from different thermometers vary considerably, intermediate temperatures deduced from these curves are in practical agreement.

II. That thermometers made and graduated as described may be used for the accurate determination of temperatures up to about 500° C.

II. "On the alleged Slipping at the Boundary of a Liquid in Motion." By W. C. DAMPIER WHETHAM, B.A., Coutts Trotter Student of Trinity College, Cambridge. Communicated by J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics, Cambridge. Received June 7, 1890.

(Abstract.)

The experiments of Helmholtz and Piotrowski* on the oscillations of a metal sphere suspended bifilarly, and filled with various liquids, gave finite values to the slipping coefficients. The inside of the sphere was gilded and polished, and the value obtained for the coefficient λ was, in the case of distilled water, 2·3534 mm. From some experiments of Girard† on transpiration through copper tubes, Helmholtz deduces the value $\lambda = 0{\cdot}3984$ mm. for water flowing past a copper surface.

In treatises on hydrodynamics, it is shown that when the motion through a tube is linear, the flux is

$$\frac{1}{8}\frac{\pi r^4}{\rho\mu}\frac{p_1-p_2}{l}+\frac{1}{2}\frac{\pi r^3}{\beta}\frac{p_1-p_2}{l},$$

or

$$\frac{1}{8}\frac{\pi(p_1-p_2)}{\mu\rho l}\left\{r^4+4\mu\rho\frac{1}{\beta}r^3\right\}.$$

In Helmholtz's notation this becomes (ρ being taken as unity)

$$\frac{1}{8}\frac{\pi(p_1-p_2)}{\mu l}\{r^4+4\lambda r^3\}.$$

Putting $r = 0{\cdot}05$ and $\lambda = 0{\cdot}23534$, we get

$$\frac{1}{8}\frac{\pi(p_1-p_2)}{\mu l}\times 117{\cdot}67\times 10^{-6};$$

whereas if there is no slip, so that λ vanishes, the flux becomes

$$\frac{1}{8}\frac{\pi(p_1-p_2)}{\mu l}\times 6{\cdot}25\times 10^{-6}.$$

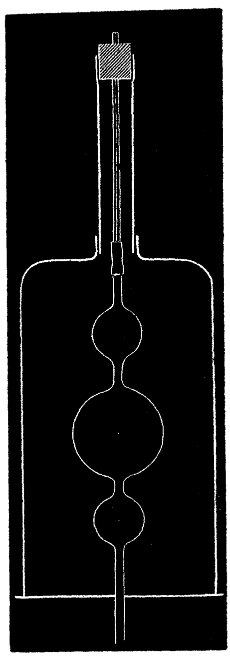
* 'Sitzungsberichte der Wiener Akademie,' vol. 40.
† 'Mémoires de l'Institut,' 1813—1815.

Thus the flow through a gilt tube of a millimetre in diameter should be twenty times as fast as through a tube where there is no slip. Poiseuille showed that for a glass tube $\lambda = 0$, and it had been generally supposed that this also held for other substances wetted by water.

Such a large effect as the above shows that the existence of the coefficient would be much better investigated by transpiration experiments than by oscillating spheres, and an investigation has been carried out on these lines.

In order to prevent absolute determinations, the time of flow of a known volume of water through a glass tube was observed, the interior of the tube silvered, and another observation taken with the same pressure and the same volume of water.

FIG. 1.

