

- II. "The Value of the Mechanical Equivalent of Heat, deduced from some Experiments performed with the view of establishing the Relation between the Electrical and Mechanical Units, together with an Investigation into the Capacity for Heat of Water at different Temperatures." By E. H. GRIFFITHS, M.A., Assistant Lecturer, Sidney Sussex College, Cambridge. Communicated by R. T. GLAZE BROOK, F.R.S. Received January 19, 1893.

(Abstract.)

The paper of which this communication is an abstract gives the particulars of an investigation which was commenced in the year 1887, and extended to the close of 1892. The object of the enquiry is sufficiently indicated on the title page.

The values of the mechanical equivalent obtained by Joule in his later determinations differ amongst themselves by as much as 1 per cent., and the differences amongst the results obtained by succeeding observers are, with the exception of Rowland's in 1880, still greater. The harmony amongst the values obtained by Rowland is marvellous; but, since his manner of investigation was the same throughout the whole series of his experiments, his conclusions stand in need of confirmation by different methods of observation. Other observers who have attempted to obtain the value of the mechanical equivalent, by means of the work done by an electric current, have been hampered by constant perplexities as to the absolute values of the electrical units adopted. The science of electrical measurements has now arrived at such a stage that its units may be regarded as sufficiently established,\* and, therefore, the time seems particularly appropriate for an enquiry into the relation between those units and the mechanical ones.

The difficulties of such an investigation are, of course, great, as is shown by the discrepancies amongst the results obtained by those observers who, in recent years, have adopted electrical methods. One cause of inaccuracy has been present in all determinations I have examined, viz., the increase in temperature of the conductor above the temperature of the medium in which it was placed, and the consequent undetermined alteration in its resistance. Rowland† writes as follows: "There can be no doubt that experiments depending on the heating of a wire give too small a value of the equivalent, seeing that the temperature of the wire during heating must always

\* 'B. A. Report,' 1892.

† 'Proceedings American Academy,' June, 1879, p. 153.

be higher than that of the water surrounding it, and hence more heat will be generated than there should be."

A short account is given in this summary of the manner in which this difficulty has been overcome, and I think it will be seen that this objection to the electrical method of investigation is now removed.

I have defined the thermal unit as the quantity of heat required to raise unit mass of water through  $1^{\circ}$  C. of the air thermometer at  $15^{\circ}$  C., and so much confusion has arisen from ambiguity as to the value of the unit, as ordinarily defined, that I have given reasons in support of the suggestion that this definition should be generally adopted.

Throughout the whole of this enquiry I have been ably assisted by Mr. G. M. Clark, B.A., Sidney College, Cambridge, and this communication should, in justice, be regarded as a joint contribution.

The value of an investigation of this kind depends, in a great measure, on the attention given to matters of detail. It is, therefore, impossible, in a short abstract, to produce the evidence on which our results are based, and we content ourselves with a brief outline of the method adopted and the conclusions arrived at, without attempting to justify those conclusions.

If a calorimeter is suspended in a chamber, the walls of which are maintained at a constant temperature, we can, by observations over a *small* range across that outside temperature, deduce the rate of rise due to the mechanical work done in the calorimeter, when the supply of heat is derived from stirring only. By repeating the observations in a similar manner over ranges whose mean temperature  $\theta_1$  differs from that of the surrounding walls  $\theta_0$ , we obtain the change in temperature due to the combined effects of the stirring, radiation, conduction, and convection at all points of our whole range of temperature. As the success of the method depends (1) on the possibility of maintaining the exterior temperature unchanged, and (2) on the regularity of the supply of heat due to the stirring, we briefly indicate our method of securing those conditions.

