"On the Relation between the Magnetic Permeability of Rocks and Regional Magnetic Disturbances." By A. W. Rücker, M.A., F.R.S. Received May 30,—Read June 19, 1890.

The investigation, of which an account is given in this paper, was undertaken with the object of throwing light on the causes of local magnetic disturbances. The two main theories which have hitherto been proposed attribute local perturbations of the needle to earth currents and to magnetic rocks respectively.

In the Bakerian Lecture for 1889 ('Phil. Trans.,' A. 1890, p. 53), Dr. Thorpe and I compared the directions of the disturbing magnetic forces found by us to exist near Melton Mowbray and Reading with the results of a survey of the local earth currents made in the neighbourhood of those places under the direction of Mr. Preece, F.R.S. No connexion could be traced between either the intensities or the directions of the currents and the magnetic forces, and the result of the investigation was thus opposed to the view that they are cause and effect. As far as I am aware, however, no attempt has hitherto been made to determine whether the mere presence in the earth's magnetic field of such iron-bearing rocks as actually exist, and which must certainly produce magnetic disturbances, suffices to account for such disturbances as are actually observed. This enquiry is obviously complementary to the comparison of the disturbing forces with the earth currents in the same neighbourhood, but the necessary data have only lately been accumulated. The recent magnetic survey has for the first time placed at our disposal facts as to the magnitude of the disturbing magnetic forces in the United Kingdom, and the measurements described below give us some idea of the order of the magnitude of the permeabilities of magnetic rocks. The present investigation is thus divided into two parts, viz.:—

(1.) A determination of the magnetic susceptibility of a number of rock specimens.

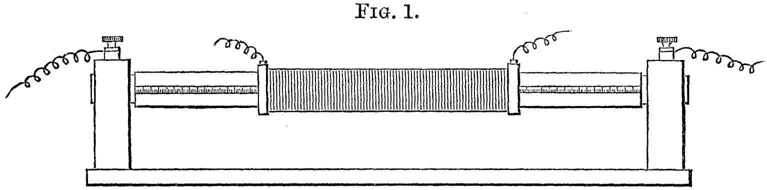
(2.) An enquiry as to the order of the magnitude of the magnetic disturbances which the presence of such rocks in the earth's magnetic field would produce.

The first part has been carried out by Mr. Highfield, Assoc. N.S.S., and Mr. Jarratt, Scholar Elect of Trin. Coll., Cambridge, both of whom are students in the Physical Laboratory of the Normal School of Science and Royal School of Mines. I am under a great obligation to these gentlemen for their share in the work. They constructed all the special apparatus required, and have made all the measurements recorded in this paper, and I am indebted to them not only for care and skill, but also for several very useful suggestions.

In selecting a method, it was important to be able to deal with small fragments of rock, and also to avoid the necessity of having to shape them into definite forms. The magnetic properties of a district can only be ascertained by the examination of a large number of specimens, and this would be practically impossible unless the above conditions were fulfilled. Great accuracy, though desirable, is not equally important. Different specimens of the same rock differ so widely in their magnetic qualities that a comparatively rough measurement is sufficient. It will, however, be seen in the sequel that the observations made are in satisfactory agreement.

To meet these requirements the following scheme was devised. A series of standard magnetic fluids were made by suspending magnetic oxide of iron in various proportions of glycerine. The susceptibilities of these mixtures were determined absolutely by the apparatus described below, and specimens of the rocks were compared with them by means of Professor Hughes' induction balance. For this purpose, equal volumes of a mixture were placed in two similar test-tubes, which were inserted in the cups of the balance, and silence was obtained by means of a compensator. The rock to be tested was

FIG. 1.



now immersed in one of the mixtures, and an equal volume of liquid having been abstracted, the zero was redetermined. Two mixtures were thus found, to the susceptibilities of which that of the rock under experiment was intermediate. The compensator used, though identical in principle, differed in form from that employed by Professor Hughes. The primary current passed through two solenoids wound in opposite directions about the two ends of a tube. Over these another larger tube could be moved in either direction, and round it another solenoid was wound which formed part of the secondary circuit. The position of this secondary solenoid was read off on a millimetre scale attached to the exterior of the inner tube. The two primary coils tended to produce induced currents in opposite directions, and thus, by moving the secondary coil in one direction or the other, silence could be obtained.

If x_1 and x_2 are the distances (measured in opposite directions) through which the compensator had to be moved to produce silence when the specimen was introduced into the first and second liquid

respectively, and if k, k_1, and k_2 are the susceptibilities of the rock and the two liquids, k may be calculated from the formula

$$\frac{x_1}{x_2} = \frac{k-k_1}{k_2-k},$$

or

$$k = k_1 + \frac{x_1}{x_1+x_2} \times (k_2-k_1).$$

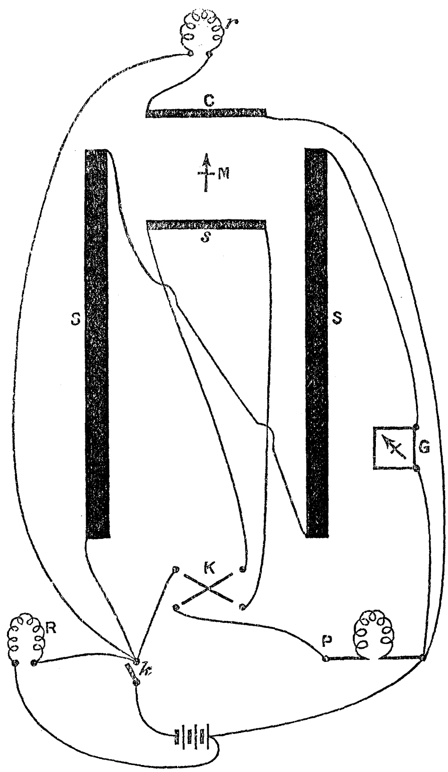
My thanks are due to Professor Judd, F.R.S., for the large number of rock specimens which he has placed at my disposal. Nearly all which have been used have been provided by him. This aid has been especially important from the fact that the attention which he has paid to the geology of the West of Scotland has made his collections rich in basaltic rocks gathered from that district, which is within the area of the recent magnetic survey, and is remarkable for the magnitude, not only of the local, but also of the regional, magnetic disturbances of which it is the seat. He has also been good enough to have sections made of a number of the rocks examined by the induction balance. Some of the results thus obtained are referred to below, but we hope to extend this part of the enquiry in the immediate future.

Part I.—*On the Magnetic Susceptibility of Rocks.*

The apparatus used for determining the absolute susceptibilities of the mixtures is shown in fig. 2.

Primarily it consisted of a magnet attached to a mirror which was delicately suspended in a glass case by means of a quartz fibre. Two large solenoids wound upon glass tubes were placed at equal distances east and west of the needle. A smaller tube, which could be filled with any of the mixtures, was arranged so as to slide into and out of either solenoid. The deflections when the tube was inserted first into one solenoid and then into the other were noted, and from these the susceptibility of the mixture could be calculated when the strength of the field in which the magnet was suspended was known. The necessity for the determination of this datum was obviated by also deflecting the magnet by a small solenoid, the moment of which was calculated from the number of coils, the length, and the strength of the current flowing through it. In the figure, M represents the magnetometer, and SS the two large solenoids into which the tube containing the liquid is capable of sliding. The ends of the solenoids projected for some 7 or 8 cm. beyond the ends of the tube, which was always brought up to the same position within the solenoids by means of fiducial marks. The sensitiveness of the magnetometer was adjusted by a control magnet placed above it, acting so as to reduce the earth's

Fig. 2.



field. In this way an oscillation-period of 25 seconds could be obtained, although it was found in practice to be unnecessary to increase it beyond about 10 seconds. At *k* there is a key which divides the current, part of which goes through the two large solenoids SS, flowing in the same direction in each, and thence through the galvanometer G back to the battery. Another part goes through the reversing key K to the small deflecting solenoid *s* and thence through the Post Office bridge P back to the battery. The resistance used in the Post Office bridge was about 200 ohms, while that of the main circuit was about 10 ohms, so that the amount of current shunted was comparatively small; nevertheless, to prevent any error being thus caused, another shunt circuit, of resistance R, equal to that in the Post Office bridge, was introduced, through which the current passed when the small solenoid was not in use.

