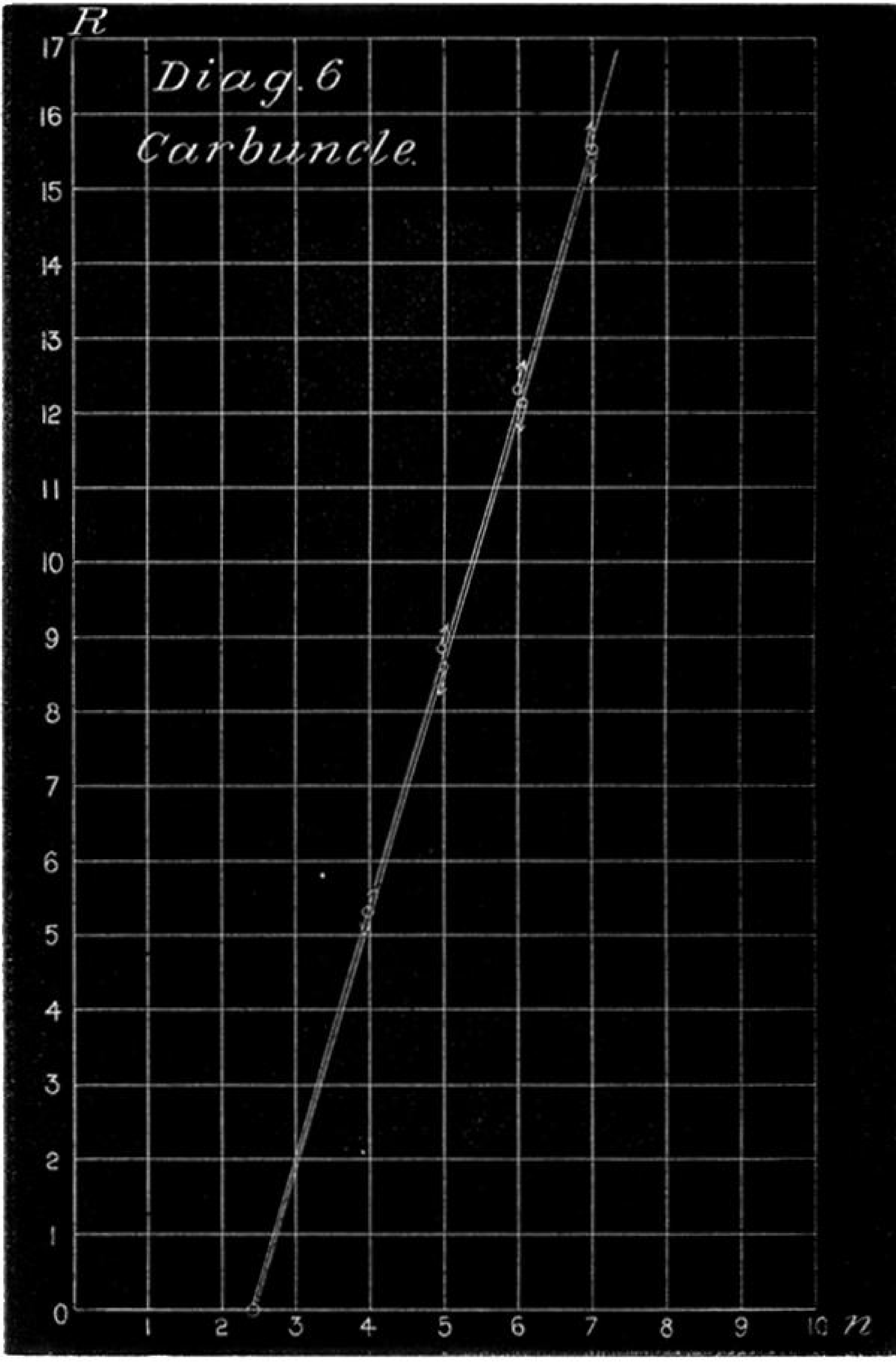
Diag. 6
Carbuncle.