1. The calorimeter\* was suspended within an air-tight steel chamber. The walls and floor of this chamber were double, and the space between them filled with mercury. The whole structure was placed in a tank containing about 20 gallons of water, and was supported in such a manner that there were about 3 inches of water both above and beneath it. The mercury was connected by a tube with a

\* The calorimeter was of cylindrical form, and suspended by three glass tubes. It was made of "gilding metal," which both internally and externally was covered with a considerable thickness of gold. All metal surfaces within the calorimeter were thickly gilded.

gas regulator of a novel form, which controlled the supply of gas to a large number of jets. Above those jets was placed a flat silver tube, through which tap water was continually flowing into the tank, all parts of which were maintained at an equal temperature by the rapid rotation of a large screw. Thus, the calorimeter may be regarded as suspended within a chamber placed in the bulb of a huge thermometer—the mercury in that bulb weighing 70 lbs. A change of  $1^{\circ}$  C. in the temperature of the tank water caused the mercury in the tubes of the regulating apparatus to rise about 300 mm. Special arrangements were made by which it was possible to set the apparatus so that the walls surrounding the calorimeter could be maintained for any length of time at any required temperature, from that of the tap water (in summer about  $13^{\circ}$  C., in winter  $3^{\circ}$  C.) up to  $40^{\circ}$  C. or  $50^{\circ}$  C. We know by observation that the temperature of the steel chamber (when once adjusted) did not vary by  $1/500^{\circ}$  C., and we believe the variations were much less.

2. We experienced great difficulty in devising a suitable form of stirrer; and we attribute the failure of our earlier experiments to defects in the ordinary forms. We find it impossible, without a lengthy description, to give a clear idea of the stirrer ultimately adopted. We can only state here that it was completely immersed when the depth of the water exceeded 1 cm., that its bearings were outside the steel chamber, and that the water was thrown from the bottom to the lid of the calorimeter.

Mere than 100 experiments were performed (many of them lasting several hours) in order to determine the value of  $\sigma + \rho (\theta_1 - \theta_0)$ ,\* when the calorimeter contained different masses of water. The harmony amongst the results was satisfactory.

These experiments proved that over our range of temperature,  $\sigma + \rho (\theta_1 - \theta_0)$  was a linear function of  $\theta_1 - \theta_0$ , and Newton's law of cooling appeared to hold strictly true over a range of  $6^{\circ}$  C. below to  $6^{\circ}$  C. above the temperature of the surrounding walls, *i.e.*, from  $14^{\circ}$  C. to  $26^{\circ}$  C.; and our experimental results were of such a nature that a very small departure would have been apparent.†

We found that with our form of stirrer  $\sigma = r^3 k$  where  $r$  was the rate of revolution, and  $k$  some constant. This relation held true for all values of  $r$  between 26 and 34 revolutions per second, and, as during our J experiments we proposed to maintain a rate of as nearly as possible 30 revolutions per second, we were able to make the necessary correction for small deviations from the normal rate.

In order to diminish the irregularities in the motor, a special

\*  $\sigma$  = rise in temperature per 1 second due to the stirring.  $\rho$  = gain or loss in temperature per 1 second due to radiation, &c., when  $\theta_1 - \theta_0 = 1^{\circ}$  C.

† If the temperatures were reckoned on the mercury thermometer scale, the curvature would be considerable.

form of pressure regulator was constructed, and found to be fairly successful, the variations in  $r$  during an experiment being generally very small.

The pressure in the space between the calorimeter and the walls of the steel chamber was reduced, as a rule, to between 0·3 and 1·0 mm.\*

If  $M$  is the capacity for heat of the calorimeter and its contents,  $M\rho$  will be the quantity of heat lost or gained per second by radiation, &c., per unit difference of temperature, and provided the pressure is unaltered, the value of  $M\rho$  should be constant whatever the weight of water. It was not until the close of our work that we were able to obtain the value of  $M$ . We then found that the value of  $M\rho$  varied greatly with small changes of pressure, and our results (although not necessary for the purposes of our investigation) are interesting, since they bear out Bottomley's conclusion† that there is a sudden decrease in the loss by radiation when the pressure falls below 0·5 mm. We extract from our paper the following table :—

Table XXVI.

