March 2, 1893.

Sir JOHN EVANS, K.C.B., D.C.L., LL.D., Vice-President and Treasurer, in the Chair.

A List of the Presents received was laid on the table, and thanks ordered for them.

In pursuance of the Statutes, the names of the Candidates for election into the Society were announced, as follows:—

Bateman, Sir Frederic, M.D.
Bateson, William, M.A.
Beevor, Charles Edward, M.D.
Boulenger, George Albert.
Bourne, Professor Alfred Gibbs, D.Sc.
Bradford, John Rose, M.D.
Brennand, William.
Burnside, Professor William, M.A.
Buzzard, Thomas, M.D.
Callaway, Charles, D.Sc.
Callendar, Hugh Longbourne.
Carter, Robert Brudenell, F.R.C.S.
Cheyne, William Watson, F.R.C.S.
Clarke, Sir George Sydenham, Major R.E.
Clowes, Professor Frank, D.Sc.
Darwin, Leonard, Major R.E.
Davis, James William, F.G.S.
Dreschfeld, Professor Julius, M.D.
Dunstan, Professor Wyndham R., M.A.
Edgeworth, Professor Francis Ysidro, M.A.
Elgar, Francis, LL.D.
Eliot, John, M.A.
Ellis, William, F.R.A.S.
Etheridge, Robert, F.G.S.
Ewart, Professor J. Cossar, M.D.
Gairdner, Professor William Tennant, M.D.
Harcourt, Professor Leveson Francis Vernon, M.Inst.C.E.
Harker, Alfred, M.A.
Hendley, Thomas Holbein, Surgeon-Major.
Hickson, Sydney John, M.A.
Hill, Professor M. J. M., M.A.
Hinde, George Jennings, Ph.D.
Hobson, Ernest William, D.Sc.
Howes, Professor George Bond, F.L.S.
Howorth, Sir Henry Hoyle, K.C.I.E.
Jones, Professor John Viriamu, M.A.
King, George.
Knobel, Edward Ball, F.R.A.S.
Lockwood, Charles Barrett, F.R.C.S.
Love, Augustus Edward Hough, M.A.
Lydekker, Richard, B.A.
Macewen, Professor William, M.D.
McConnell, James Frederick Parry, Surgeon-Major, F.R.C.P.
MacMunn, Charles, M.D.
Mansergh, James, M.Inst.C.E.
Martin, John Biddulph, M.A.
Martin, Sidney, M.D.
Matthey, Edward, F.C.S.

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Mott, Frederick Walker, M.D.
Newton, Edwin Tully, F.G.S.
Notter, James Lane, Surgeon-Lieut.-Col.
Oliver, John Ryder, Major-General R.A.
Ord, William Miller, M.D.
Reade, Thomas Mellard, F.G.S.
Roberts, Ralph A., M.A.
Rutley, Frank, F.G.S.
Salomons, Sir David, M.A.
Sherrington, Charles Scott, M.B.
Stebbing, Rev. Thomas Roscoe Rede, M.A.
Stevenson, Thomas, M.D.
Stewart, Professor Charles, M.R.C.S.
Stirling, Edward C., M.D.
Thomson, Professor John Millar, F.C.S.
Thornycroft, John Isaac, M.Inst. C.E.
Trail, Professor James William Helenus, M.D.
Tuke, Daniel Hack, M.D.
Ulrich, Professor George Henry Frederick, F.G.S.
Veley, Victor Hubert, M.A.
Wallace, Alfred Russel, LL.D.
Waterhouse, James, Colonel.
Watkin, Henry Samuel Spiller, Lieut.-Col. R.A.
Webb, Francis William, M.Inst. C.E.
Woodward, Horace Bolingbroke, F.G.S.
Worthington, Professor Arthur Mason, M.A.
Young, Professor Sydney, D.Sc.

The following Papers were read:—

I. "Harmonic Analysis of Hourly Observations of Air Temperature and Pressure at British Observatories. Part I. Temperature." By Lieut.-General R. STRACHEY, R.E., F.R.S. Received January 20, 1893.

(Abstract.)

This paper is a discussion of the results of the computations of the harmonic constants contained in a volume recently published by the Meteorological Office. The tables in this volume give the constants of the harmonic components of the first four orders, for each month for twenty years, of the daily curves of temperature and pressure at Greenwich; and the constants for the first three orders, for the temperature and pressure, for each month for twelve years, at the seven observatories maintained by the Meteorological Office.

The tables supply the values of the coefficients of the cosines and sines of the several terms of the usual harmonic series, representing any hourly value:—

$$A_n = p_0 + p_1 \cos n \,.\, 15^\circ + q_1 \sin n \,.\, 15^\circ + \&c. \quad \ldots\ldots\ldots \quad (1).$$

They also give the amplitudes of the several components, and the epoch of maximum derived from the formula

$$A_n = p_0 + P_1 \sin(n \,.\, 15^\circ + T_1) + P_2 \sin(2n \,.\, 15^\circ + T_2), \&c. \ldots \quad (2).$$

In these tables, and the present discussion, the coefficients of the cosines of the arcs for the several components are designated by the letter p, and those of the sines by the letter q. The amplitude for the several components is designated by P, and the epoch of the first maximum that occurs after midnight is designated by the letter μ.

By the introduction of the epoch of maximum, the connexion of the component with the hour of the day and the sun's place is directly indicated, which for the purpose of these discussions is more convenient than the method usually adopted, of stating the value of the angle T in formula (2).

