Washington (Major F. P.) Lecture on the Methods and Processes of the Ordnance Survey. 8vo. 1890. The Author.

Woodward (A. S.) and C. D. Sherborn's Catalogue of British Fossil Vertebrata. Supplement for 1890. [Extracted from Geol. Mag.] 8vo. *Hertford* 1891. The Authors.

Wolf (R.) Astronomische Mittheilungen. No. 76. 8vo. [*Zürich*] 1890. Dr. Wolf.

April 9, 1891.

Sir WILLIAM THOMSON, D.C.L., LL.D., President, followed by the Treasurer, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "On Electrostatic Screening by Gratings, Nets, or Perforated Sheets of Conducting Material." By Sir WILLIAM THOMSON, D.C.L., P.R.S. Received April 2, 1891.

I. *Grating.*

1. Maxwell, in his "Theory of a Grating of Parallel Wires" ('Electricity and Magnetism,' Arts. 203—205,* and Plate XIII), gives a very valuable and interesting two-dimensional investigation of electrostatic screening, and a most instructive diagram of "Lines of Force near a Grating," which powerfully helps to understand and extend the theory, and to acquit it of an accusation wrongly made against it in the last two sentences of Art. 205. It is only on the supposition of the grate-bars being circular cylinders that the investigation is less than rigorous: and that supposition nowhere enters into the investigation; it merely appears in the word "radius," in the first line of the last sentence but one of Art. 204, and it is contradicted in lines 3 and 4, and by the rest of the sentence, and by the next sentence. (See § 6 below.)

2. The conclusion, "$\alpha = -0{\cdot}11\,a$," in the last sentence of Art. 205, condemned as "evidently erroneous," is quite correct, and very interesting. It shows that a corrugated metal plate agreeing with

* In formula (7) of Art. 204, delete λ; in Art. 204, delete 2 in last line of p. 250 (Edition 1873); and delete 2 in lines 6 and 16 from foot of the page.

the equipotential surface $c = \frac{1}{2}a$, exceeds in electrostatic capacity a plane metal surface through the poles of the diagram (Plate XIII, reproduced in § 9 below), with the surroundings described in Art. 204, and supplies the datum requisite for finding the exact amount of the excess. The reason for the greatness of the excess clearly is that the surface $c = \frac{1}{2}a$, which just touches the plane through the poles of the diagram midway between the poles, is everywhere nearer than this plane to the other plate of the condenser. (See § 7 below.)

3. For $c = a/6$ we have, by (11) of Art. 205, $\alpha = 0$, and the corresponding equipotential, partially shown in Maxwell's diagram, is a set of curves concave towards $z = -\infty$, and asymptotic to the lines $x = (i \pm \frac{1}{4})a$, i denoting any integer. (See §§ 10 to 13 below.) For every value of c less than $a/6$, the equipotential is a row of ovals; and the grating formed by constructing these ovals in metal has less electrostatic capacity in the circumstances described in Art. 205 than a plane through the poles or the ovals (this being no doubt what is meant by "a plane . . . in the same position" as the grating).

4. For every value of c exceeding $a/6$ the equipotential, instead of being the boundary of a grating, is a continuous corrugated surface, and its electrostatic capacity exceeds that of the plane through the poles.

5. Begin now afresh, and let it be required to find the electric force in the air on either side of an infinite row of parallel bars at equal consecutive distances, a, each uniformly charged with electricity. Let ρa be the quantity per unit length on each bar, so that ρ would be the surface density, if the same quantity were uniformly distributed over the plane of the bars. Taking O in one of the bars, OX perpendicular to the bars, and OZ perpendicular to their plane, we find (by Fourier's method) for the z-component of force at any point (x, z) for which z is positive,

$$Z = \frac{4\pi\rho}{a}\left(\tfrac{1}{2} + \epsilon^{-mz}\cos mx + \epsilon^{-2mz}\cos 2mx + \text{\&c.}\right) \ldots\ldots (1),$$

where

$$m = 2\pi/a \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Summing this we find

$$Z = \frac{2\pi\rho}{a}\,\frac{\epsilon^{mz} - \epsilon^{-mz}}{\epsilon^{mz} - 2\cos mx + \epsilon^{-mz}} \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