As the effects produced by the two principal solenoids on the magnetometer were not exactly balanced when they were placed at equal

distances from it, it was found necessary to introduce another small compensating solenoid (C) which could be so adjusted as to neutralise the residual effect.

The mirror was raised sufficiently above the horizontal plane through the axes of the two solenoids to permit a beam of light being thrown upon it and reflected to a scale. An experiment was performed as follows :—

The small solenoid circuit s was first thrown out of connexion, and the main circuit, together with the shunt R, put in. The zero reading, as given by the magnetometer, having been taken, the apparatus was so arranged that the zero did not alter when the current was put on, or off, or reversed. The tube was now inserted into each of the solenoids in turn, the deflections were noted, and afterwards the zero was again taken to show that no change had occurred during the experiment. The shunt R was then thrown out, and the circuit containing the small solenoid s put in, the resistance in the Post Office bridge having been previously arranged so as to give a deflection of much the same magnitude as that due to the introduction of the tube containing the mixture of magnetic oxide and glycerine. Knowing the deflection which is given by a solenoid of known moment, and that produced by the introduction of a definite amount of mixture under the same conditions as regards the sensibility of the magnetometer, we are able to calculate the susceptibility of the mixture, as follows :—

Let $2b$ be the length of the tube.

ξ, η, ζ the co-ordinates of the centre of the magnet referred to three rectangular axes, passing through the middle point of the axis of the tube, parallel to its length, and perpendicular to its length in the horizontal and vertical directions respectively.

σ, σ_1, the distances from the centre of the magnet of the feet of the perpendiculars let fall from the ends of the tube on the horizontal plane through the magnet.

It is then easy to show that if the length of the magnet is small as compared with σ and σ', and if p be the strength of the pole induced at the end of the tube,

$$p = \frac{\mathrm{F} . d}{(\mathrm{M}d + 2\mathrm{N} . \eta . \mathrm{D})},$$

where F is the strength of the field in which the magnet is placed,

d the deflection produced by the introduction of the tube containing the liquid into the solenoid,

D the distance of the scale from the magnet,

$$M = \frac{\xi - b}{(\sigma^2+\zeta^2)^{\frac{3}{2}}} - \frac{\xi + b}{(\sigma_1^2+\zeta^2)^{\frac{3}{2}}}$$

and
$$N = \frac{1}{(\sigma^2+\zeta^2)^{\frac{3}{2}}} - \frac{1}{(\sigma_1^2+\zeta_1^2)^{\frac{3}{2}}}.$$

With the distances actually adopted in one experiment,

$$M = -0{\cdot}000273, \quad d < 10\text{cm.},$$

$$N = 0{\cdot}000170, \quad \eta = 17{\cdot}0 \text{ cm.}, \quad D = 103 \text{ cm.}$$

Hence, the term $M.d$ is small, and may be neglected, and, therefore,

$$p = \frac{Fd}{2.N.\eta.D}.$$

If I be the intensity of magnetisation of the liquid,
A the area of the tube,
n the number of turns per cm. in the solenoid,
C the strength of the current in absolute units,
κ the susceptibility of the liquid,

$$\kappa = \frac{I}{4.\pi.n.C} = \frac{F.d}{8.\pi.n.C.N.\eta.D.A},$$

where $8.\pi.n.N.\eta.D.A$ is constant for any one position of the apparatus, and $= N_1$ say;

therefore
$$\kappa = \frac{F.d}{N_1.C}.$$

The solenoids were practically identical in construction, and if $d_1 - d_2$ be the algebraical difference of the deflections right and left when the tube is introduced into the two solenoids,

$$\kappa = \frac{F(d_1 - d_2)}{2.N_1.C}.$$

To determine F the auxiliary solenoid was used. It was placed perpendicular to the axis of the magnet, which bisected it. Its length being $2a$, number of turns per cm. n', area A', the distance of its poles from the centre of the magnet u, and the deflections being d'_1 and d'_2 to the left and to the right respectively, when the current C' circulated through it,

$$F = \frac{8.a.n'.C'.A'.D}{u^3(d'_1 - d'_2)}.$$

Hence,
$$\kappa = \frac{1}{2\pi}\cdot\frac{(d_1 - d_2)}{(d'_1 - d'_2)}\cdot\frac{C'}{C}\cdot\frac{n'}{n}\cdot\frac{A'}{A}\cdot\frac{a}{\eta}\cdot\frac{1}{u^3}\cdot\frac{1}{N},$$

or, since the currents varied inversely as the resistances of the principal and shunt circuits,

$$\kappa = \mathrm{B}\,\frac{d_1 - d_2}{d'_1 - d'_2}$$

where B is a constant depending on the dimensions and resistances of the various parts of the apparatus.

In one series of experiments, chosen haphazard for illustration, the values of the different quantities were as follows :—

$$\sigma = 17{\cdot}16 \text{ cm.}, \qquad \sigma_1 = 40{\cdot}31 \text{ cm.}, \qquad \zeta = 3{\cdot}7 \text{ cm.};$$

therefore N $= 0{\cdot}0001696$.

The resistance of the principal circuit = 10·9 ohms,

„ „ shunt „ = 141 „

$$n' = 30{\cdot}81, \qquad n = 11{\cdot}25,$$

$$\mathrm{A}' = 1{\cdot}5836 \text{ sq. cm.}, \qquad \mathrm{A} = 6{\cdot}7424 \text{ sq. cm.},$$

$$a = 13{\cdot}0 \text{ cm.}, \qquad \eta = 17{\cdot}1 \text{ cm.},$$

$$u = 22{\cdot}61 \text{ cm.},$$

$$(d_1 - d_2) = 4{\cdot}3 \text{ cm.}, \qquad (d'_1 - d'_2) = 8{\cdot}5 \text{ cm.};$$

therefore $\kappa = 0{\cdot}00158$.

In each experiment a deflection was the mean of two readings taken with the current direct, and reversed when the tube was in each solenoid, and with the current direct and reversed in the case of the auxiliary solenoid. The effect of the earth's magnetic field was thus eliminated.

Various possible sources of error had to be investigated. In the first place, the suspended magnetic oxide might gradually settle in the tubes, or, under the influence of the current, the particles might tend to set themselves with their axes parallel to the axis of the solenoid, as in Sir William Grove's well-known lecture experiment. These effects were most to be feared in the case of the strong mixtures. It was found that if the magnetic oxide were dried it caked and it was impossible afterwards to suspend it in a state of fine and equal division in the glycerine. Hence the mixtures were made by mingling known volumes of one standard mixture of magnetic oxide and water with glycerine. Thus the stronger mixtures were the more aqueous, and therefore the less viscous. To investigate the possible error due to settling, the following experiments were made :—

The mixtures were allowed to remain in the tube for 30 minutes : no difference in the deflection could be detected in the case of the weaker mixtures; but for the strongest we obtained the following results. The deflections are throughout measured in cm.

