

§ 30 (with  $\sin i\theta$  and  $\cos i\theta$  instead of spherical harmonics). But it deserves more complete synthetic investigation, not limited by the approximations of §§ 21, 22, if for no other reason, because of Hertz's mouse-mill. This must, however, be reserved for a future communication. Meantime, it is worth saying that sudden variations of electric current, or alternating electric currents, distribute themselves between different straight parallel conductors in the same proportion as static electrification is distributed in corresponding electrostatic arrangements, whenever the suddenness, or the frequency, is sufficient to cause the impedance by mutual induction of the separate parallel conductors (and therefore, *a fortiori*, the impedance by self-induction of each) to be very large in comparison with ohmic resistance. Hence Hertz's mouse-mill screening follows (though by utterly different physical action), simply the electrostatic law, except in any case in which his wave-length is less than a considerable multiple of the diameter of his mouse-mill.

## II. "On Variational Electric and Magnetic Screening." By Sir W. THOMSON, P.R.S. Received April 1, 1891.

1. A screen of imperfectly conducting material is as thorough in its action, when time enough is allowed it, as is a similar screen of metal. But if it be tried against rapidly varying electrostatic force, its action lags. On account of this lagging, it is easily seen that the screening effect against periodic variations of electrostatic force will be less and less, the greater the frequency of the variation. This is readily illustrated by means of various forms of idiostatic electrometers. Thus, for example, a piece of paper supported on metal in metallic communication with the movable disc of an attracted disc electrometer annuls the attraction (or renders it quite insensible) a few seconds of time after a difference of potential is established and kept constant between the attracted disc and the opposed metal plate, if the paper and the air surrounding it are in the ordinary hygrometric condition of our climates. But if the instrument is applied to measure a rapidly alternating difference of potential, with equal differences on the two sides of zero, it gives very little less than the same average force as that found when the paper is removed and all other circumstances kept the same. Probably, with ordinary clean white paper in ordinary hygrometric conditions, a frequency of alternation of from 50 to 100 per second will more than suffice to render the screening influence of the paper insensible. And a much less frequency will suffice if the atmosphere surrounding the paper is artificially dried. Up to a frequency of millions per second, we may safely say that, the greater the frequency, the more perfect is the annulment of

screening by the paper; and this statement holds also if the paper be thoroughly blackened on both sides with ink, although possibly in this condition a greater frequency than 50 to 100 per second might be required for practical annulment of the screening.

2. Now, suppose, instead of attractive force between two bodies separated by the screen, as our test of electrification, that we have as test a faint spark, after the manner of Hertz. Let two well insulated metal balls, A, B, be placed very nearly in contact, and two much larger balls, E, F, placed beside them, with the shortest distance between E, F sufficient to prevent sparking, and with the lines joining the centres of the two pairs parallel. Let a rapidly alternating difference of potential be produced between E and F, varying, not abruptly, but according, we may suppose, to the simple harmonic law. Two sparks in every period will be observed between A and B. The interposition of a large paper screen between E, F, on one side, and A, B, on the other, in ordinary hygrometric conditions, will absolutely stop these sparks, if the frequency be less than, perhaps, 4 or 5 per second. With a frequency of 50 or more, a clean white paper screen will make no perceptible difference. If the paper be thoroughly blackened with ink on both sides, a frequency of something more than 50 per second may be necessary; but some moderate frequency of a few hundreds per second will, no doubt, suffice to practically annul the effect of the interposition of the screen. With frequencies up to 1000 million per second, as in some of Hertz's experiments, screens such as our blackened paper are still perfectly transparent, but if we raise the frequency to 500 million million, the influence to be transmitted is light, and the blackened paper becomes an almost perfect screen.

3. Screening against a varying magnetic force follows an opposite law to screening against varying electrostatic force. For the present, I pass over the case of iron and other bodies possessing magnetic susceptibility, and consider only materials devoid of magnetic susceptibility, but possessing more or less of electric conductivity. However perfect the electric conductivity of the screen may be, it has no screening efficiency against a steady magnetic force. But if the magnetic force varies, currents are induced in the material of the screen which tend to diminish the magnetic force in the air on the remote side from the varying magnet. For simplicity, we shall suppose the variations to follow the simple harmonic law. The greater the electric conductivity of the material, the greater is the screening effect for the same frequency of alternation; and, the greater the frequency, the greater is the screening effect for the same material. If the screen be of copper, of specific resistance 1640 sq. cm. per second (or electric diffusivity 130 sq. cm. per second), and with frequency 80 per second, what I have called the "mhoic effective

thickness"\* is 0.71 of a cm.; and the range of current intensity at depth  $n \times 0.71$  cm. from the surface of the screen next the exciting magnet is  $e^{-n}$  of its value at the surface.

