resulting in the production of several very interesting and very beautiful metallurgical specimens.

The great advantage of this process is, like that of the foregoing separation from arsenic, its extreme simplicity, the low temperature which renders it possible to work upon very large quantities at one time, and the very small amount of time necessary for this separation in comparison with the process hitherto adopted, and the absence of loss in the bismuth operated upon by volatilisation. It is obvious that where metals can be so easily treated in large quantities, the labour and skill hitherto necessary is very considerably reduced, and there is the additional advantage that the loss attending large operations is minimised.

In this and in my previous papers upon this beautiful metal bismuth, I have been able to point to simple dry processes for its separation from gold, lead, copper, arsenic, and antimony, and all these processes are available for treating with care large quantities at one time. When it is remembered what is involved in having to dissolve any quantity of bismuth in acids, and its subsequent precipitation from solution, it surely will be admitted that much of the difficulty in purifying crude bismuth has been effectively removed, as the methods given have been found possible in practice, and advantageous.

I have introduced upon the diagram the points at which arsenic is volatilised, and also the point at which antimony separates from bismuth under the conditions described in this paper.

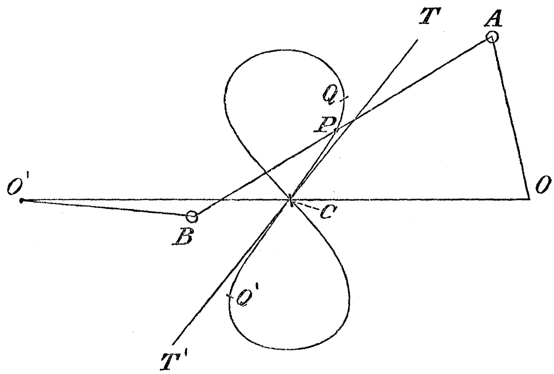
III. "On the Three-Bar Motion of Watt." By William Brennand. Communicated by C. B. Clarke, F.R.S. Received January 2, 1893.

(Abstract.)

The figure represents a simple form of "Watt's Parallel Motion." OA = O′B = r are the arms that can turn freely about O,O′, fixed centres, in one plane. The link AB = $2l$ is pivoted at A and B. As the arms move, P the middle of the link, traces out a portion of the curve, viz., from Q to Q′, backwards and forwards, nearly in a right line.

OC = O′C = d.—Of the three parameters d, r, l, any one can be taken as unit (in this paper l is taken 10 units); then d and r are independent parameters. The problem Watt had to solve was to discover numerical values of d and r that should give the tracing point P the smallest deviation from a right line.

Watt gave a series of values for d and r which are employed by engineers, with small thumb-rule ameliorations, to this day. They are



"good" values, *i.e.*, they give the path of P nearly rectilinear; but they are all subject to the relation $d^2-r^2 = l^2$. The curve above is one of this class; in these the tangent at the origin lies wholly outside the curve, and has the closest possible contact with it, while at the moment the tracing point P passes through the origin, the arms are at right angles to the link. The question arises, may there not be better values of d and r (not subject to the relation $d^2-r^2 = l^2$) which give a more nearly rectilinear motion to P than any of the Watt values?

The equation to the curve traced by P, in common polar coordinates, is (taking CO initial radius vector)—

$$(\rho^2+l^2-r^2-d^2)^2+4d^2(\rho^2-r^2)\sin^2\theta = 0 \quad \ldots\ldots \quad \text{(A)}$$

Willis ('Principles of Mechanism,' p. 401) says that the full equation is so exceedingly involved and complex as to be of no use in obtaining the required practical results. And Willis accordingly follows the preceding writers in "approximate methods."

The present paper takes up the subject at this point, and the general substance of the paper and its results may be stated under three heads, viz.:—

1. The nature and properties of the curve (A) are worked out so that a complete idea of it for all values of d and r is obtained.

2. Hence are derived numerous values for d and r which give good results; the deviation in these from the right line is calculated, and in some of them shown to be less than in any of the arrangements given by Watt.

3. The more complex arrangements, where the radii are not equal or where the tracing point divides the rod unequally, are also dealt with.

In the first head, large use has been made of plotting the para-

meters d and r as if they were x and y coordinates, and supposing at each point of the "chart" thus resulting that the curve (A) is existent at that point.

This method is capable of great extension; it is used in the present paper in the heading (3), and has been found of great service in dealing with other complex curves, as when the tracing point P is not in the straight line AB.

Watt gave a rule for "fixing" in the particular set of cases where $d^2-r^2 = l^2$. A very simple practical rule for fixing has been found for all values of d and r.

The 'English Mechanic' of December 29, 1882, published my first attempts on this curve; and the chart is there employed.

The numerous writings of late years on Three-Bar Motion, by Mr. Roberts, Professor Cayley, and others, do not appear to invade the narrow area of the present paper, which especially aims at obtaining numbers for d and r of practical value.

Presents, January 26, 1893.

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