This has equal positive and negative values for equal positive and negative values of z, and it therefore gives the value of the z-force, not only for positive, but also for negative, values of z. Taking now $-\int Z dz$, with constant assigned to make the integral zero for $z = \pm D$, we find

$$V = \rho a\left(\log\frac{1}{\epsilon^{mz} - 2\cos mx + \epsilon^{-mz}} + mD\right) \ldots\ldots\ldots (4)$$

as the potential due to the grating, and two parallel planes at equal distances, D, on its two sides, each uniformly electrified with half the quantity of electricity of opposite sign to that on the grating.

6. If now we construct in metal, C, any one complete equipotential surface, V_0, of this system, and electrify it with the same quantity of electricity as that which we gave originally to the infinite row of infinitely thin bars; and if we place metal planes, B, B′, at the two places of zero-potential ($z = \pm D$), we have an insulated conductor at potential V_0, between two planes, B, B′, at zero potential, and at distance 2D asunder, on each of which the electric density is $\frac{1}{2}\rho$. For brevity, I shall denote the insulated conductor by I.

Its electrostatic capacity per unit area of its medial plane (the plane of the original infinitely thin bars) is ρ/V_0.

7. This conductor, I, is symmetrical on each side of its medial plane, and consists either of an infinite number of isolated parallel bars, each surrounding one of the original infinitely thin bars, or of a plate symmetrically corrugated on its two sides, with maximum and minimum thicknesses respectively at the places of the infinitely thin bars, and the lines midway between them. For the case of isolated bars, let $2c$ be the diameter of each, in the medial plane. Then, to find V_0, we must put $x = \pm c$ and $z = 0$, in (4). Thus we find

$$V_0 = 2\pi\rho\left(\frac{a}{2\pi}\log\frac{1}{4\sin^2\frac{\pi c}{a}} + D\right) \qquad \text{(5).}$$

Hence the electrostatic capacity of I in the circumstances is

$$1\Big/\left\{2\pi\left(D + \frac{a}{2\pi}\log\frac{1}{4\sin^2\frac{\pi c}{a}}\right)\right\} \qquad \text{(6),}$$

which is greater or less than $1/(2\pi D)$, the electrostatic capacity that it would have if reduced to its medial plane, according as $c >$ or $< \frac{1}{6}a$. The conductor I, to be a grating, implies $c < \frac{1}{2}a$, or $\sin^2\frac{\pi c}{a} < 1$, and therefore requires that

$$V_0 > 2\pi\rho\left(D - \frac{a}{2\pi}\log 4\right) = 2\pi\rho\,(D - {\cdot}22a) \qquad \text{(7).}$$

When V_0 exceeds this critical value, the conductor I is the continuous plate corrugated on each side, which was described in § 7. The critical value corresponds to an intermediate case of a plate so deeply furrowed on each side as to be just cut through by its two surfaces crossing at right angles; and (7) shows that the electrostatic

capacity of the conductor I so constituted is equal to that of a plane sheet of thickness

$$2a \log [2^2]/(2\pi), \text{ or } \cdot 44a \quad \ldots\ldots\ldots\ldots\ldots\ldots (8),$$

insulated midway between the two earth plates B, B′, at the same distance asunder as they had with I between them.

8. By (4), (5), and (7), we have for the equation of the surface constituting the two sides of I in this critical case,

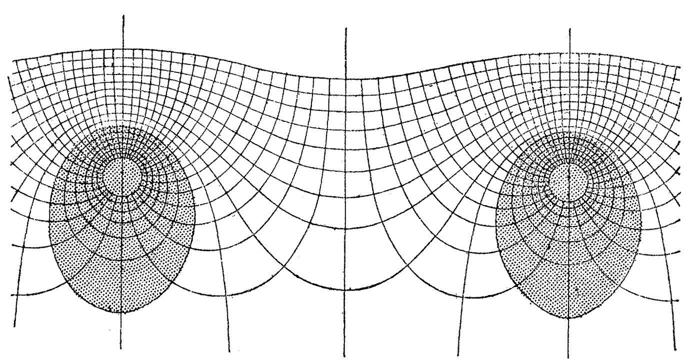
$$\epsilon^{mz} - 2\cos mx + \epsilon^{-mz} = 4 \quad \ldots\ldots\ldots\ldots\ldots\ldots (9).$$