Thus (as  $e^3 = 20.09$ ) the range of current-intensity at depth 2.13 cm. is  $\frac{1}{20}$  of its surface value. Hence we may expect that a sufficiently large plate of copper of  $2\frac{1}{4}$  cm. thick will be a little less than perfect in its screening action against an alternating magnetic force of frequency 80 per second.

4. Lord Rayleigh, in his "Acoustical Observations" ('Phil. Mag.,' 1882, first half-year), after referring to Maxwell's statement, that a perfectly conducting sheet acts as a barrier to magnetic force ('Electricity and Magnetism,' § 665), describes an experiment in which the interposition of a large and stout plate of copper between two coils renders inaudible a sound which, without the copper screen, is heard by a telephone in circuit with one of the coils excited by electromagnetic induction from the other coil, in which an intermittent current, with sudden, sharp variations of strength, is produced by a "microphone clock" and a voltaic battery. Larmor, in his paper on "Electromagnetic Induction in Conducting Sheets and Solid Bodies" ('Phil. Mag.,' 1884, first half-year), makes the following very interesting statement:—"If we have a sheet of conducting matter in the neighbourhood of a magnetic system, the effect of a disturbance of that system will be to induce currents in the sheet of such kind as will tend to prevent any change in the conformation of the tubes [lines] of force cutting through the sheet. This follows from Lenz's law, which itself has been shown by Helmholtz and Thomson to be a direct consequence of the conservation of energy. But if the arrangement of the tubes [lines of force] in the conductor is unaltered, the field on the other side of the conductor into which they pass (supposed isolated from the outside spaces by the conductor) will be unaltered. Hence, if the disturbance is of an alternating character, with a period small enough to make it go through a cycle of changes before the currents decay sensibly, we shall have the conductor acting as a screen.

"Further, we shall also find, on the same principle, that a rapidly rotating conducting sheet screens the space inside it from all magnetic action which is not symmetrical round the axis of rotation."

Mr. Willoughby Smith's experiments on "Volta-electric induction," which he described in his inaugural address to the Society of Telegraph Engineers of November, 1883, afforded good illustrations of this kind of action with copper, zinc, tin, and lead, screens, and with different degrees of frequency of alternation. His results with iron are also very interesting: they showed, as might be expected, comparatively little augmentation of screening effect with augmentation

\* 'Collected Papers,' vol. 3, Art. cii, § 35.

of frequency. This is just what is to be expected from the fact that a broad enough and long enough iron plate exercises a large magneto-static screening influence; which, with a thick enough plate, will be so nearly complete that comparatively little is left for augmentation of the screening influence by alternations of greater and greater frequency.

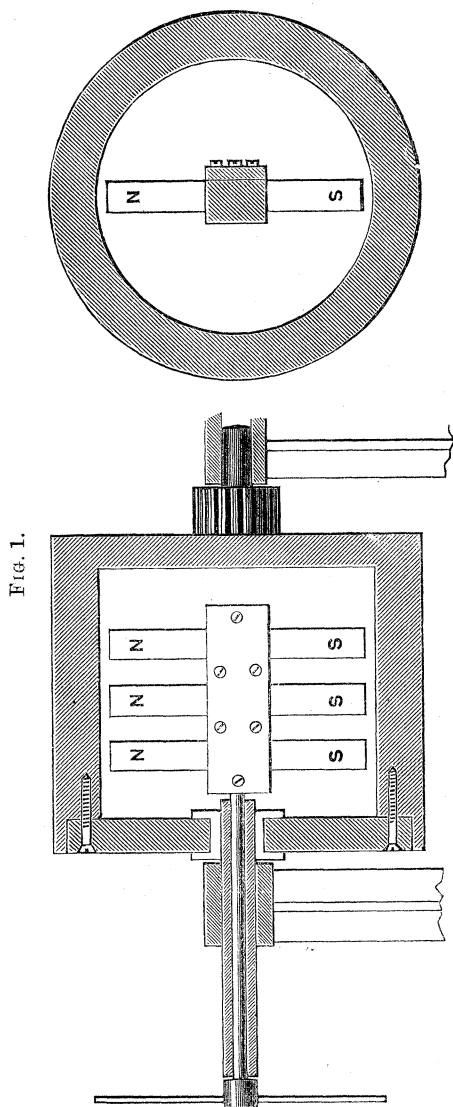
5. A copper shell closed around an alternating magnet produces a screening effect which on the principle of § 3 we may reckon to be little short of perfection if the thickness be  $2\frac{1}{4}$  cm. or more, and the frequency of alternation 80 per second.