R
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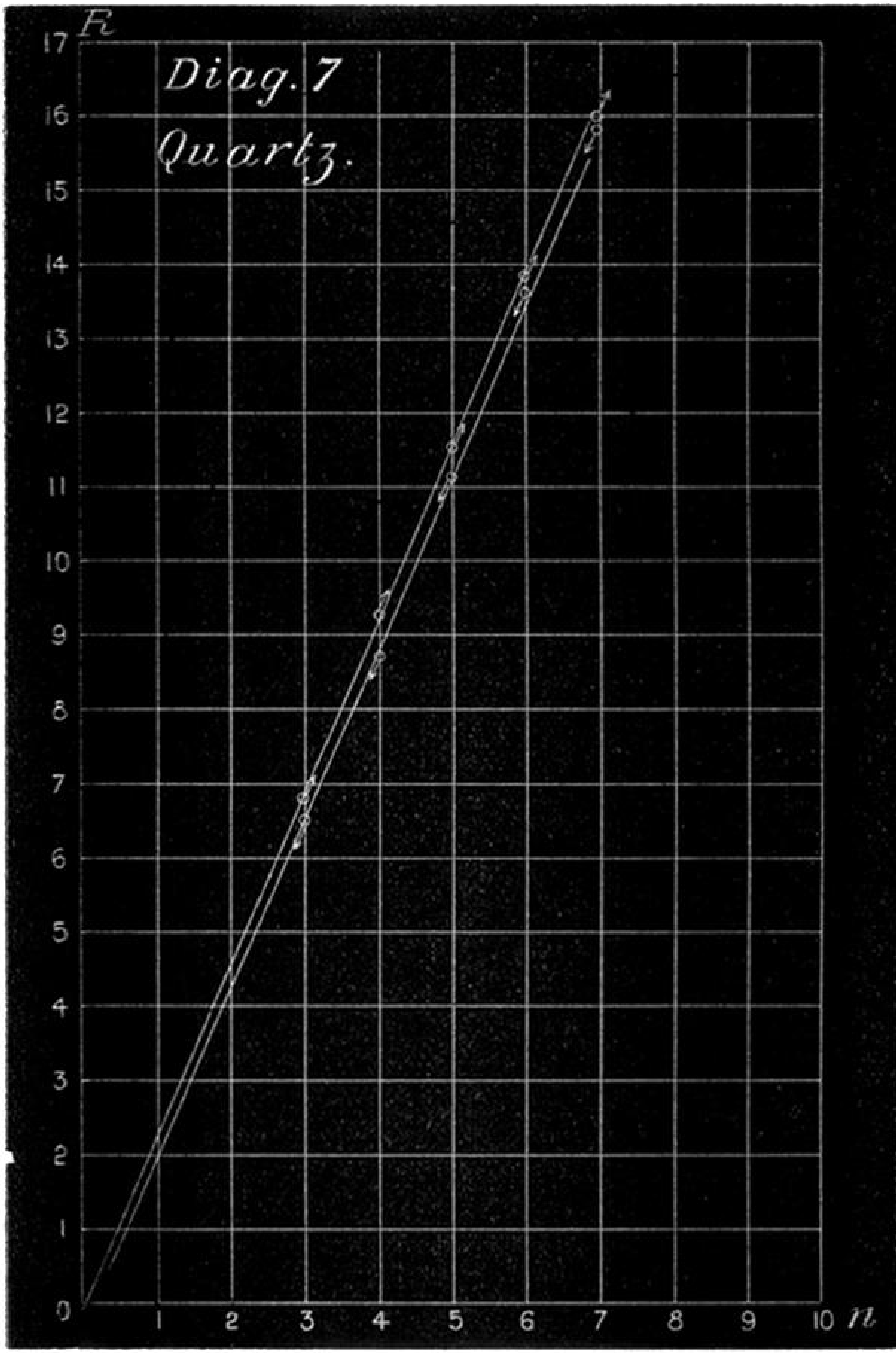
Diag. 7
Quartz.
R
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