Taking double the positive value of z which this gives when $x = 0$, we find

$$2a \log [(1+\sqrt{2})^2]/(2\pi), \text{ or } \cdot 562a \quad \ldots\ldots\ldots\ldots (10)$$

as the maximum thickness of I. This is $\log(1+\sqrt{2})/\log 2$, or 1·273, times the amount shown in (8) for the thickness of the plane-sided plate of equal electrostatic capacity; which is just such a relation as is expected before calculation!

9. If $\phi(z, x)$ denote what V becomes when in place of mD we substitute $-mz$ in (4), we have the potential due to a uniform electrical force ρam, or $2\pi\rho$, added to the z-component, of the force due to the grating with its given charge of ρa quantity per unit length of each bar; and it is the equipotentials and lines of force of this system that are represented in Maxwell's diagram of Plate XIII, reproduced here.



In it the resultant force for infinitely large positive values of z is parallel to OZ, and of constant value $4\pi\rho$; and it is zero for infinitely large negative values of z. The approximation to these values is very close, at only so moderate a distance as a on either side of the grating.

10. Choosing, in the system of §6, any one of the multiple-oval

equipotentials around the infinitely thin bars, that indicated by the shading, for example which I have added to Maxwell's diagram, let c be the distance from the infinitely thin primary bar within it, at which it is cut by the plane of the primary bars. By putting, in the expression for $\phi(z, x)$, $z = 0$, and $x = c$, we find

$$\phi(0, c) = \rho a \log \frac{1}{4 \sin^2 \frac{\pi c}{a}} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

as the potential at the surface of each of these chosen ovals. Construct now each of these ovals in metal, and let the supposed uniform force, $2\pi\rho$, be produced by uniform electrification of density $-\rho$, on a metal plane, B, at any great distance, b, on the positive side of the grating. We thus construct a grating of thick bars of oval-shaped cross section which, when electrified with the same quantity of electricity as that which we gave initially to the infinitely thin bars, and subjected to the influence of the equal quantity of negative electricity on B, has $\phi(z, x)$ for potential through external space from B ($z = b$), to infinite distance on the other side of the grating ($z = -\infty$), and has for potential through all the portions of space within the surfaces of the grate-bars the constant value expressed by (11). In this system the potential, for positive values of z great in comparison with a, is, by (4) with $-mz$ instead of $+m$D,

$$\phi(z,x) \doteq -4\pi\rho z \quad \ldots\ldots\ldots\ldots\ldots (12).$$

The difference of potentials between B and the grating is, by (6) and (5),

$$\phi(0, c) - \phi(b, x) \doteq 4\pi\rho \left(b + \frac{a}{4\pi} \log \frac{1}{4 \sin^2 \frac{\pi c}{a}} \right) \quad \ldots\ldots (13).$$

Hence the electrostatic capacity of the mutually insulated system, B, and the grating of oval-shaped bars is equal to the capacity of a pair of parallel planes, B and a plane at a distance beyond the plane of the primitive infinitely thin bars equal to

$$\frac{a}{4\pi} \log \frac{1}{4 \sin^2 \frac{\pi c}{a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14).$$

11. If in (4) we put $-nz$ in place of $+m$D, we have the potential of a system in which besides the electricity of the primary bars there is distant electricity such as all in all to give at great enough distances on the two sides of the primitive bars uniform fields of z-force respectively equal to

$$\rho(m+n), \text{ for } z \doteq +\infty; \text{ and } \rho(m-n), \text{ for } z \doteq -\infty \ldots\ldots (15).$$

If, in (4) with $-nz$ instead of $+m\text{D}$, we put

$$\epsilon^{-\text{V}/\rho a} = \text{C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

we find, as the equation of the equipotential surfaces,

$$-2\cos mx + \epsilon^{mz} + \epsilon^{-mz} = \text{C}\epsilon^{-nz} \ldots\ldots\ldots\ldots (17).$$

By taking $n = 0$, or $n = m$, we fall back on the cases of §§ 5–8, and §§ 9, 10, respectively.