Date.	Experiments.	Mass of water.	Pressure in mm.	Thermal grams per second.
September 10—13....	83—93	139·78	1·15	0·0140
August 8—10 .....	41—50	103·01	1·15	0·0140
„ 11 .....	51—54	103·01	0·98	0·0140
September 14—16....	94—102	199·67	0·48	0·0138
„ 16—18....	103—110	259·50	0·48	0·0138
August 26.....	79—80	277·93	0·44	0·0136
„ 14—16 .....	58—59	188·07	0·40	0·0134
„ 24, 25 .....	62—65	188·07	0·37	0·0032
„ 30, 31 .....	74—78	277·93	0·37	0·0132
„ 26, 27 .....	66—71	277·93	0·37	0·0131
„ 17, 18 .....	60—61	188·07	0·37	0·0131
September 4.....	81—82	140·27	0·36	0·0130

The critical point of the curve deduced from the above table occurs at a higher pressure, and the bend is somewhat sharper than is the case with the curve given by Bottomley.

If  $\left(\frac{\partial\theta_1}{\partial t}\right)_{\sigma,\rho}$  is the rate of rise due to the non-electrical supply, and

$\left(\frac{\partial\theta_1}{\partial t}\right)_e$  that due to the electrical supply,

then 
$$\frac{\partial\theta_1}{\partial t} = \left(\frac{\partial\theta_1}{\partial t}\right)_e + \left(\frac{\partial\theta_1}{\partial t}\right)_{\sigma,\rho} \dots\dots\dots (1).$$

\* The pressures were ascertained by a McLeod's gauge.

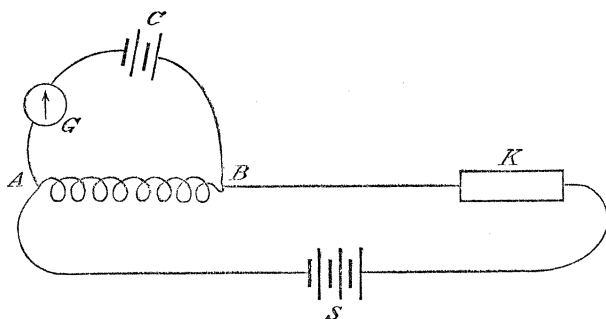
† 'Phil. Trans.,' 1887, A.

We have described the manner in which we determined the last term of this equation, and thus, by direct observation of  $\frac{\delta\theta_1}{\delta t}$ , we were able to obtain the value of  $\left(\frac{\delta\theta_1}{\delta t}\right)_e$  and

$$\left(\frac{\delta\theta_1}{\delta t}\right)_e = \frac{E^2}{J.R'.M'} \dots\dots\dots (2),$$

where  $R'$  is the resistance of the coil, and  $M'$  the capacity for heat of the calorimeter and its contents at a temperature  $\theta_1$ .

We have now to indicate the methods by which we ascertained the electrical value of the energy supplied by the current.



E.—The extremities of the wires CA, CB will (when the galvanometer G shows no deflection) be maintained at a difference of potential equal to that due to the cells at C.

Let K be an adjustable resistance placed in the circuit which communicates with S (the storage cells), and let R be the resistance of the calorimeter coil AB. Whatever variations may take place in R and in the potential difference at S, it is always possible (provided the adjustment of K is sufficiently under control) to maintain, by close attention to the indications of the galvanometer, the points A and B at a constant difference of potential.

A special rheochord was designed by means of which it was found possible, in spite of variations in the resistance of AB, to maintain the potential difference unchanged throughout each experiment, and we believe that in no case did the variations exceed 1/10,000 of the mean difference of potential during each experiment.

The Clark cells (which were placed at C) were constructed by us according to the directions of Messrs. Glazebrook and Skinner, and have, on several occasions, been directly compared with the Cavendish, and, indirectly, with the Berlin, standard. Their differences from the

standard are small, and their mean E.M.F. at  $15^{\circ}$  C. = 1.4344 volts.\*

R'.—Had it been possible to maintain a constant value for R, it would have greatly simplified both the experimental work and the calculations. In the year 1890 we devoted much time to the examination of the various copper-manganese-nickel alloys, and we performed a series of determinations with a coil whose temperature coefficient was practically zero. The reasons which led us to reject these alloys and adopt a platinum wire will, we think, be found sufficient.

The value of R was first determined by a dial-box (legal ohms) constructed by Messrs. Elliott.† Mr. Glazebrook has been so kind as to perform a complete standardisation of this box by means of the B.A. standards. The resulting corrections have been applied, and the values of R expressed in true ohms, as defined by the 'B.A. Report,' 1892.