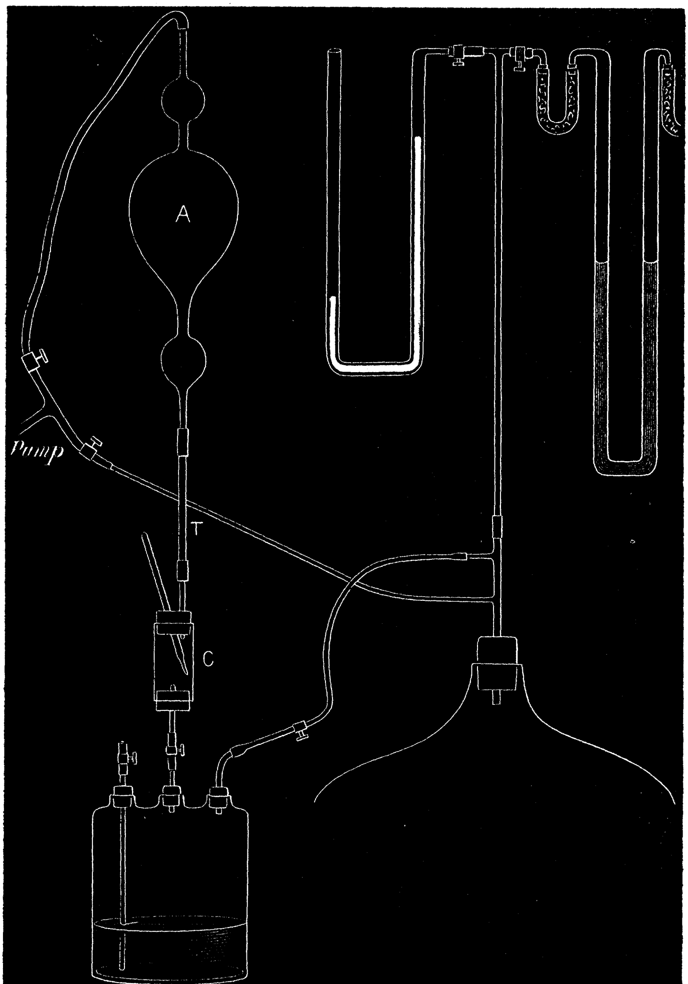


I. The water was allowed to run out of a bulb through the transpiration tube under its own pressure.

As the result of four series of observations on three different tubes, it was found that the times of flow for the glass tubes were the same to within 0·7 per cent. as the times for the silvered tubes, corrections being made for changes of temperature, and for the decrease

in diameter due to the silver layer. Different thicknesses of silver were used. The differences in the times were all within the limits of experimental error.

Fig. 2.



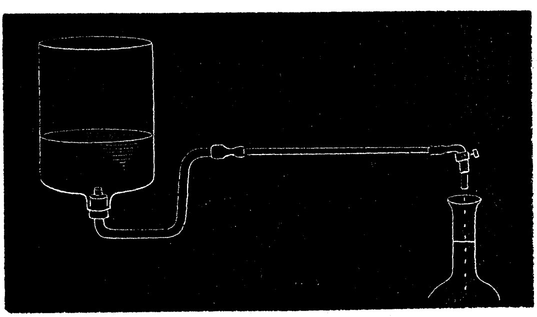
II. In order to determine whether any slipping occurred when the gradient of velocity was pushed near the limits of linear motion, as investigated by Prof. Osborne Reynolds,* the difference of pressure

* 'Phil. Trans.,' 1886.

was increased by placing a partially exhausted space in connexion with the lower end of the tube, the size of the bulb being also increased. The pressure was read by a kathetometer on a gauge of sulphuric acid or mercury, and a small correction (always less than 0·2 per cent.) made in order to compare the times of flow under the same pressure. The temperature of the water was observed immediately after it had passed the transpiration tube, by reading a thermometer immersed in the cylinder C, at equal intervals, while the water was running. The smallest tubes which could conveniently be silvered had a diameter of rather less than a millimetre, and in this series of experiments the tubes had diameters of 0·084 and 0·072 cm. The results of four series of experiments showed that in three cases the times for the silver and glass surfaces agreed to less than 0·5 per cent., and in one case, where the silver was rather too thick, and probably a little irregular, the time of flow for the silvered tube was 1 per cent. greater.

Even if the slipping effect was only half that deduced by Helmholtz and Piotrowski for gold, the times of flow should have been for these small tubes from fifteen to twenty times less for silver than for glass. The two series of experiments given above may therefore be considered a quite satisfactory proof of identity.

Fig. 3.



III. Girard's experiments on copper tubes give $\lambda = 0·3984$ mm. The times of flow for a tube of 1·83 mm. in diameter were about three times less than that given by Poiseuille's law. These experiments were repeated with some solid drawn tubes, kindly made for me by Messrs. Elliott, of Selly Oak, Birmingham. The diameters were estimated to about 0·1 per cent., by weighing the tube first empty, and then full of water. By some subsidiary determinations with glass tubes, I showed that this gave the same result as the usual method with mercury. The results in all cases showed agreement with Poiseuille's observations. The times of flow were always a little

greater, obviously owing to irregularities in the tubes, but never less.

The surfaces of the tubes were then modified in various ways: by cleaning with acids and alkalis, by polishing with emery powder, by coating with a film of oil, and by amalgamating with mercury. In no case, however, could any deviation from Poiseuille's laws be detected. Girard gives no account of the method he employed for determining the diameters, and this may explain his results; any constant error would of course be more important in smaller tubes than in larger ones, and produce the same result as a slipping coefficient. The tubes I used were some of them smaller than those of Girard, and any slipping would have produced an even greater effect.

IV. At the beginning of Helmholtz and Piotrowski's paper, the latter claims to have shown by experiments on a glass flask, plain and silvered, that the friction exerted on it when oscillating by the contained water depended on the nature of the surface. These experiments were repeated, care being taken to make corrections for temperature, and to prevent alterations in the bifilar suspension, which were very apt to occur. Both of these precautions were neglected by Piotrowski, who only took two observations of the logarithmic decrement and the time of swing for each state of the flask, but deduced a 4 per cent. difference in the frictions.