12. To find an approximate equation for the equipotentials at distances around primitive bars small in comparison with a, the distance from bar to bar, let x and z be so small that we may neglect all powers of mx, mz, and nz, above the square, which implies that C is small of the same order as $(mx)^2$ and $(mz)^2$, (11) becomes

$$\left\{ \begin{array}{c} x^2 + \left(\dfrac{z+\frac{1}{2}nr^2}{1+\frac{1}{4}n^2r^2}\right)^2 = r^2\left(1+\frac{1}{4}n^2r^2\right) \\ \text{where } r^2 \text{ denotes } m^{-2}\text{C}. \end{array} \right\} \ldots\ldots\ldots (18).$$

This shows that, to the degree of approximation in which we neglect cubes and higher powers of mx, mz, nz, the equipotential is a row of elliptic cylinders of eccentricity $nr/\sqrt{2}$, with their greater diametral planes perpendicular to the plane of the primitive bars. When $n = 0$, the equipotential is a row of circular cylinders having the primitive bars for their axes; and this is true to the higher approximation in which we need only neglect powers above the cubes of mx and mz, as we see by going back to equation (17), with $n = 0$.

13. The conclusions of § 12 are useful for detailed investigation of the screening effect of plane gratings of circular or elliptic, straight parallel bars electrified with given quantities of electricity and placed with their planes perpendicular to the lines of force in a uniform field of force, and to corresponding problems in which potentials are given, as in Maxwell's §§ 203—205.

14. Instead of a single row of parallel equidistant infinitely thin bars in one plane, let us take for primitives two or more such rows, parallel or not parallel, all in one plane or not in one plane. We may thus form an endless variety of force-systems available for illustrating or helping to solve problems which may occur. Towards the several problems of electric screening we find important contributions by considering in two parallel planes rows of primitive lines parallel to one another for one case and perpendicular to one another for another case. The consideration of three rows of primitive lines in one plane, dividing it into equal and similar triangles alternately oriented in

opposite directions, leads to a complete theory of electrostatic screening by a triangular lattice of metallic wire or ribbon. The fundamental potential formula for this system obtained by summation of expressions, each given by an application of (4) to one of the three rows, is

$$V = \log \frac{1}{(\epsilon^{lz}-2\cos lp+\epsilon^{-lz})^{\varpi a}(\epsilon^{mz}-2\cos mq+\epsilon^{-mz})^{\rho b}(\epsilon^{nz}-2\cos nr+\epsilon^{-nz})^{\sigma c}} + 2\pi(\varpi+\rho+\sigma)D \quad \ldots\ldots (19);$$

where a, b, c denote the intervals between the successive lines of the three systems; ϖa, ρb, σc, the quantities of electricity per unit length of bar in the three systems; p, q, r, z, special coordinates of the point for which (19) expresses the potential, viz., z, its distance from the plane of the primitive bars, and p, q, r its distances from three planes drawn perpendicular to this plane through a bar of each of the three systems; D the value of $\pm z$ for planes on the two sides of the net for which the potential is zero; and, lastly,

$$l = 2\pi/a,\ m = 2\pi/b,\ n = 2\pi/c \quad \ldots\ldots\ldots\ldots (20).$$

For the present, however, we may confine ourselves to the case of two rows of primitive lines dividing a plane into squares and charged, both rows, with equal quantities, $\frac{1}{2}\rho a$, of electricity per unit length. The potential formula, a particular case of (19), is

$$V = \tfrac{1}{2}\rho a \left[\log \frac{1}{(\epsilon^{mz}-2\cos mx+\epsilon^{-mz})(\epsilon^{mz}-2\cos my+\epsilon^{-mz})} + 2mD\right] \quad (21).$$

15. The consideration of the equipotentials of this surface is very interesting. The equipotential lines in the plane of the primitive bars are given by the equation

$$16\left(\sin\frac{\pi x}{a}\sin\right)\frac{\pi y^2}{a} = \epsilon^{-2V/\rho a} \quad \ldots\ldots\ldots\ldots (22).$$

16. Considerations quite analogous to those of §§ 6, 7, 8, and again the other considerations analogous to those of §§ 9—13, are, after the full explanations there given, easily completed so as to formulate a full theory of electrostatic capacity and electrostatic screening for square nets of wire exposed to electric action giving uniform fields of force at distances on each or on one side of the plane of the net considerable in comparison with a, the side of each square.