6. Suppose now the alternation of the magnetic force to be produced by the rotation of a magnet *M* about any axis. First, to find the effect of the rotation, imagine the magnet to be represented by ideal magnetic matter. Let (after the manner of Gauss in his treatment of the secular perturbations of the solar system) the ideal magnetic matter be uniformly distributed over the circles described by its different points. For brevity call *I* the ideal magnet symmetrical round the axis, which is thus constituted. The magnetic force throughout the space around the rotating magnet will be the same as that due to *I*, compounded with an alternating force of which the component at any point in the direction of any fixed line varies from zero in the two opposite directions in each period of the rotation. If the copper shell is thick enough, and the angular velocity of the rotation great enough, the alternating component is almost annulled for external space, and only the steady force due to *I* is allowed to act in the space outside the copper shell.

7. Consider now, in the space outside the copper shell, a point *P* rotating with the magnet *M*. It will experience a force simply equal to that due to *M* when there is no rotation, and, when *M* and *P* rotate together, *P* will experience a force gradually altering as the speed of rotation increases, until, when the speed becomes sufficiently great, it becomes sensibly the same as the force due to the symmetrical magnet *I*. Now superimpose upon the whole system of the magnet, and the point *P*, and the copper shell, a rotation equal and opposite to that of *M* and *P*. The statement just made with reference to the magnetic force at *P* remains unaltered, and we have now a fixed magnet *M* and a point *P* at rest, with reference to it, while the copper shell rotates round the axis around which we first supposed *M* to rotate.

8. A little piece of apparatus, constructed to illustrate the result experimentally, is submitted to the Royal Society and shown in action. In the copper shell is a cylindric drum, 1.25 cm. thick, closed at its two ends with circular discs 1 cm. thick. The magnet is supported on the inner end of a stiff wire passing through the centre of a perforated fixed shaft which passes through a hole in one end of

the drum, and serves as one of the bearings ; the other bearing is a rotating pivot fixed to the outside of the other end of the drum. The accompanying sections, drawn to a scale of three-fourths full size, explain the arrangement sufficiently. A magnetic needle outside, deflected by the fixed magnet when the drum is at rest, shows a great diminution of the deflection when the drum is set to rotate.



If the (triple compound) magnet inside is reversed, by means of the central wire and cross bar outside, shown in the diagram, the magnetometer outside is greatly affected while the copper shell is at rest; but scarcely affected perceptibly while the copper shell is rotating rapidly.

9. When the copper shell is a figure of revolution, the magnetic force at any point of the space outside or inside is steady, whatever be the speed of rotation; but if the shell be not a figure of revolution, the steady force in the external space observable when the shell is at rest becomes the resultant of the force due to a fixed magnet intermediate between  $M$  and  $I$  compounded with an alternating force with amplitude of alternation increasing to a maximum, and ultimately diminishing to zero, as the angular velocity is increased without limit.

10. If  $M$  be symmetrical, with reference to its northern and southern polarity, on the two sides of a plane through the axis of rotation,  $I$  becomes a null magnet, the ideal magnetic matter in every circle of which it is constituted being annulled by equal quantities of positive and negative magnetic matter being laid on it. Thus, when the rotation is sufficiently rapid, the magnetic force is annulled throughout the space external to the shell. The transition from the steady force of  $M$  to the final annulment of force, when the copper shell is symmetrical round its axis of rotation, is, through a steadily diminishing force, without alternations. When the shell is not symmetrical round its axis of rotation, the transition to zero is accompanied with alternations as described in § 8.

11. When  $M$  is not symmetrical on the two sides of a plane through the axis of rotation,  $I$  is not null; and the condition approximated to through external space with increasing speed of rotation is the force due to  $I$ , which is an ideal magnet symmetrical round the axis of rotation.

12. A very interesting simple experimental illustration of screening against magnetic force may be shown by a rotating disc with a fixed magnet held close to it on one side. A bar magnet held with its magnetic axis bisected perpendicularly by a plane through the axis of rotation would, by sufficiently rapid rotation, have its magnetic force almost perfectly annulled at points in the air as near as may be to it, on the other side of the disc, if the diameter of the disc exceeds considerably the length of the magnet. The magnetic force in the air close to the disc, on the side next to the magnet, will be everywhere parallel to the surface of the disc.

FIG. 1.