If R is the resistance of the coil‡ when at the standard temperature  $\theta$ , then  $R' = R\{1 + \kappa(\theta_1 + \beta - \theta)\}$ , where  $\kappa$  is the temperature coefficient of the wire and  $\beta$  is the excess of its temperature above  $\theta$ , the temperature of the calorimeter. It is difficult to describe in a few sentences the manner in which we determined the value of  $\beta$ , but the following explanation may serve to indicate the method of procedure.

Suppose P, Q, R, and S to be the arms of a Wheatstone's bridge of which S is the coil. Let the arms P and Q be equal, not only in resistance but in mass and dimensions, and let  $R = S$  when the coil is at a certain temperature  $\theta$ , the reading of the thermometer in the calorimeter being  $\theta_1$ . Let R be built up of a large mass of metal having a small temperature coefficient and a considerable cooling surface.§ If the bridge is balanced when a certain current is passed through it, the balance will be destroyed if the temperature of the coil S be raised as the current is increased, for the increase in temperature of R may be neglected, and P and Q will remain equal, however their values alter, since they are traversed by equal currents and their capacities for heat are the same. Equilibrium can, however, be restored by cooling the calorimeter to a certain temperature,

\* A full description of these cells will be found in Messrs. Glazebrook and Skinner's paper ('Phil. Trans.,' 1892, pp. 622—624).

† Particulars of this box have been given in a previous paper ('Phil. Trans.,' A, 1891, p. 44).

‡ The wire had a thin coating of amber varnish, and the insulation appeared to be sufficient. In order to test this, a series of observations of R were taken when the calorimeter was filled with pure pentane. The increase in R did not exceed 1 in 22,000.

§ The mass of German silver used by us in the arm R weighed several pounds and contained about 1800 feet of single wire in triple and double strands.

say,  $\theta_2$ . We then know that S has resumed its former value, and is therefore again at the temperature  $\theta_s$ ; thus the value of  $\beta = \theta_s - \theta_2$ . By changing E, the potential difference of the ends of the coil, we can find values of  $\beta$  corresponding to values of E. By applying to the ends of the coils the potential balance previously described, the potential difference at the extremities of S can, by increasing the E.M.F. at the ends of the bridge, be raised to that of 1, 2, &c., Clark cells.

The following table gives the results obtained by this method. During our observations, the rate of stirring was the usual one, but we also investigated the effect of small changes in the rate. The last column gives the values deduced from the parabola  $\delta R = 0.00422n^2$  (where  $n$  is the number of Clark cells), as we found it convenient to express the differences in temperature by the corresponding differences in R. A difference of 0.0010 between the last two columns corresponds to a change of 1 in 8600 in R.

The correction is a most important one, and the neglect of it by previous investigators sufficiently accounts for their failure to obtain satisfactory results by observation of the heat developed in a wire by an electric current.

Table IX.

The following numbers were plotted.		$\delta R$ (legal ohms).	$\delta R$ deduced from $\delta R = 0.00422n^2$ .
No. of cells ( $n$ ).	Increase.		
0	$-x$	0	0
1	0	0.0042	0.0042
2	0.0120	0.0163	0.0168
3	0.0333	0.0376	0.0378
4	0.0638	0.0681	0.0675
5	0.1023	0.1066	0.1055
6	0.1478	0.1516	0.1519

*t, Time.*—An electrical clock with a seconds pendulum was used as our standard for time. It was carefully compared at intervals with a chronometer by Dent. A chronograph was controlled by this clock. The rate of the clock was a losing one until August 21, but after that date its error was less than 1/25,000, and no correction was necessary.

*w, Mass.*—A balance sensitive to a change of 1/100,000 of the least mass measured by us and a set of Oertling's weights\* were used in our determination of mass.

\* These weights were re-standardised by Messrs. Oertling in August, 1892.

*θ*, *Temperature*.—At the time of writing (December, 1892) our thermometry is based on measurements made by platinum thermometers. We propose to make, at an early date, a direct comparison of our standard thermometer with the air thermometer by means of the apparatus described by Mr. Callendar,\* who has been so kind as to promise his assistance.