17. In what follows we shall for brevity call any thin sheet, whether plane or not plane, which answers to the description contained in the title of this paper, a *perforated sheet* or a *perforated surface;* understanding that its radii of curvature are everywhere large in com-

parison with its thickness. The diameters of holes must be large in comparison with the thickness in order that the approximations which we use below may be valid. We shall call the electric density of a perforated sheet the total quantity of electricity with which it is electrified, reckoned per unit area of continuous surface approximately agreeing with it, and passing through the middle of cage bars, bosses, &c. This continuous surface I shall call the medial surface, or sometimes, for brevity, *the medial.*

18. In what precedes we have virtually a complete investigation of the screening effect of a homogeneous plane perforated sheet against the electric force of a uniform field with lines of force perpendicular to the plane. Let it now be required to find the screening effect of a non-plane perforated sheet against a uniform field of electrostatic force, and of a perforated sheet S, plane or not plane, against the electrostatic force of any given electrified bodies.

19. Let ϕ be the potential of the given electrified bodies at any point (x, y, z) of the space occupied by S, and let ρ be the unknown electric density of S at (x, y, z), under the influence of those bodies. To make the problem of finding ρ determinate, we might suppose either the total quantity of electricity on S, or the potential at which its metal is kept, to be given. We shall take the latter supposition, and call the given potential C.

20. Let ϕ denote the potential which would be produced by the electricity of S if it were spread continuously over the medial with electric density equal to ρ at (x, y, z); and let

$$\phi+\mu\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

denote the potential in the metal of S, due to the actual distribution of electricity on its surface.

21. To understand the meaning of this notation (μ), consider a large area around (x, y, z), so large that its border is very distant from (x, y, z) in comparison with the thickness of the sheet, and with the diameters of its apertures, but not so large as to deviate sensibly from the tangent plane at (x, y, z). Let the electricity of all the surface of S beyond A be changed from the imagined continuous distribution to the actual distribution on the surface of the perforated metal. This change will make no sensible difference in the potential at (x, y, z). Next, let the imagined continuous distribution of uniform electric density ρ, over the continuous area A, be changed to the actual distribution of the same quantity over the surface of the perforated metal of the porous sheet A. The augmentation of potential at (x, y, z) produced by this charge is what we denote by $\mu\rho$, where μ is a coefficient depending on the shapes and magnitudes of the perforations, that is to say, on the complex surface of the perforated metal. It would be zero if there were no perforations, and we shall see that the

greater it is the less is the screening efficiency. We shall therefore call μ the electric permeability, and μ^{-1} the electric screening efficiency of the perforated sheet. The sheet is homogeneous as to permeability or screening efficiency if μ has the same value for all parts of it, but we need not assume this to be the case; on the contrary, we shall suppose μ to be any known function of (x, y, z). In §§ 5—16 we have the explanations necessary for determining μ in the various cases of gratings and nets there described. For similarly perforated surfaces, the values of μ are as the linear dimensions of a perforation or of the bars or bosses of the structures.

22. The equation of electric equilibrium is

$$\phi + \mu\rho = \mathrm{K} \text{ (a constant)} \quad \ldots\ldots\ldots \quad (24),$$

when S, being insulated and electrified, is not under the influence of any other electrified matter.

It is

$$\phi + \mu\rho = \mathrm{K} - \mathrm{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25),$$

when S is under the influence of any given electrified bodies producing a given potential, V, at (x, y, z).