The results of my observations are

Silvered surface, time of swing ..	8·806 sec.	log. dec. 0·142335
Unsilvered surface ,, ..	8·779 ,,	,, 0·142217

By the theoretical part of Helmholtz's paper, this makes the ratio of the friction on glass to the friction on silver

$$1{\cdot}0022 : 1.$$

The change, if any, is less than 0·3 per cent., and the ratio is unity within the limits of experimental error.

The figures given above are the means of twelve observations for the silver, and of twenty-three observations for the glass, some being taken before, and some after, those for the silver.

V. A modification of Piotrowski's experiment was then tried. Instead of filling the oscillating flask with water, it was filled with sand, and oscillated as a rigid body in a beaker of water. The ordinary investigation for such cases was then applicable, and it is easily shown that if k and k' are the frictions, λ and λ' the log. decrements, and T and T′ the times of swing for the two cases,

$$\frac{k}{k'} = \frac{\lambda T}{\lambda' T'}.$$

The outside of the bulb was silvered, observations taken, and the silver then dissolved without touching the suspension, and observations again taken.

Silvered surface ..	9·898 sec.	log. dec.	0·20718
Glass „ ..	9·938 „	„	0·20751

Ratio of frictions 1 : 1·00564.

The change is thus less than 0·6 per cent., and is within the limits of experimental error.

The main part of Helmholtz's paper is taken up with the consideration of experiments on the oscillations of an accurately worked sphere. It is remarkable that he deduces a value for the coefficient of viscosity which is about a quarter greater than that given by Poiseuille. This seems to suggest that a slight change in the application of theory to the results of experiment is needed, which will reduce the coefficient for the viscosity of the liquid, and increase the value for its adhesion to the walls of the vessel to that required for the condition of no slip. The existence of any effect approaching in magnitude that given by Helmholtz would produce, as I have shown, such an enormous change in the time of flow through a silvered tube, that the result of my experiments must be considered quite conclusive. The argument from the differences in friction due to differences in surface, in favour of the contact theory of E.M.F. is now seen to be worthless; and it must be admitted that no slip occurs, at any rate with solids that are wetted by the liquid.

III. "Re-determination of the True Weight of a Cubic Inch of Distilled Water." By H. J. CHANEY. Communicated by the President. Received February 4, 1890.

(Abstract.)

Recent investigations as to the value of the metric unit of volume —the cubic decimetre—appear to show, indirectly, that the present weight of a cubic inch of distilled water (252·458 grains, $t = 62°$ F., $b = 30$ in.)—the hitherto accepted unit of volume in this country—is appreciably too high. This weight (252·458 grains) is based on weighings made by Shuckburgh in 1798, and on linear measurements by Kater in 1821; but their results are affected by uncertainty as to thermometric and linear measurements, and as to the condition of the water used. Hence a direct re-determination of the unit of volume in this country appeared now to be desirable.

FIG. 1.