Zero		0·00
Deflection with tube in		7·31
,, ,, ,,	after 30 mins..	7·00
Zero		0·00
Deflection with tube in		8·38
,, ,, ,,	after 30 mins..	8·11

During the first few minutes no appreciable falling off was observed, so that, as the time required for an experiment does not exceed a minute or two, any error arising from this cause may be neglected. Care was also taken to rotate the tubes frequently, and to empty them and pour the liquid in again at short intervals. In order to test the accuracy of the whole arrangement, several series of experiments were made with the apparatus set up in different positions, and the values obtained were such as to show that no serious discrepancy existed. In the following table the fractions in the first column are the ratios of the volumes of the standard mixture of water and magnetic oxide to that of the glycerine, and they may therefore be called the strengths of the mixtures. In the other columns are the susceptibilities obtained in each case with a completely different arrangement of the apparatus.

s = Strength.	κ.	κ.	κ.
$\frac{1}{5}$	0·00126	0·00115	0·00125
$\frac{2}{9}$	..	144	137
$\frac{1}{4}$	158	153	155
$\frac{2}{7}$	..	174	168
$\frac{1}{3}$	218	214	209

Six weeks after the last of the preceding series had been taken, another set of experiments was made in order to test the invariability of the magnetic properties of the mixtures. No change whatever could be detected, as will be seen from the following table of results:—

Strength = s.	κ.
$\frac{1}{5}$	0·00120
$\frac{2}{9}$	137
$\frac{1}{4}$	159
$\frac{2}{7}$	163
$\frac{1}{3}$	208

If now we take the means of the results of these four series of experiments as giving the values of the susceptibilities, and divide each number by the strength of the mixture, the ratio is found to be nearly constant. It must be remembered that errors which cause deviations from the mean value are in part, and probably in large part, due to uncertainty as to the exact composition of each mixture. As the magnetic oxide settles quickly in water, the amounts added to the glycerine were probably only approximately proportional to the volumes used; but, as the susceptibility of each mixture is absolutely determined without reference to its supposed composition, this will not affect the accuracy of the results.

Strength = s.	Mean value of κ.	$\frac{\kappa}{s}$.
$\frac{1}{5}$	0·001215	0·00607
$\frac{2}{9}$	1393	627
$\frac{1}{4}$	1562	625
$\frac{2}{7}$	1683	589
$\frac{1}{3}$	2122	637

The numbers in the last column are consistent with the view that the susceptibility of any mixture varies directly as the percentage of magnetic oxide which it contains. The matter may be further tested by means of the sets of experiments described immediately below, in which the susceptibilities of three of the liquids (including the weakest and the strongest mixtures) were again measured. The mean results are as follows :—

Strength = s.	Mean value of κ.	$\frac{\kappa}{s}$.
$\frac{1}{5}$	0·00124	0·00620
$\frac{1}{4}$	159	636
$\frac{1}{3}$	204	612

In this case the strongest mixture gives the smallest result. On the whole, then, and for the purposes of this investigation, the values of κ/s must be considered as independent of the strength of the mixture. Any difference which exists could only be certainly detected by a very careful determination of the quantity of magnetic oxide present in the unit of volume in each case. Since this law holds good within the limits of the series of liquids the suscepti-

bilities of which could be accurately determined, it may be safely inferred that for weaker mixtures, at any rate, it will still be true, and so a number of mixtures weaker than one-fifth were prepared, and their susceptibilities calculated by proportion from those of the stronger ones. Values so obtained will, however, be subject to the error of mixing.

Finally, a series of experiments, referred to above, was made to determine whether the permeabilities of the mixtures varied with the magnetic force. The susceptibilities obtained when 2, 3, and 6 Grove's cells were used in turn are given in the following table:—

Strength.	Six cells.	Three cells.	Two cells.
$\frac{1}{5}$	0·00122	0·00127	0·00123
$\frac{1}{4}$	164	162	150
$\frac{1}{3}$	209	199	203

The sums of the significant figures in the three columns are 495, 488, and 476 respectively, thus indicating a slight increase of permeability with the magnetic force. The differences between the individual observations are, however, too great to allow us to rely on this result, and the table can only be considered as proving that no serious error will occur if we assume that the permeabilities of the mixtures are independent of the magnetic force.

[Added Sept. 12, 1890.—Experiments made afterwards confirmed this view and extended the range over which its accuracy was tested. The weakest field employed was about twice the earth's field in the United Kingdom.]

It would have been difficult to obtain accurate results with a weaker field, but as the law of proportionality between the magnetic force and the induction appears to hold very approximately for forces between 5·0 and 1·7 C.G.S. units, it is probable that it is also valid for smaller values. It is true that Silow ('Wiedemann's Annalen,' vol. 11, 1880, p. 330) has stated that the susceptibility of ferric chloride is a maximum when the inducing force is about 0·4 C.G.S. unit, and that for that value it is between two and three times as great as for fields at strengths such as those at which we have experimented, but the change in a range of magnetic force much less than 1·0 to 5·0 was very marked. Thus, between forces of about 1·4 and 2·5, the susceptibility altered by about 13 per cent. An effect such as this could not possibly have escaped our notice, and there can be no doubt that for forces such as those with which we have dealt the susceptibility of magnetite changes

very slowly, and that the variation is not sufficient to affect seriously the argument of this paper.

We now turn to the method of comparing the susceptibilities of the rock specimens with those of the liquid by means of Professor Hughes' induction balance.

In the first place it was necessary to make certain that the effects observed were due only to the permeabilities, and not to the conductivities, of the bodies under investigation. The weaker liquids were practically non-conductors, but the stronger ones conducted feebly. When, however, a solution of salt in water, of rather greater conductivity than the strongest mixture, was introduced into the balance, which had previously been adjusted, no sound whatever could be detected, thereby proving that the very different effects obtained with the magnetic oxide were not, in any way, due to Foucault currents in the mixture. Two of the rocks which produced the greatest effect in the balance were also chipped out into the form of horse-shoes, and by dipping the ends into two mercury cups or into two cups containing acid and water, they were used to complete circuits, in which a mirror galvanometer was included. They appeared, as thus tested, to be non-conductors. We are, therefore, confident that the experiments are not vitiated by Foucault currents set up within either the liquids or the rocks.

The first test applied to the method was to measure by the aid of the balance the susceptibilities of the different mixtures relatively to each other. Thus in the case of three liquids a, b, and c, say, the susceptibility of b was found by using the values of the susceptibilities of a and c which had been obtained by the absolute method. In the following table the numbers thus obtained are compared with those given directly by the absolute method :—

s.	κ measured by—	
	Magnetometer method.	Induction balance.
$\frac{2}{9}$	0·00139	0·00135
$\frac{2}{7}$	168	172
$\frac{1}{3}$	212	208

The agreement between the last two columns is sufficient to justify the induction balance method. The strengths of the fields in the balance were different from those employed in the absolute method, and Foucault currents might affect the results. It is clear, however, that neither of these possible causes of difference produces any

appreciable error. Even if the magnetic force and the induction are not strictly proportional, the ratio of the permeabilities of the liquids is, to the degree of accuracy attained, the same in the balance as in the solenoid.

Finally, the method was applied to a specimen of basalt which Professors Thorpe and Rücker had brought from the Island of Canna, in the West of Scotland. A piece of this had been cut into the form of a rectangular bar, and its magnetic properties had been investigated by Dr. Hoffert. His experiments are described in the published account of the magnetic survey ('Phil. Trans.,' A, 1890). The permanent magnetisation was determined by three methods, and the susceptibility was found from the times of vibration when the bar was suspended in a known magnetic field with the directions of permanent and induced magnetisation alternately coincident and opposed. Dr. Hoffert found that the value of κ was about 0·0015. Unfortunately, the particular bar used by him has been mislaid, but we have measured the permeability of another fragment of the same specimen, and find—

$$\kappa = 0{\cdot}00132.$$

Observations to be described below prove that differences such as this exist between specimens of the same rock. We do not therefore regard these numbers as giving any test of the accuracy either of Dr. Hoffert's or our own observations, but the agreement between them is sufficient to prove that there can be no doubt as to the order of the magnitudes of the quantities under discussion.

The range over which the instrument could be employed was between susceptibilities about five times greater and ten less than that of Canna basalt, *i.e.*, from about 0·00792 to 0·00013. The higher of these values could only be obtained by extrapolation, as liquids of such great permeability could only be formed by using so small a quantity of glycerine that the magnetic oxide settled too quickly to enable us to obtain reliable results. In the tables given below, the statement that the magnetic susceptibility of a substance is zero means only that it is distinctly less than that of the weakest liquid employed, *i.e.*, than about 0·00013. The details of a single experiment are given by way of example. The readings of the compensator scale are in centimetres.