23. As a first example, going back to (24), let μ be such that ϕ shall be constant. This makes, if we denote by k a constant,

$$\mu = k\phi/\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (26),$$

(k being a constant), which means that the screening efficiency is, in different places of S, inversely proportional to the electric density at similarly situated places of a continuous electrified conductor of the same shape as S. Let, for instance, S be an ellipsoid; then, if the sizes of the perforations be inversely proportional to the perpendicular from the centre to the tangent plane, (26) is satisfied. Generally, to fulfil this condition, the net must be finer in the more convex and more projecting parts, and coarser in the flatter and less projecting parts.

24. If any perforated conductor or cage, S, fulfilling the condition of § 23, be electrified and insulated away from the disturbing influence of other conductors, or electrified bodies, the charge distributes itself so as to have in every part the same quantity per unit area of the medial, as a smooth continuous metallic surface agreeing with the medial and electrified with the same total quantity. When the medial is a closed surface, the electricity on the perforated surface does not confine itself to the parts of it outside the medial: on the contrary, when the apertures are very wide in comparison with diameters of cage-bars, bosses, &c., the electricity distributes itself almost equally on the parts of the complex surface inside and outside the medial.

25. Seeing that the electric density (as defined in § 17) is the

same for a perforated surface fulfilling the condition of § 23 as for the medial constructed in continuous metal, we naturally ask the question, what then is the difference between the two cases, if any, besides the fact of the electricity being equally but very unequably distributed over the outer and inner portions of the complex surface in one case, and equably over the outside of the smooth medial in the other? There is a very important and interesting difference. The electrostatic capacity of the perforated conductor, S, is less, in the ratio of 1 to $1 + k$, than that of the medial constructed in continuous metal; as we see by (23) and (26).

26. As a sub-example, suppose S to be a spherical surface. If homogeneously perforated, it will fulfil the condition of § 23: and if its screening efficiency is the same as that of a grating of parallel bars (circular cross section of diameter $2r$; distance from centre to centre a), we have, by (5) of § 7, when $\pi c/a$ is very small,

$$\mu = 2a \log \frac{a}{2\pi c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (27).$$

Now, S being spherical, if R denotes its radius, we have (§ 20)

$$\phi = 4\pi R\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots (28).$$

Hence, by (26) and (27),

$$k = \frac{a}{2\pi R} \log \frac{a}{\pi c} = \frac{1}{N} \log \frac{a}{2\pi c} \quad \ldots\ldots\ldots\ldots (29),$$

where N denotes the number of bars in the equatorial belt of the cage of § 27 below.

27. To illustrate a realisation of § 26, let a spherical cage be made up of a narrow equatorial belt of approximately straight parallel bars of diameter $2c$, and distance from middle of one bar to middle of next, a; completed by polar caps (nearly hemispheres) of thin metal perforated so as to have everywhere the same effective electric screening efficiency $1\{2a \log (a/2\pi c)\}$.

Suppose, for instance, the bars to be of "No. 18 gauge" ($2c = 0{\cdot}122$ cm.) and $a = 5$ cm. We have

$$\log (a/2\pi c) = \log 13 = 2{\cdot}57.$$

Hence, for this case, and any other in which the ratio a/c is the same, we have, by (27) and (29),

$$\mu = 5{\cdot}14\, a \quad \ldots\ldots\ldots\ldots\ldots\ldots (30),$$

$$k = 0{\cdot}409 \frac{a}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (31).$$

Thus, if $a = 5$ cm., and $R = 50$ cm., $k = 0{\cdot}0409 \doteq \frac{1}{24}$; and (§ 25) the electrostatic capacity of the spherical cage $\frac{24}{25}$ of that of a simply continuous spherical surface of the same magnitude.

28. Let now an electrified metal globe, or globe of insulating material uniformly electrified, G, be insulated concentrically within S. It may be of any magnitude, large or small, provided only that the interval between the two surfaces be at least two or three times the diameter of the largest of the perforations of S. Let S be connected with the earth, and let Q denote the quantity of (positive) electricity with which G is electrified, and Q′ the quantity of the opposite electricity which it induces on S. The potential in the metal of S due to Q′ is, by (23),

$$-\left(\frac{Q'}{R}+\mu\frac{Q'}{4\pi R^2}\right) \qquad (32).$$

This, added to Q/R, the potential due to G, must be zero, and therefore

$$Q = Q\left(1+\frac{\mu}{4\pi R}\right) \qquad (33),$$

or, by (26),

$$Q = Q'(1+k) \qquad (34).$$

Hence, in the particular case of § 27 (31),

$$Q = Q'\left(1+0{\cdot}409\frac{a}{R}\right) \qquad (35);$$

and when $R = 10a$, we find $Q - Q' \doteq \frac{1}{25}Q$, and conclude that the effect of S, earthed, with G electrified and insulated within it, is just 4 per cent. of the effect of G unscreened.