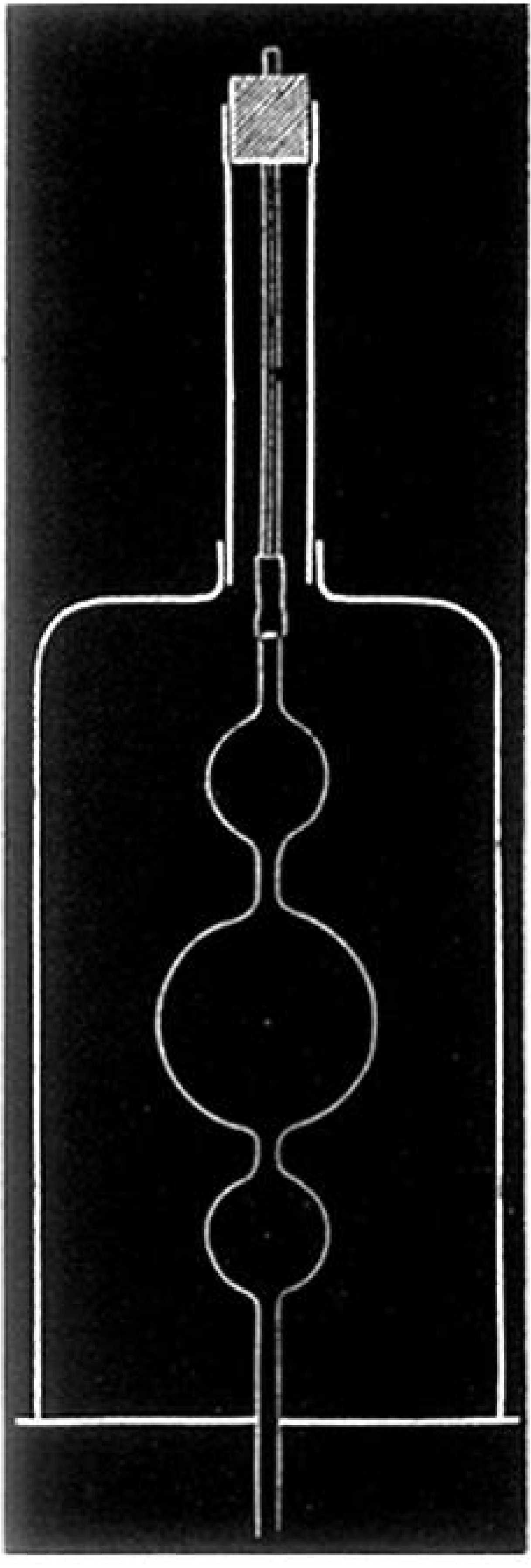


Fig. 2.

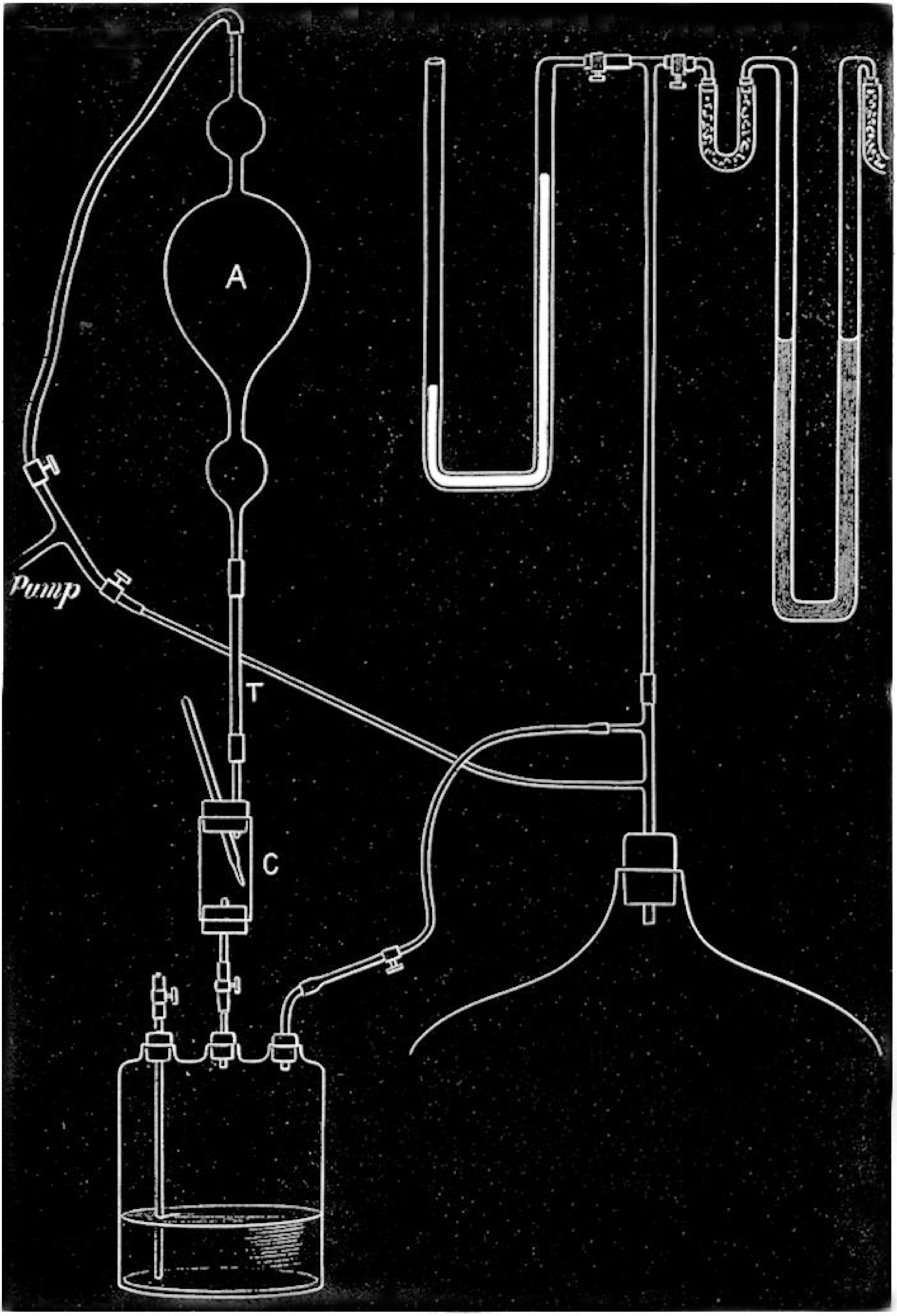


Fig. 3.