Experiment.

Equal volumes of the liquid of strength 2/9 were placed in test-tubes in the two arms of the balance.

$$\text{Zero readings}\begin{cases}10{\cdot}1\\10{\cdot}2\\10{\cdot}0\\10{\cdot}15\end{cases}\qquad \text{Mean} = 10{\cdot}11 = z_1.$$

A specimen of Canna basalt was placed in the left-hand test-tube, and liquid abstracted until the volume was the same as before.

$$\text{Readings} \ldots \begin{cases} 9{\cdot}8 \\ 9{\cdot}9 \\ 9{\cdot}8 \end{cases} \qquad \text{Mean} = 9{\cdot}83 = r_1;$$

therefore $x_1 = (z_1 - r_1) = 0{\cdot}28.$

Equal volumes of the liquid of strength 1/5 were now taken.

$$\text{Zero readings} \begin{cases} 10{\cdot}15 \\ 9{\cdot}9 \\ 10{\cdot}0 \\ 10{\cdot}2 \end{cases} \qquad \text{Mean} = 10{\cdot}06 = z_2.$$

The same specimen of rock was now placed in the left-hand test-tube, after having been carefully washed and dried.

$$\text{Readings} \ldots \begin{cases} 10{\cdot}0 \\ 10{\cdot}2 \\ 10{\cdot}0 \\ 10{\cdot}1 \end{cases} \qquad \text{Mean} = 10{\cdot}08 = r_2.$$

$$x_2 = (z_2 - r_2) = 0{\cdot}02.$$

Hence, by the formula given above,

$$\kappa = 0{\cdot}001227.$$

On the whole then, we think that the various tests which have been applied to it prove that the method employed fulfils the required conditions very satisfactorily. It is not capable of giving results of the last degree of accuracy, but it enables us to measure quickly and certainly, with only a small percentage error, the permeabilities of rock specimens without the labour and expense involved in shaping them into definite geometrical forms.

The method, too, has the advantage that, when once the permeabilities of the standard liquids are determined, the apparatus can be used anywhere. If therefore it were desirable to institute a close comparison between the magnetic disturbances and the magnetic permeabilities of the rocks in a given district, and it were important that the investigator should become at once acquainted with his results, it would be quite practicable to transport the apparatus required to the scene of the investigation, and to determine the magnetic properties of the specimens in any convenient room within a few hours of their collection.

Our observations on rock specimens may be divided into three

groups, according as they were (α) non-magnetic, (β) magnetic but not basaltic, (γ) basaltic.

Of the first group, we tried a number of specimens, many of which were *a priori* certain not to be magnetic. Some of them, however, were just as likely to be conductors as the magnetic rocks; and the fact, therefore, that they have been tried, and produce no effect, strengthens the view that the measurement of the permeabilities was not affected by the conductivities of the specimens. Among those submitted to experiment were limestones, sandstones, mica- and hornblende-schists, granite with tourmaline, red granite, trachyte, felsite, rhyolite, gabbro, muscovite granite, luxullianite, various diorites, and hæmatite.

Two specimens of Archæan gneiss, brought by Dr. Thorpe from Loch Maddy, in the Outer Hebrides, were found to be practically non-magnetic.

We have also, through the kindness of Professor Judd, had the opportunity of testing the specimens of Silurian rocks and red sandstones obtained from the Palæozoic ridge by deep borings near London. Fragments from Kentish Town, Richmond, Meux's Brewery, and Ware were tried and found to be non-magnetic.

Turning next to specimens of other than basaltic rocks which were found to be magnetic, we obtained the following results:—

	κ.
Phonolite	0·00070
Dolerite	94
Trachyte..........	39
Melaphyre	39
Tourmaline granite .	24
Syenite	104

It will be noticed that several of these are rocks of the same kind as those of which other specimens were found to be non-magnetic. This is an example of the fact that the permeabilities of different portions of the same rock are very various, and that no conclusion can be drawn unless a large number of specimens have been examined.

Special attention having been given in the recent magnetic survey to the magnetic disturbances produced by the Malvern Hills, it was thought that a detailed investigation of their magnetic properties would be interesting. Mr. Highfield, therefore, paid a visit to Malvern for the purpose of collecting specimens. The position at which any specimen was found was marked on the spot on an Ordnance map carried for the purpose. It is, perhaps, hardly neces-

sary to reproduce this map here, but notes are appended to the table given below which indicate the point on the range from which the specimen was obtained. Care was taken that the specimens should not be weathered. It may be well to add that the Malvern Hills are a range of hornblendic rocks, bounded on the east by a great fault, which divides them from the red marls of the Valley of the Severn. On the western side the igneous rocks emerge from under Wenlock

No. of specimen.	Position of station.	Mean value of κ.
1*a*, *b*, *c*	Quarry at North Malvern, near the tank	0·00012, 0, 0
2*a*, *b*	On the crest at the extreme north end of the range	46, 25
3*a*, *b*, *c*	On the ridge between the peaks of the North Hill	0, 12, 59
4*a*, *b*	Near to the summit of the North Hill	0, 102
5*a*, *b*	Near St. Ann's Well	0, 0
6*a*, *b*	Near to the top of the Worcestershire Beacon	69, 90
7*a*, *b*, *c*	Near to the footpath, considerably below and a little to the south of the top of the Worcestershire Beacon	0, 0, 12
8*a*, *b*	On the ridge about ¼ mile south of the top of the Worcestershire Beacon	0, 0
9*a*, *b*	On the ridge above the railway tunnel	0, 0
10*a*, *b*	In a cutting through the ridge above Malvern Wells	113, 0
11*a*, *b*	On the ridge nearly due east of Brand Hall (Ordnance map)	30, 0
12*a*, *b*	On the ridge on the north side of the Ledbury Road	139, 0
13	On the Herefordshire Beacon, about a furlong south-east of the encampment	0
14	On the western side of the Herefordshire Beacon, about 300 yards south of 13	0
15	At the south end of the Herefordshire Beacon, above Hill Farm	0

limestone and Silurian rocks, which are bent upwards on their flanks. The collection of specimens was composed entirely of the crystalline rocks: it was begun at the extreme north end of the range, and continued as far south as the Herefordshire Beacon, a distance of about 5 miles. In most cases two specimens were taken at each spot, and in the table these are indicated by the same number followed by different letters.

As the susceptibilities of the rocks varied so considerably, Professor Judd was good enough to have sections made of some of those which differed most widely. The following report made by him shows a satisfactory agreement between the indications of the induction balance and microscopic examinations :—

Specimen.	κ.	Remarks.
12*a*	0·00139	Large amount of magnetite, well crystallised; much pyrites.
10*a*	113	Rather smaller quantity of magnetite, and in smaller crystals.
11*a*	30	Magnetite, small in quantity, and sporadically distributed.
7*c*	12	Magnetite, very small in quantity (possibly only titanoferrite present).
12*b*	zero	Magnetite, very small in quantity (possibly only titanoferrite present).

As was to be expected, basic rocks proved to be the most strongly magnetic; but it is well at once to emphasise the fact that powerful permanent magnetisation affords no proof of high permeability. Thus a specimen of rock from the Peak of the Island of Ascension, kindly supplied to us for examination by Captain Creak, F.R.S., strongly attracted and repelled the pole of a compass needle. Its susceptibility, however, was only moderately large, being 0·00122.