In a previous paper† the details of a careful comparison of the platinum and the air thermometer have been given. It was then shown that the platinum-air difference curve  $\delta(t|100|^2 - t|100)$  gave values of *t* at all temperatures from 0° to 100° C. accurate within 0·01° C. of the real value of *t*, and that discrepancies appeared to have an experimental origin. The experience of two years' work with platinum thermometers has but increased our confidence in them.‡ Should a direct comparison with the air thermometer modify our determinations of differences of temperature (and it is only differences which are important to us), our results will be modified accordingly. This will not, however, necessitate any repetition of the experimental work, as the corrections involved will be of a numerical order only.

A mercury thermometer by Hicks (labelled *E<sub>m</sub>*§) was standardised by direct comparison with different platinum thermometers, the observations being taken under conditions similar to those prevalent during our "J experiments," and the temperatures corresponding to the readings given in column 1, Table XVII, were thus ascertained.

We found it necessary to calibrate the mercury thermometer not only for irregularities in the bore, but for certain strictly recurrent changes in "lag," which we have found to be invariably associated with a rising mercury thermometer. We are unable to briefly describe the method adopted, and we confine ourselves to stating that it was based on observations of time. The results of this investigation, which extended over several months, prove that a calibration of the ordinary kind is insufficient if accurate observations have to be obtained with a rising mercury thermometer. As an illustration we give the following table, which shows the difference in the value of certain ranges on thermometer *E<sub>m</sub>* when steady and when rising at the normal rate of our experiments.

\* 'Roy. Soc. Proc.' January, 1891.

† See 'Phil. Trans.,' 1891, A, p. 155.

‡ 'Phil. Mag.,' December, 1892.

§ The stem of this thermometer was graduated in mm.: about 40 mm. = 1° C.



Table XVII.\*

Range $E_m$ .	Range C. Thermometer steady.	Range C. Thermometer rising.
87·5—127·5	0·975	0·973
127·5—177·5	1·233	1·235
177·5—217·5	0·989	0·987
217·5—257·5	0·993	0·995
257·5—297·5	1·016	1·016
297·5—347·5	1·278	1·278
347·5—397·5	1·281	1·282
397·5—447·5	1·306	1·307
447·5—507·5	1·601	1·598
507·5—537·5	0·809	0·811

We have now indicated how the various quantities in equation (2) were determined, with the exception of  $J$  and  $M'$ ; we can therefore deduce from equation (2) the time ( $T$ ) of rising  $1^\circ$  C. at any point of our range when  $R = 1w$  and  $E$  is the potential difference of one Clark cell.

We thus get

$$\frac{J}{E^2} M' = T \dots\dots\dots (3).$$

If  $w$  be the weight of water, and  $w_x$  the water equivalent of the calorimeter at the standard temperature, and if  $f$  and  $g$  be the temperature coefficients of their specific heats, then

\* [Note, February 18, 1893.—On February 14, ult., I received a mercury thermometer (by M. Tonnelot) which had been under observation by Dr. Guillaume for the previous two months. I have made a direct comparison of this thermometer with  $E_m$  at three points, observing all the precautions enjoined by Dr. Guillaume, and the results are as follows :—

Reading $E_m$ , thermometer steady.	$\theta_1$ by Paris hydrogen thermometer.	$\theta_1$ as given by our platinum thermometer.	Range, Paris standard.	Range from Table XVII, <i>supra</i> .
87·5	13·975	13·990	} 6·483	6·484
347·5	20·458	20·474		
507·5	24·641	24·662		4·188

Thus, although we differ by  $0\cdot015^\circ$  C. in actual elevation at  $14^\circ$  C., our agreement as to the value of the above ranges is close. Want of time has prevented a complete comparison, and the observations have been somewhat hurried. They serve, however, to indicate that our thermometric errors are small.—E. H. G.]

$$M^1 = w(1+f\overline{\theta_1-\theta}) + w_x(1+g\overline{\theta_1-\theta}) ;$$

hence 
$$\frac{J}{E^2}\{w(1+f\overline{\theta_1-\theta}) + w_x(1+g\overline{\theta_1-\theta})\} = T \dots\dots (4).$$