29. If S is connected with the earth, and supported at a height above the earth equal to at least six or eight times its diameter, the quantity of electricity (positive in fine weather) induced on it will be $1/(1+k)$ of that which would be induced on a simply continuous metal globe of the same size. Hence the potential at any point of the air within S at not less distance inwards than $2a$ will be $k/(1+k)$ of the undisturbed atmospheric potential at the same height above the ground, or 5 per cent. in our particular case. This is quite in accordance with the imperfectness of the screening effect against atmospheric electricity found by Roiti* within earthed wire cages, supported at a considerable height above the ground, by a bracket attached to the top of a wall of a building in Florence, tested by a water-dropper with its nozzle inside the cage con-

* "Osservazioni Continue della Elettricità Atmosferica" ('Pubblicazioni del R. Istituto di Studi Superiori in Firenze'), Florence, 1884.

nected by an insulated wire with a quadrant electrometer in the buildings.

30. The problem of finding the distribution of electricity on a spherical cage, of equal electric permeability, μ, in all parts of its surface, formulated in (25) of § 22, is easily solved by aid of spherical harmonics. Confining ourselves for brevity to the case of external influencing bodies, let their potential at any point, P, within S be

$$V = -\Sigma S_i \frac{r^i}{R^i} \quad \dots\dots\dots\dots\dots\dots (36),$$

where S_i denotes a given spherical surface-harmonic of order i, and r the distance of P from the centre of S. And ρ_i, denoting an unknown surface-harmonic of order i, let

$$\rho = \Sigma \rho_i \quad \dots\dots\dots\dots\dots\dots (37)$$

be the harmonic expression for ρ, the required electric density. Going back to § 20 for the definition of ϕ, we find, by the elements of spherical harmonics,

$$\phi = \Sigma \frac{4\pi\rho_i}{2i+1} \frac{r^i}{R^{i-1}} \quad \dots\dots\dots\dots\dots\dots (38).$$

Hence, by (25),

$$\rho_0 = \frac{K+S_0}{4\pi R+\mu} \quad \dots\dots\dots\dots\dots\dots (39),$$

$$\rho_i = \frac{(2i+1)S_i}{4\pi R} \frac{1}{1+\frac{(2i+1)\mu}{4\pi R}} \quad \dots\dots\dots\dots\dots\dots (40),$$

and

$$\phi = K + \Sigma \frac{1}{1+\frac{(2i+1)\mu}{4\pi R}} \frac{S_i r^i}{R^i} \quad \dots\dots\dots\dots\dots\dots (41).$$

In (39) we have virtually the same result as in (33). The approximation on which we are founding in §§ 17—29 is valid in (40) and (41) only for values of i small in comparison $2\pi R/a$: but, as in virtue of greatness of the logarithm for the case formulated in (27), μ may be great in comparison with a; and therefore the denominator of (40) need not be only infinitesimally greater than unity, and may be any numeric however great.

31. Taking $S_1 r = x$, $S_2 = 0$, $S_3 = 0$ ...., we see by (41) that if an insulated unelectrified spherical cage be brought into a uniform field of electric force, X (that of atmospheric electricity, for example, at any height above the ground exceeding five or six diameters of the cage), the force within the cage is

$$X \frac{X}{1+\frac{3\mu}{4\pi R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42),$$

or, according to (27), and (29),

$$X \frac{X}{1+\frac{3a}{2\pi R}\log\frac{a}{2\pi c}}, \quad \text{or} \quad X \frac{X}{1+\frac{3}{N}\log\frac{a}{2\pi c}} \ldots\ldots (43).$$

This result is also applicable to a hemispherical screen of radius R, simply placed on the ground. For the particular proportions of § 27, it makes the force under the hemispherical cage $\frac{1}{9}$ of the undisturbed force outside. A cage of ordinary gardener's (anti-rabbit) hexagonal wire-net (of $5\frac{1}{2}$ cm. from parallel to parallel) cannot be very different from this. If, instead of the radius being 50 cm. it be 200, but the cage still of the same net, the force inside would be only 3 per cent. of the undisturbed force outside.