We have collected in the following table the results of our measurements on basic rocks :—

Specimen of—	Locality.	κ.	Mean value of κ.
Dolerite	Ratho, Edinburgh	0·00113	
Enstatite-andesite	Newport, Fife	59	
" "	Durham	134	
Porphyritic basalt	Schemnitz, Hungary	109	
Basalt	Faroe Islands	116	
Olivine-diabase	Nabe, Rhine	47	
Basalt	Unkel-on-Rhine	45	
"	Rowley Regis	118	
"	Giant's Causeway	27	0·00024
"	" "	21	
Olivine-gabbro	Skye	697	0·00561
" "	Cuilin Hill, Skye	747	
Fine-grained gabbro	Skye	246	
Olivine-gabbro	"	553	
Gabbro	Deer Forest, Ardnamurchan	660	0·00420
"	" "	632	
"	" "	307	
"	" "	83	
Dolerite	Tobermorey, Mull	0·00147	0·00163
Porphyritic basalt	" "	231	
Olivine "	" "	74	
" "	" "	184	
Basalt	Tobermorey Harbour	209	
Porphyritic basalt	Fishguard "	61	
" dolerite	Mull	155	
Platy basalt	"	113	
Gabbro	Mhaim Clackaig, Mull	100	
"	Ben More "	146	
Basalt	Dumfrin "	114	
Olivine gabbro	Ben More "	429	
Dolerite	Dun-da-gaioth "	156	
Basalt	Staffa	0·00048	0·00062
"	"	77	
Gabbro	Loch Coruiskh, Skye	0·00049	0·00237
"	"	164	
"	"	628	
"	"	27	
"	"	362	
"	"	82	
"	"	153	
"	"	284	
"	"	75	
"	"	684	
"	"	99	

These results are sufficiently numerous to justify their employment in the calculations which follow. They prove that, in spite of great variations between individual specimens, the average susceptibility of

basic rocks is relatively very high. The average of all the specimens from the west of Scotland and from Ireland is 0·00245. If we exclude the Giant's Causeway and Staffa, it is 0·00271, which is thus the average of all the specimens tested from a district nearly 70 miles in length. There would, therefore, be nothing absurd in the supposition that equally large values obtained over equally large areas elsewhere; but in the calculations the assumed susceptibility is the much smaller value given by the Mull specimens, viz., 0·0016.

An experiment was made on the effects of temperature on the permeability of magnetite. It was only of a rough preliminary kind, but the result was quite clear, and further and more elaborate experiments on the same point are about to be undertaken in the laboratory at South Kensington.

The interior of one of the cups of the Hughes' induction balance was lined with asbestos cloth, and a fragment of non-magnetic granite which had been heated to incipient redness in the flame of a Bunsen burner was introduced into it. The balance which had been previously obtained was quite undisturbed. The same experiment was then repeated with a piece of magnetite. The introduction of the rock at once caused the telephone to "speak," but silence was quickly obtained by turning the screw by which the parallelism of the primary and secondary coils is secured. The compensator used in the previous experiments had not sufficient range, and readings were taken by a paper scale of degrees attached to the screw head. As the magnetite cooled the zero altered, and, in order to maintain silence, it was necessary to keep turning the screw in the direction which indicated that the permeability of the specimen was decreasing. The stone was allowed to cool for half an hour, and the total alteration of the zero measured. It was then removed altogether, and the new position of silence found. In one experiment the following values were obtained. The figures in the second column indicate the number of degrees through which the screw was turned from the first zero obtained after the introduction of the hot magnetite :—

	Reading.
Magnetite, hot.....	0°
,, cold....	320
,, removed	790

This result proves that, as in the case of iron, the permeability of magnetite increases as the temperature rises, the increase in the experiment just described being about 70 per cent. A second experi-

ment gave about 60 per cent. It is, of course, probable that if the temperature is raised sufficiently the permeability of magnetite, like that of iron, will rapidly diminish, and that, after a certain temperature is reached, it will cease to be magnetic. This point also we hope to investigate further.

PART II.—*On Regional Magnetic Disturbances.*

In attempting to base calculations upon the permeabilities measured by Messrs. Highfield and Jarratt, it is necessary to make some assumptions as to the magnetic state of the earth's crust.

The average increase of temperature with depth is about 1° C. for every 90 feet, and if this rate obtains for a depth of several thousand feet, the temperature would be 700° C. at 12 miles, or about 20 kilos. from the surface. Iron ceases to be magnetic between 700° C. and 800° C., and it seems, therefore, fair to assume that below this depth magnetic matter does not exist. Whether this be so or not, it is necessary to suppose that at some given distance from the surface the earth may be regarded as magnetically uniform. In selecting such a distance for purposes of calculation the relations between the magnetic properties of iron and temperature afford perhaps the most trustworthy guide.

Let then a level surface be regarded as homogeneous. It may be called the *magnetic floor*. Let matter, magnetised by the earth's induction, be supposed to be placed upon it, and let all calculations be based on the hypothesis that the permeabilities with which we have to deal differ but little from unity.

The disturbance produced by the magnetic mass will be the same whether the magnetic floor is magnetic or non-magnetic; for the same coating of south hemisphere magnetism, which in the latter case will represent the effects of the earth's induction on the lower surface of the mass, will represent the modification it produces in the magnetisation of the floor if the latter is magnetic.

The disturbances which have to be explained are of two kinds, viz., those in which a very great range of vertical force disturbance occurs within a limited area, and those in which a moderately high value of vertical disturbance occurs over a large area.

In all cases the effects of the upper and lower surfaces of the disturbing masses will be opposed, and the force observed will be the same if the disturbance is produced by similar masses, the distances of which from the attracting point are proportional to their linear dimensions. Even supposing, therefore, the shape and the magnetic properties of the mass are known, we learn nothing as to its proximity to the surface from the mere magnitude of the disturbing force at a single station.

On the other hand, the absence of vertical disturbing force does not necessarily prove that no magnetic matter exists between the magnetic floor and the point of observation. The upper and lower surfaces of a plate, of which the horizontal dimensions are very large compared with the distance of either from the surface, would produce equal and opposite effects over the central parts.

On approaching the plate from a distance the vertical disturbing force would increase near the edge, and then die out as the centre was approached, the phenomena so far corresponding with those which occur when the observer crosses an underground ridge of magnetic rocks. The two cases, however, could be distinguished by the fact that the direction of the horizontal forces would be the same on both sides of the line of maximum vertical force if the disturbance were due to the edge of a plate, but different if it were produced by a magnetic ridge.

I now propose, therefore (1), to describe the distribution of vertical force disturbance over England and Wales; (2) to show that the presence beneath the surface of rocks which possessed *in situ* the same magnetic properties as basic rocks on the surface would produce disturbing forces of the same order as those which are actually observed.

In discussing the first point, it will be convenient to measure disturbances or departures of the magnetic force from its normal calculated value as terms of 0·00001 C.G.S. or 0·0001 metric unit, which may be regarded as the *unit* of *disturbing force.* Vertical disturbing forces are positive when they urge a north-seeking pole downwards.

It is fully explained in the published account of the magnetic survey that vertical force disturbances are measured from an arbitrary datum, and that there is no proof that this is uniform all over the kingdom.

The largest area of positive disturbance occurs in the east and south-east of England. It extends from the English Channel to the north of Yorkshire, *i.e.*, about 230 miles north and south, and in parts it is about 110 miles wide.

It is, however, deeply cut into by a narrow gulf-like region of negative disturbance in the Midlands, and by others in Kent, Sussex, and South Lincolnshire.

Taking this into account, and also the fact that in the north it is very narrow, it is fair to assume that it may be approximately represented by a rectangular figure 180 miles long and 108 miles wide. The regions of negative vertical force which bound it are relatively small, say from 15 to about 50 miles wide in the west, while to the north they are wider.

To the east and south the sea prevents our determining the exact limits of the district. It may, however, be taken as a rough but

adequate representation of the actual state of things to assume that the district has no magnetic matter to the north and south; but it is separated from other similar regions on the east and west by a non-magnetic trench 36 miles wide.