By repeating observations with different weights of water,  $w_1$  and  $w_2$ , and observing  $T_1$  and  $T_2$ , the corresponding times, we obtain by subtraction

$$\frac{J}{E^2}(w_2-w_1)(1+f\overline{\theta_1-\theta}) = T_1-T_2 \dots\dots\dots (5).$$

Hence when  $\theta_1 = \theta$  (*i.e.*, at the standard temperature) we can find  $J$  without first ascertaining the values of  $f$ ,  $g$ , or the water equivalent of the calorimeter, and by repeating the observations over different ranges we can find  $f$  without previously obtaining  $J$ ; or, having obtained  $f$ , we can find  $w_x$  and  $g$ , and then by equation (4) deduce the value of  $J$  from a single experiment. We have adopted both methods as a check upon the calculations, which involve much arithmetic. The latter method is the more convenient, as it enables us to ascertain the results of separate experiments, but it cannot be applied until the values of  $f$ ,  $g$ , and  $w_x$  have previously been obtained by observations on two different weights at two different temperatures.

The following table shows a few of the results given in Table XL of our paper. We have divided our experiments into Series I and II, and we have given reasons why more weight should be attached to the latter series. We here give a summary of the values of  $T$  deduced from Series II. By "group" we denote all experiments conducted with the same weight of water, and in every case a group contains experiments performed with different values of  $n$  (where  $n$  is the number of Clark cells which determine the potential difference). As  $n$  was in some cases changed from 2 to 6, the rate of production of heat was increased 9 times. The agreement amongst the results of experiments performed with different values of  $n$  is not shown in the portion of the table here given, but it is very close, and affords a satisfactory proof of the accuracy of the values assigned to  $\sigma$  and  $\rho$ , and the validity of the method employed to ascertain the actual temperature of the coil.

The number of experiments performed in each group is shown by the figure under the heading "mean." The extent of our experimental irregularities is clearly indicated by this table.

The "smooth curve" was in each case so drawn that the sum of the positive and negative areas included between it and the slightly irregular experimental curve (given by the numbers in the columns headed "mean") was zero.

Table XL.—Values of T (results of Series II only).

Temp.	Group A. <i>w</i> = 139·776.		Group C. <i>w</i> = 199·674.		Group D. <i>w</i> = 259·500.	
	Mean (6).	From curve.	Mean (4).	From curve.	Mean (5).	From curve.
14·477	458·7	458·8	580·7	580·9	702·7	702·9
15·581	459·1	458·9	581·0	581·0	703·3	703·0
16·682	459·1	459·0	581·1	581·1	703·0	703·0
17·683	459·0	459·1	581·3	581·1	703·1	703·0
18·688	459·4	459·2	581·3	581·2	703·1	703·0
19·835	459·3	459·3	581·9	581·3	703·6	703·1
21·115	459·4	459·5	581·1	581·4	703·3	703·1
22·409	459·7	459·6	581·6	581·4	702·9	703·1
23·862	459·7	459·7	581·1	581·5	702·7	703·2
25·006	459·9	459·8	581·5	581·6	703·4	703·2
No. of } col. } I	XIV	XV	XVIII	XIX	XXII	XXIII

The values of T at integral values of the temperature over our range were read off from the smooth curves. We give the values at 15°, 20°, and 25° C.

Table XLI.—Values of T at 15°, 20°, and 25° C.

Temp.	Series I.		Series II.		
	Group B. <i>w</i> = 188·065.	Group E. <i>w</i> = 277·931.	Group A. <i>w</i> = 139·776.	Group C. <i>w</i> = 199·674.	Group D. <i>w</i> = 259·500.
15·000	557·14	740·46	458·87	580·95	702·91
20	557·62	740·60	459·35	581·25	703·05
25	558·09	740·75	459·81	581·55	703·20
No. of } col. } 1	2	3	4	5	6