32. In every case the force at any distance from the perforated surface, on either side of it, more than the diameter of a perforation, is, as is easily proved by Fourier's methods, very nearly the same as if the electricity were spread equably over the medial surface, with the same quantity per unit area of the medial as the grating has in each part of it. Hence, in the case of § 31, the force is uniform throughout the interior of the cage, except within distances from the net of two or three times the aperture. Hence a second screen, similar but slightly smaller, placed inside the first will reduce the force farther in the same ratio; so that, if eX denote the force inside the single screen, the force inside the inner screen when there are two will be e^2X, provided the distance between the two is nowhere less than the diameter of the perforation. Thus, with screens such as those in the last particular case of § 31, the force inside the inner screen would be only 9/10,000 of the undisturbed force far enough outside the outer. The two screens, if placed close together, so as to narrow the apertures as much as possible, would have little more than double the screening efficiency of either singly, as we may judge from (27) of § 26, and from (21) of § 14. The principle that, to duplicate a screen with best advantage, the two screens should be placed, not in one surface but in two, with not less distance between them than the diameter of their apertures, is not only theoretically interesting, but is of great practical importance in the screening of electrometers against disturbing electric force.

33. Questions analogous to those of §§ 26—32, but for circular cylindric (mouse-mill) cages of equidistant parallel bars, instead of the spherical or hemispherical cage which we have been considering, are readily answered by the simpler work corresponding to that of

§ 30 (with sin $i\theta$ and cos $i\theta$ instead of spherical harmonics). But it deserves more complete synthetic investigation, not limited by the approximational conditions of §§ 21, 22, if for no other reason, because of Hertz's mouse-mill. This must, however, be reserved for a future communication. Meantime, it is worth saying that sudden variations of electric current, or alternating electric currents, distribute themselves between different straight parallel conductors in the same proportion as static electrification is distributed in corresponding electrostatic arrangements, whenever the suddenness, or the frequency, is sufficient to cause the impedance by mutual induction of the separate parallel conductors (and therefore, *a fortiori*, the impedance by self-induction of each) to be very large in comparison with ohmic resistance. Hence Hertz's mouse-mill screening follows (though by utterly different physical action), simply the electrostatic law, except in any case in which his wave-length is less than a considerable multiple of the diameter of his mouse-mill.

II. "On Variational Electric and Magnetic Screening." By Sir W. THOMSON, P.R.S. Received April 1, 1891.

1. A screen of imperfectly conducting material is as thorough in its action, when time enough is allowed it, as is a similar screen of metal. But if it be tried against rapidly varying electrostatic force, its action lags. On account of this lagging, it is easily seen that the screening effect against periodic variations of electrostatic force will be less and less, the greater the frequency of the variation. This is readily illustrated by means of various forms of idiostatic electrometers. Thus, for example, a piece of paper supported on metal in metallic communication with the movable disc of an attracted disc electrometer annuls the attraction (or renders it quite insensible) a few seconds of time after a difference of potential is established and kept constant between the attracted disc and the opposed metal plate, if the paper and the air surrounding it are in the ordinary hygrometric condition of our climates. But if the instrument is applied to measure a rapidly alternating difference of potential, with equal differences on the two sides of zero, it gives very little less than the same average force as that found when the paper is removed and all other circumstances kept the same. Probably, with ordinary clean white paper in ordinary hygrometric conditions, a frequency of alternation of from 50 to 100 per second will more than suffice to render the screening influence of the paper insensible. And a much less frequency will suffice if the atmosphere surrounding the paper is artificially dried. Up to a frequency of millions per second, we may safely say that, the greater the frequency, the more perfect is the annulment of