The difficulty in accounting for a high average difference of vertical disturbing force increases with the size of the district, as has already been explained. We are, therefore, choosing regions which afford the most severe test of the theory that the disturbing forces are due to the magnetism induced in iron-bearing rocks. If we confine our attention to a line crossing the two trenches and the plateau half-way between their northern and southern boundaries, no important error will be introduced if we consider the length of the trenches and plateau to be indefinitely extended north and south. The values obtained are, however, affected by the assumptions we make as to the magnetic character of the ground east and west of the district under consideration. The first hypothesis made is that the external boundaries of the trench are narrow plateaux, and that beyond them there is no magnetic matter. Afterwards, we will suppose that they extend to infinity at a uniform height above the magnetic floor. We may also, at first, assume that the edges of the magnetic masses are vertical, and that they are magnetised by the vertical component of the earth's field only. It will be convenient to express all distances in kilometres, and, as above stated, all disturbing forces in terms of 0·00001 C.G.S. or 0·0001 metric unit.

The mean values of all the positive and all the negative vertical force disturbances in England and Wales, recorded by Dr. Thorpe and myself, are +59 and −80 respectively, giving an average range of about 140.

A difference of this order can be obtained if we suppose that the slabs of magnetic matter are 16 kilom. (or about 10 miles) thick, and that thus the upper surfaces are 4 kilom. (or about $2\frac{1}{2}$ miles) from the surface of the earth.

The components of the attraction exerted on a point by a plane rectangular plate consisting of an uniform layer of attracting matter of density σ may be expressed as follows:—

Let planes be drawn through the point perpendicular to the plate and parallel to its edges. Let the points in which these meet the edges or edges produced of the rectangle be joined to the point, and let these make angles ϕ_2 and ϕ_1, θ_2 and θ_1 with the normal.

Then the components perpendicular to the plate and parallel to it and to the plane of the ϕ's are—

$$\sigma\{\sin^{-1}(\sin\phi_2\sin\theta_2)-\sin^{-1}(\sin\phi_2\sin\theta_1)-\sin^{-1}(\sin\phi_1\sin\theta_2)+\sin^{-1}(\sin\phi_1\sin\theta_1)\},$$

and
$$\sigma\{\log_e(\cos\phi_1\sin\theta_2+\sqrt{1-\sin^2\theta_2\sin^2\phi_1})$$
$$-\log_e(\cos\phi_2\sin\theta_2+\sqrt{1-\sin^2\theta_2\sin^2\phi_2})$$
$$-\log_e(\cos\phi_1\sin\theta_1+\sqrt{1-\sin^2\theta_1\sin^2\phi_1})$$
$$+\log_e(\cos\phi_2\sin\theta_1+\sqrt{1-\sin^2\theta_1\sin^2\phi_2})\}.$$

The component parallel to the plane of the rectangle and to that of the θ's is obtained from the last by writing ϕ for θ and *vice versâ*.

In the particular case for which $\phi_1=\theta_1=0$ and $\theta_2=\pi/2$, the expressions reduce to

$$\sigma\phi_2 \text{ and } -\sigma\log_e\cos\phi_2=\sigma\log_e\sec\phi_2.$$

In the case under consideration these must be doubled, as the plate is supposed to extend both north and south of the point.

If we assume the magnetic susceptibility of the mass to be 0·0016, which is about the mean value of the Mull basic rocks, and take the vertical component of the earth's field as 0·46 C.G.S. units, we get—

$$2\sigma = 2\times0\cdot46\times0\cdot0016 = 0\cdot00147,$$

so that if ϕ_2 is expressed in degrees

$$2\sigma\phi_2 = 0\cdot0000257\,\phi_2.$$

also
$$2\sigma\log_e\sec\phi_2 = 0\cdot00338\log_{10}\sec\phi_2.$$

Hence the disturbance produced is 2·57 ϕ_2 and 338 $\log_{10}\sec\phi_2$ units of disturbing force respectively.

By means of these expressions we may calculate the attractions exerted at points on the surface 10 kilom. apart by horizontal plates of magnetic "matter" of opposite kinds 60 kilom. wide, and of indefinite length at depths of 4 and 20 kilom. respectively.

Subtracting the numbers given by the lower from those deduced from the upper plate, we get the resultant vertical and horizontal forces due to the opposite magnetisation of the upper and lower surfaces.

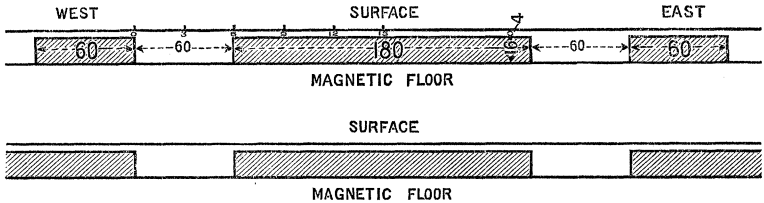
The minor plateaux are supposed to consist of two such plates, the principal plateaux of three plates side by side.

The resultant force at any point is obtained by adding, algebraically, the components due to each mass.

Fig. 3 represents the two cases which have been submitted to calculation. They differ only in that the magnetic matter outside the central plateau is supposed, in the one case, to be limited, and in the other, to be unlimited, in an east and west direction.

The forces are determined for points on the surface 10 kilom. apart.

FIG. 3.



Thus 0 being over the outer edge of the trench, 3 is over the middle of the trench, 6 over the edge of the plateau, and 15 over its centre. The forces are expressed in terms of units of disturbing force or 0·00001 C.G.S. unit. Horizontal forces are positive which urge a north-seeking pole towards the central plateau. Vertical forces are positive which act downwards as above stated.

	Case I.		Case II.	
	Vertical.	Horizontal.	Vertical.	Horizontal.
0	21	−209	− 41	−223
1	− 98	− 80	−155	− 93
2	− 90	− 23	−144	− 34
3	− 87	9	−136	0
4	− 94	41	−144	34
5	−108	99	−155	93
6	6	227	− 41	223
7	117	102	72	99
8	101	49	55	46
9	81	20	40	20
10	73	12	30	14
11	66	8	24	9
12	62	4	20	5
13	59	1	17	3
14	57	0	16	1
15	56	0	16	0
Mean positive.	+64	..	+ 32	..
,, negative	−95	..	−117	..
,, range ..	159	..	149	..

Other disturbances may be imagined which give similar results. If the central plateau stood alone, surrounded by non-magnetic matter, the mean range, as calculated above between the vertical forces over it and those observed within the 60 kilom. (36 miles) of its edge, would be 127. If the edge of the valley, instead of being vertical, were vertical for 10 kilom. only, and then sloped upwards with an inclination of 1 in 10, the range would be 107.

Although, therefore, the method of testing the theory is necessarily very rough, it is nevertheless evident that the range of the vertical forces over a mass of magnetic matter of almost the same area as the region of positive vertical force disturbance in England, and within 60 kilom. of its edge, might agree fairly with the facts if the edges of the mass were steep, and if its magnetic properties were the same as those of the Mull basaltic rocks. The observed range is 140, and widely differing hypotheses give calculated values between 107 and 159.

It is not, however, sufficient to account for the differences of vertical force over large areas. Within there are smaller but still large districts in which the vertical force disturbance considerably exceeds the mean, and the explanation of the phenomena by rock magnetism would be imperfect if the calculated forces were insufficient to explain the characters of these. It might easily be that, to produce an average vertical force disturbance equal to that which obtains over the whole plateau, it was necessary to bring the upper surface so near to the surface of the earth that the remaining depth was insufficient to account for the additional forces in play on the area of greater disturbance within it.

The largest area of very high vertical force which has at present been mapped in detail lies in Lincolnshire and South Yorkshire.

The following table exhibits the names of stations within it, and the vertical force disturbance at each:—

Sutton-on-Derwent ..	229	Thorne	230
Market Weighton	389	Butterwick..........	295
Howden	245	Brigg..........	251
Doncaster..........	225	Market Rasen	277
Mean of all, 267.		Excluding three highest, 236.	