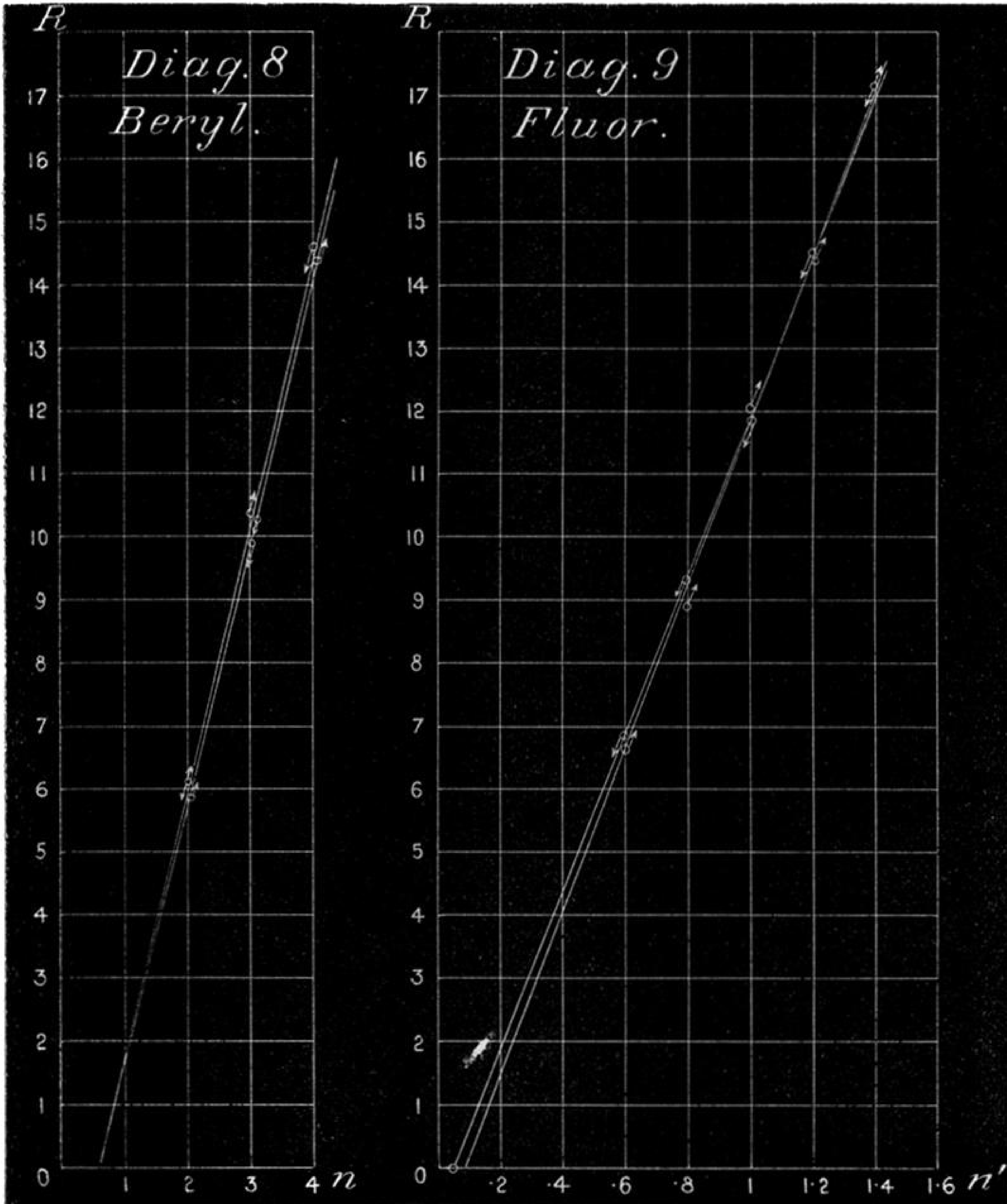

Diag. 8
Beryl.
Diag. 9
Fluor.
R
R
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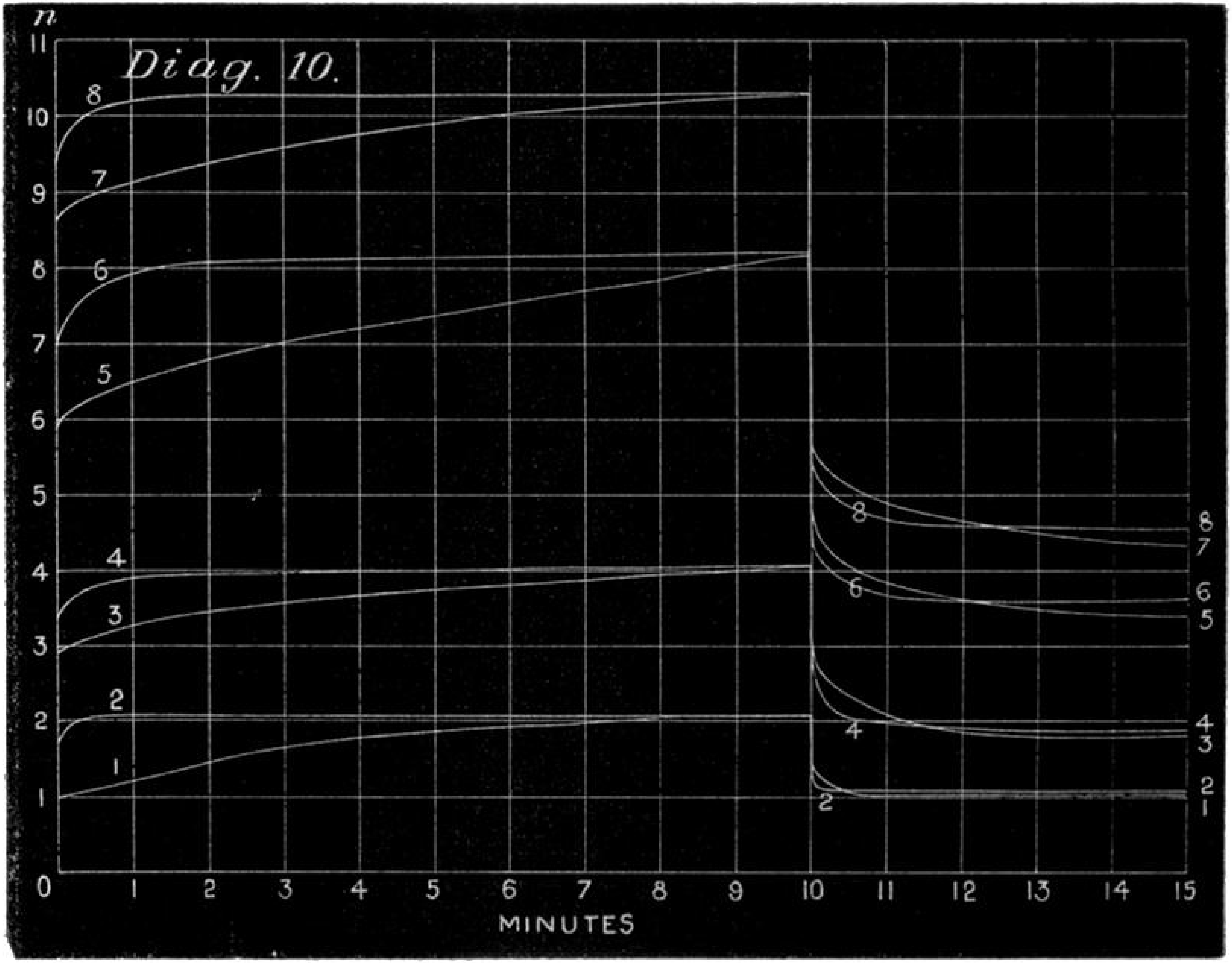

Diag. 10.