From this table we obtain the following results:—

Specific heat of water at 25° in terms of its  
specific heat at 15°, deduced from columns 4

and 6..... = 0·99734

Ditto from columns 4 and 5..... = 0·99722

Ditto from columns 5 and 6..... = 0·99746

Mean ..... = 0·99734

Hence, adopting 15° C. as the standard temperature, the

$$\text{SPECIFIC HEAT OF WATER} = 1 - 0.000266 (t - 15).*$$

Also by means of equation (15) we get the following values of J : —

$$\begin{array}{llll} \text{Columns 4 and 6} & \dots\dots\dots & J = 4.1939 \times 10^7 \\ \text{,,} & 4 \text{ ,, } 5 & \dots\dots\dots & J = 4.1940 \times 10^7 \\ \text{,,} & 5 \text{ ,, } 6 & \dots\dots\dots & J = 4.1940 \times 10^7 \\ \text{Mean} & \dots\dots\dots & J = \underline{4.1940 \times 10^7} \end{array}$$

This value of J, as previously pointed out (equation 5), is entirely independent of the value assigned to the water equivalent of the calorimeter.

And we find the water equivalent of the calorimeter at 15° C. in terms of water at 15° C. = 85.340 grams. The water equivalent of the calorimeter at 25° C. in terms of water at 15° C. = 86.174 grams.

Hence water equivalent =  $85.340\{1 + 0.000977(t - 15)\}$ .

We can now find the capacity for heat of the calorimeter and contents for any weight of water at 15°, 20°, and 25° C., and deduce the value of J from each group separately. The Groups B and E are experiments performed on 188.065 and 277.931 grams† respectively.

Table XLIII.—Values of J.

Group.	15°	20°	25°	Mean.
A	$4.1940 \times 10^7$	$4.1940 \times 10^7$	$4.1939 \times 10^7$	4.1940
B	4.1930 "	4.1941 "	4.1949 "	4.1940
C	4.1939 "	4.1938 "	4.1937 "	3.1938
D	4.1940 "	4.1939 "	4.1940 "	4.1940
E	4.1938 "	4.1940 "	4.1943 "	4.1940
				4.1940

We have in the above table given the values resulting from the calculation at different temperatures, for the limit of our experimental errors is thus clearly indicated, since the values of J ought (in the absence of experimental errors) to be identical at all temperatures. The close agreement between the values from different groups, and from the same group at different temperatures, is a satisfactory proof of the accuracy of our determination of the water

\* Over the range 14° to 26° C.

† All weights are reduced to *vacuo*.

equivalents of the calorimeter, and of the changes in it and in the capacity for heat of the water. If we reject Group B (and we have already stated that it has little value), the results are practically identical.

Hence (the thermometry depending on comparisons with platinum thermometers) if we assume

1. The unit of resistance as defined in the 'B.A. Report,' 1892;
2. That the E.M.F. of the Cavendish Standard Clark cell at 15° C.  
= 1.4342 volts;\*
  3. That the thermal unit = quantity of heat required to raise  
1 gram of water through 1° C. at 15° C.,

the most probable value of

$$J = 4.1940 \times 10^7 \dagger$$

This, by reduction, gives the following:—

$$J = 427.45 \text{ kilogramme-metres in latitude of Greenwich } (g = 981.17).$$

$$J = 1402.2 \text{ ft.-lbs. per thermal unit C. in latitude of Greenwich } (g = 32.195).$$

$$J = 778.99 \text{ ft.-lbs. per thermal unit F. in latitude of Greenwich } (g = 32.195).$$

The length of this abstract is already unduly great, and we will, therefore, not enter on any discussion of the results beyond remarking that if we express Rowland's value of  $J$  in terms of our thermal unit we exceed his value by 1 part in 930, and we exceed the mean of Joule's determination by 1 part in 350.‡

The difference between Rowland's value of the temperature coefficient of the specific heat of water and ours would, however, cause both his and our values of  $J$  to be identical if expressed in terms of a thermal unit at 11.5° C.

\* If we assume the E.M.F. of our Clark cells to be the same as that of the Cavendish standard (and we are inclined to think we have over-estimated the difference), we get  $J = 4.1930 \times 10^7$ .

† The value obtained by us in 1891 =  $(4.192 +) \times 10^7$ .

‡ Rowland obtained the mean value of Joule's determinations by assigning values to different experiments, and the above comparison refers to the numbers thus obtained. If, however, we attach equal weight to all Joule's results, as given by Rowland, the mean *exceeds* our value by 1 in 4280, assuming our expression for the temperature coefficient of the specific heat of water.