If these places are connected by straight lines, an irregular figure of eight is produced, the area of which is about 425 square miles, or 1100 square kilom.

It is, of course, unlikely that the stations are all close to the edge of the attracting mass, but, on the other hand, it may be deeply cut into by valleys or regions of less disturbance. The latter hypothesis, if correct, would make it easier to explain the high forces, whereas an extension of the district beyond the limits actually observed makes any such explanation more difficult. The distance between the most northerly and most southerly stations (Sutton-on-Derwent and Market Rasen) is 45 miles, or 75 kilom. The distance east and

west between Doncaster and Brigg, which are near the centre of the southern loop of the figure of eight is about 27 miles, or 45 kilom.

It may, therefore, fairly be assumed that the order of the forces within the district will be the same as that of those over a rectangle 60 kilom. in length and 30 in breadth. The linear dimensions of such a figure are somewhat less than the extreme length and breadth of the district, but its area is 1·6 times greater. Let us then superpose upon the plateau a rectangular mass, 60 kilom. long by 30 broad, and 3·5 deep, the upper surface of which is, therefore, at a depth of 0·5 kilom. or 1638 feet.

The vertical disturbing forces due to this mass along a line passing over its centre and parallel to its longer edge, are as follows:—

Distances are expressed in kilometres.

Distance.		Force.
Middle	0	76
	10	79
	20	98
	25	123
Edge	30	37
	40	−34
	50	− 9
	60	− 4

Taking the mean of the numbers at the edge and at a distance of 5 kilom. from it as applying to the district between them, the mean of the positive vertical disturbances is 83.

If this minor mass were placed on the larger one, so that their longer edges were parallel, and that the median line of the smaller mass was 20 kilom. from the edge of the larger one, thus corresponding with the position 8 in fig. 3, p. 527, and if, lastly, the distribution of magnetic matter were as is shown in that figure, then the mean vertical disturbance along the median line of the small mass would be $101 + 83 = 184$.

In comparing the observed and calculated values it is convenient to take the mean force in the negative districts as a datum; so that the disturbance as calculated is $95 + 184 = 279$, and as observed $80 + 236 = 316$.

This latter number is obtained by excluding the high values at Market Weighton, Butterwick-on-Trent, and Market Rasen, but these can easily be imitated by placing small masses on the upper surface.

A plate, 5 kilom. square and 0·25 kilom. thick, approaching the surface to within 0·25 kilom., or 820 feet, would increase the force by

40. A mass of magnetic matter in the shape of a frustum of a cone, of which the vertex was in the surface, and the angle between the generating and median lines $\tan^{-1} \sqrt{2}$, and of which the upper plane surface was 450 feet from the surface of the earth, would exert at the vertex a vertical force 231, thus bringing the total calculated disturbance up to $279 + 231 = 510$, as against $384 + 80 = 464$, observed at Market Weighton. Such a cone gives a maximum effect; but it must be remembered that, although no magnetic rocks exist near the surface at Market Weighton, yet the fact that the older rocks approach the surface near that town led Professor Judd to advise its being selected as a station, and it may well be that the underlying magnetic masses rise steeply in its neighbourhood.

It is a little difficult to summarise an argument of this kind, but if we start from the near mean negative disturbance in the "valley" as a datum, and distinguish the other stations as being over an underground plateau, mountain, and peak respectively, we get the following results:—

Station.	Depth of nearest point of magnetic matter.	Mean vertical disturbance in terms of 0·00001 C.G.S.	
		Observed.	Calculated.
Valley....	No large mass of magnetic matter above the magnetic floor, *i.e.*, within 20 kilom., or 12 miles, of the surface	0	0
Plateau...	4 kilom. = 2·5 miles	140	159
Mountain .	0·5 kilom. = 1640 feet..................	318	279
Peak.... .	0·137 kilom. = 450 feet................	464	510 max.

It is thus possible to imagine a distribution of magnetic matter at depths between 450 feet and 12 miles, and of no greater permeability than Mull basalt, the mere presence of which in the earth's magnetic field would produce vertical disturbing forces of the same order as those actually observed.

The maximum calculated horizontal forces are larger than any of those which have been measured in England or Wales. Case I (see Table, p. 527) has been adopted as the basis of calculation, and the horizontal disturbing force at the edge of the plateau is thus 227. Forces of this magnitude are found in Scotland, but the largest value in England is 124 at Melton Mowbray.

At the middle of the edge of the smaller mass, on the side on which

its horizontal attraction strengthens that due to the plateau as a whole, the resultant force would be about 450, a value which is surpassed only by such stations as Canna and Soa, which are islands in the West of Scotland.

While, then, in order to account for the high vertical forces, we have been obliged to make favourable assumptions as to the shape of the masses, as to the position of the smaller on the larger mass, and as to the magnetic properties of the rocks, we now find that these lead to possible values of the horizontal disturbing force considerably greater than any which have been measured.

The most obvious explanation of the discrepancy is the assumed verticality of the sides of the magnetic masses. If the sides of the valley slope for half the height at an inclination of one in ten, as described above (p. 527), the largest horizontal force due to the plateau at any of the points for which the calculation has been made is 123, as against 227 when the sides are vertical. It must, however, be remembered that in this case the mean range of vertical forces is reduced from 159 to 107. It is not, therefore, convenient to assume that the slopes are very gradual.

The supposition that the main mass is surrounded by magnetic matter of less permeability than itself reduces the horizontal forces, but also reduces the vertical forces below the observed values.

An irregular outline, on the other hand, might tend to increase the range of the vertical force at points near the edge, while it would diminish the horizontal forces.

It appears on the whole, however, that the gentle slopes or gradual changes of permeability which would reduce the horizontal forces to the observed values, would give vertical forces about one-third too small.

I do not think that this can be considered an unsatisfactory result, but I will defer comment upon it in order to turn to another point. So far, we have been discussing districts of widespread disturbance. In Scotland the forces are more localised, but more intense. The most rapid change of vertical force disturbance which has been measured is in the Southern Hebrides, where it varies from −736 at Loch Boisdale to +369 at Bernera, which is only 20 miles distant.

Dr. Thorpe and I have proved ('Roy. Soc. Proc.,' vol. 47) that a very intense centre of disturbance exists in this neighbourhood, and it is remarkable as being near the highly magnetic rocks of Mull, but, as its effects appear to be more far reaching than those of that island, it is legitimate to assume that it is produced by rocks of exceptional magnetic power. An equal range of force could, however, be produced by the mean permeability of basalt in the west of Scotland, *i.e.*, 0·00271.

If, as before, we take the attracting mass to be the frustum of a

cone, of which the vertex is in the surface, the vertical angle is $2 \tan^{-1}\sqrt{2}$, and the upper and lower faces are y_0 and y kilom. from the surface respectively, the force exerted at the vertex is

$$2\pi\sigma \times 0{\cdot}887 \log_{10} (y/y_0)$$

where σ is the density on the horizontal surface.

If $y = 20$ and $y_0 = 0{\cdot}5$ kilom., this gives a vertical disturbing force of 1110. By thus assuming a favourable form for the rock mass, it is possible to account for the force by means of ordinary basalt, which nowhere approaches the surface nearer than a depth of 500 metres, or about 1600 feet.

Some difficulty may be felt about Ireland, over a large part of which the vertical force disturbance is negative.

If this be regarded as a real upward force, it could only (on the hypothesis under consideration) exist in the neighbourhood of magnetic matter, which would probably cause more widespread positive disturbance than has been registered.

I am inclined to account for this by a shift in the datum from which the disturbances are measured. If the calculated vertical forces are all 0·00100 C.G.S. unit too large in Ireland, the vertical disturbance would be nearly everywhere positive. An error of this sort would be accounted for by an error of 0·00040 C.G.S. unit in the *horizontal force*, and this again would correspond to a displacement of the lines of equal horizontal force through 6 miles. If, however, the error was due to an unfavourable combination of inaccuracies, both in the lines of equal dip and equal horizontal force, this displacement of the lines might be reduced, so that, on the whole, it is not impossible that the datum in Ireland may be 100 units of vertical force disturbance different from that in England. If this is so, and if the change is gradual across England, the difference between the means of the negative and positive vertical disturbance would be diminished, and the calculated would be brought into closer accord with the observed range of vertical forces.