n
MINUTES
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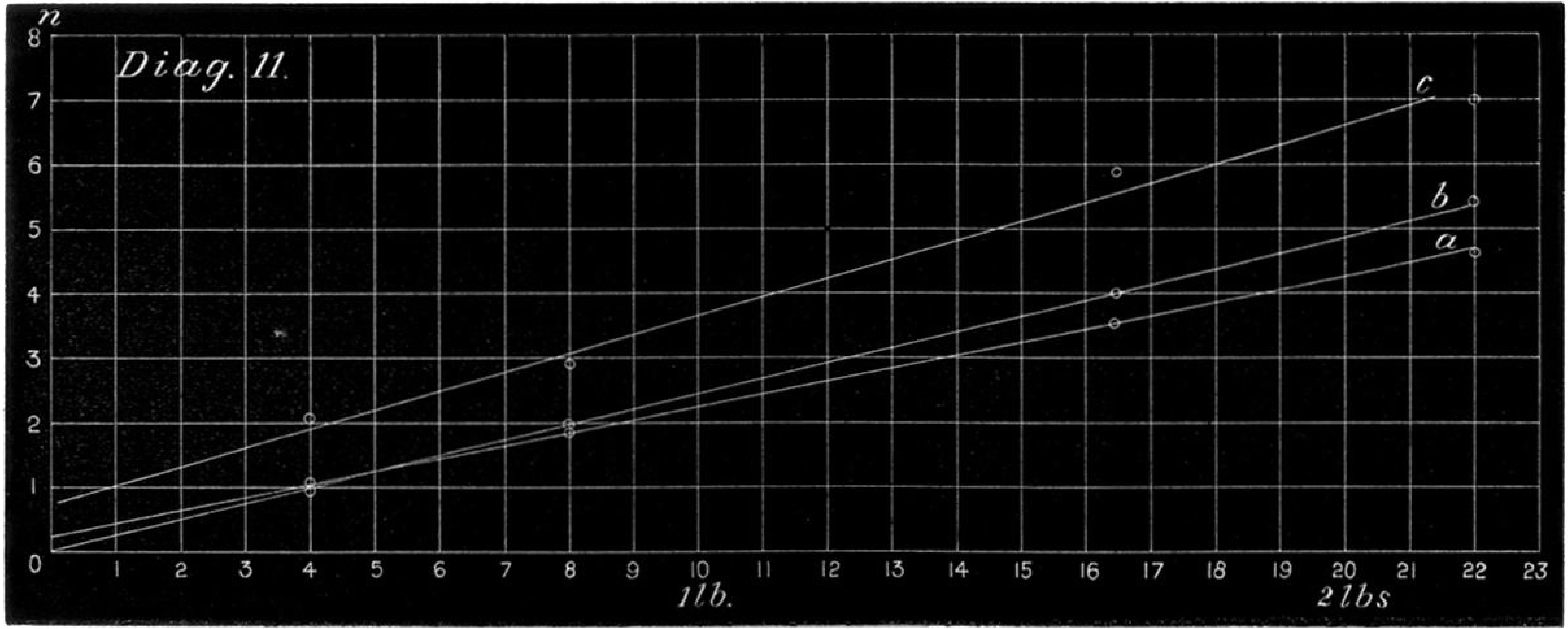

Diag. 11.
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17
18
19
20
21
22
23
a
b
c
1lb.
2lbs