As the Malvern Hills have been so carefully studied, it seemed worth while to see whether the observed deflections of the needle towards the hills could be accounted for by their magnetic nature.

It is, however, at once evident that the problem is beset with difficulties. The permeability of the range appears to be different in different parts, and a mean value will not give accurate results. The visible mass of igneous rock, supposed to be of mean permeability, is certainly insufficient to account for the observed effects, and if we attempt to base calculations on assumed underground extensions of the mass, they are of course founded on pure hypothesis.

The easiest way of attacking the problem is to calculate the sum of

the two attractions exerted on opposite sides of the hills towards them.

Observations have been made at four stations, two (Great Malvern and Malvern Wells) on the eastern, and two (Mathon and Colwall Green) on the western, side of the hills. Great Malvern and Mathon are near the north end of the range, and the sum of the two attractions is 0·00243 C.G.S. unit of force. Malvern Wells and Colwall Green are near the middle of the range, and the sum of the attractions is 0·00118. It will thus be seen that at what would, *primâ facie*, have appeared the most favourable position the forces are smaller, but the result accords with the fact that the specimens collected at the north end of the range contained the largest quantity of magnetite. Sections across the range have been published by the Geological Survey, and from these it seems that on the eastern side the wall of igneous rock is nearly vertical, while on the western side it slopes more gradually under the sedimentary rocks, and if continued as far from the range as our stations (about a mile and a quarter, or 2 kilom.), it must be at a depth of 2000 feet, or, say, 600 metres. Assuming, then, that our stations were near the edge of the horizontal extension of the igneous rocks, and that the latter extend north and south for 8 miles (13 kilom.), I find that the sum of the attractions on opposite sides of the range near its centre, and at points as distant from it as our stations were, is 0·00291, 0·00154, or 0·00067, according as we assume the mean permeability to be—

(1) That of the three most favourable rock specimens;
(2) That of all the specimens which possessed measurable permeabilities;
(3) That of all the specimens obtained.

If these are compared with the observed forces, viz., 0·00243 and 0·00118, it is seen that, while they are of the right order of magnitude, an exact numerical agreement could only be obtained if we supposed the mean permeability of the range to be somewhat greater than that of its surface, as judged by the specimens collected by Mr. Highfield.

The results may, I think, be considered to support the view that the igneous rock extends laterally at a moderate depth from the surface, at least on the western side of the hills, to a distance of a mile or a mile and a quarter, but probably not much more, from the axis of the range.

In conclusion, then, I am anxious that the purport of the calculations above described should not be misunderstood. In particular, I do not attempt to specify the depth at which magnetic matter exists where none appears on the surface, in the east of England or elsewhere. But, in spite of this uncertainty about every detail, the

investigation, I think, supplies for the first time a definite answer to the enquiry whether the mere presence in the earth's magnetic field of concealed magnetic rocks, such as those which exist on the surface, would suffice to account for the observed local or regional magnetic disturbances in districts where the superficial deposits are non-magnetic.

The question is not answered by pointing to the large disturbances produced close to basic rocks, for these may be, and probably often are, due to permanent magnetism. But, as this is very irregularly distributed in the surface rocks, we cannot regard it as a probable cause of widespread disturbance, though locally it may produce very intense effects.

It is, I think, answered in the affirmative by the above discussion.

In gauging the value of the answer, it must be remembered that only one of the various constants involved has been at our disposal, and that it would not have been possible to imitate the observed results by assigning to it appropriate values, whatever arbitrary assumption had been made as to the others.

Thus, in the simple case of a rectangular magnetic slab, the average vertical force produced over it in virtue of the earth's inductions, depends on five variables, viz., its length and breadth, the distance of its upper and lower horizontal surfaces from the surface of the earth, and its permeability. Of these, the first two have been defined by the observed magnitude of the areas of high vertical force. The depth of the lower surface has not been fixed to suit the exigencies of the argument, but deduced from the temperature at which iron ceases to be magnetic. The permeability is at most that given by experiment on the specimens of basic rocks from the west of Scotland. Only one disposable constant remained, viz., the depth of the upper surface, and by shifting this we can do no more than raise the average disturbing force to a certain maximum, which might have been much less than the observed disturbances. As a matter of fact, however, by choosing suitable depths we are able to obtain forces of exactly the right order of magnitude. With the constants chosen there appears to be some difficulty in obtaining correct relative values of the vertical and horizontal disturbing forces, though even here the order of their magnitude is unaffected. This is exactly the kind of difficulty which might almost certainly be expected in calculations based only on more or less probable assumptions. It will be diminished by any change which increases the intensity of the calculated forces, and there are several possible causes which might produce such an effect.

Thus no account has been taken of the increase in the permeability of magnetite with temperature. This is certainly a *vera causa*, and will tend to bring observation and calculation into closer agreement.

Again, the high specific gravity of the earth, as compared with that

of the surface rocks, makes it probable that the interior is largely metallic, and it is possible that, even at depths less than that assigned to the magnetic floor, iron may exist in large quantities with the very high permeability it possesses at high temperatures.

Lastly, the basic rocks of Mull give a much smaller mean value of κ than those of Skye and Ardnamurchan. If the underground rocks were as permeable as these, all difficulty would vanish.

On the other hand, there are some considerations which point to the opposite direction. Thus, Professor Judd informs me that he leans to the opinion that in igneous magmas subject to the great pressures which obtain at considerable depths iron tends to form silicates rather than magnetic oxide. If this is so, the permeability of the surface rocks may be a maximum rather than a minimum, unless native iron exists in large quantity at great depths. The effect of pressure on permeability is an unknown factor, which might support or weaken the argument.

Points such as these are, however, outside the scope of this paper. But, though it is obviously unwise to be dogmatic on a question which is still surrounded by difficulties, I think that the result of the present enquiry is much in favour of the rock-magnetism theory of regional magnetic disturbances.

FIG. 1.

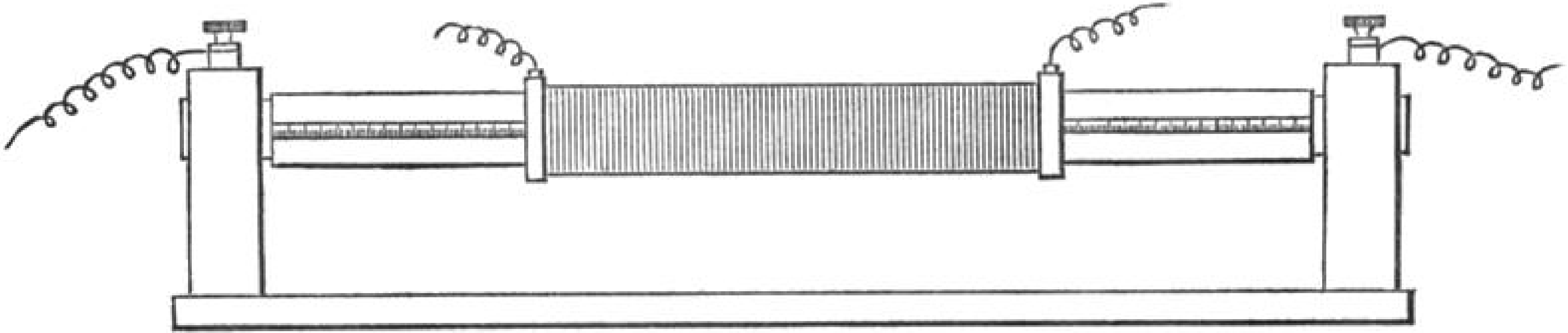


FIG. 2.