Reference is made to difficulties and uncertainties that occasionally arise in computing the mean values of some of these constants. Where there are periodical variations of value which lead to changes of sign, the arithmetical mean will tend to obliterate variations which may, in truth, be strongly marked. Also difficulty at times arises in respect to the epoch of maximum, from uncertainty, in dealing both with casual irregularities, and periodical changes, in saying whether the epoch has been thrown forward or backward.

The absolute *magnitudes* of the coefficients p and q indicate the amplitude of the component, and their *signs* the phase, or the epoch of maximum. It will readily be seen that the combinations of coefficients, $+p+q$; $-p+q$; $-p-q$; $+p-q$, correspond respectively to epochs of maximum in the first, second, third, and fourth quadrants of the period of the component; and the mutual destruction of a series of positive and negative values of p and q in a mean value will therefore only signify that there is no true mean epoch of maximum, and that all positions are alike probable or uncertain.

The foregoing remarks apply to the whole series of computations; what follows refers only to the temperature tables, to which the present communication is limited.

1. *Greenwich Temperature.*

The examination of the tables shows that, with very considerable variations of absolute magnitude, there is on the whole very marked consistency in the main characteristics of the components.

Taking as a test the position of the epoch of maximum, which may be regarded as more directly dependent on the sun's action, and on his position, it will be seen that the values of μ indicate very clearly the closeness of this connexion.

In all the components a truly periodical variation of the value of μ is apparent, and the period of maximum always travels backwards, that is, it becomes earlier as the year passes from winter to summer,

while it returns in the opposite direction in the change back to winter.

For the first component the variation of the five years' mean of μ from the twenty years is in no month more than $2\frac{1}{2}°$ or ten minutes of time, and the average for all months is less than half that amount.

In the second component the epoch of maximum, which during the winter months is always *after* midnight, falls back in the summer, when it is at times *before* that hour. The variation of the five-year mean from the twenty-year mean is in no month more than 6°, and the average is only 2°·3, or nine minutes of time.

In the winter months the maximum of the third component is always between 4 A.M. and 5 A.M.; in March it changes rapidly, in the summer being found invariably between midnight and 1 A.M., while after September it returns to its winter position. The variation of the five-year from the twenty-year mean in no month exceeds 5°, and the average in all months is only 2°·1, or $8\frac{1}{2}$ minutes of time.

The fourth component shows double maxima and minima, the former at the equinoxes, the latter at the solstices. The largest variation of the five-year mean of any month from the twenty-year mean is 10°, and the average for all months is 4°·3, or seventeen minutes. Considering how small are the absolute values of the coefficients p_4 and q_4, on which the value of μ_4 depends, the average being a little less than $\frac{1}{10}$th of a degree Fahrenheit, it is rather a matter of surprise that the variations should be so small than that they should reach their actual amounts.

It may be noticed that the total amplitude of the components being $\sqrt{(p^2+q^2)}$, a considerable variation of its value is quite consistent with invariable or slightly varying values of μ, which depend on the ratio p/q.

The component of the first order, which in the winter is more than double the magnitude of any of the others, and in summer more than ten times as great, gives the dominant character to the daily curves of temperature. In the series of twenty years variations in different years of as much as 100 per cent. are to be found for almost every month, but for the most part even these irregularities disappear in the mean of a series of five years, and the monthly means for the twenty years are remarkably consistent.

The progression of the value of P, in the course of the year, follows approximately the sine of the sun's meridional altitude, and the empirical formula

$$P = 10 \cos z - 0{\cdot}91$$

gives a close approximation to the values shown in the tables, if a "lagging" of eight or ten days is allowed in reckoning the sun's place.

The second component has two clearly marked *maxima* about the time of the equinoxes, and a principal *minimum* at midsummer.

The component of the third order varies in a converse manner, having two well-marked *minima* at the equinoxes, with a principal *maximum* at midsummer.

The component of the fourth order appears to combine the characters of the two previous ones, having two *maxima* about the time of the equinoxes, and a principal *minimum* in the winter.

The following empirical formulæ give close approximations to the values of P_2 and P_3:

$$P_2 = 1{\cdot}08 + 0{\cdot}20 \cos(\lambda + 126^\circ) + 0{\cdot}41 \cos(2\lambda - 2^\circ),$$

$$P_3 = 0{\cdot}42 + 0{\cdot}16 \cos(\lambda + 260^\circ) + 0{\cdot}10 \cos(2\lambda - 172^\circ),$$

in which λ is the sun's longitude.

The mean value of μ for the first component is 214°, corresponding to 2 h. 26 m. P.M., the variation due to season being 12° or 48 m. of time, by which the maximum is earlier in summer than in winter.

In the second order the first maximum in June is 24°, or 1 h. 20 m. earlier than in January.

In the third order the difference in the same direction is 63°, or 4 h. 12 m. of time.

In the fourth order, there is some doubt as to the manner in which the change of epoch of the summer and winter maxima is brought about. From March, when the first maximum occurs about 60° after midnight, or 4 A.M., there is a continued retrogression till June, when the maximum is at 16° after midnight, or 1 h. 4 m. A.M. This is followed by a progression from June till October, when the maximum again occurs at about 60°, or 4 A.M.

In passing from October to November, a sudden change takes place by which the maximum is established at about 10° after midnight. There is a like sudden change between January and February in the opposite direction, which again brings the maximum to 60° after midnight. From the component in November and February being very small, it is not improbable that these sudden changes may coincide with the component becoming zero.

Remembering that the fourth component includes four series of undulations, the most probable explanation of these changes is to be found in a change of the position of these undulations, during which, between January and February, the first recedes, and its place is taken by the second, which leads to sudden appearance of a maximum about 60°, or 4 A.M. A similar change between October and November in an opposite direction would introduce the maximum at 10° after midnight.

In the summer months (May, June, and July) the temperature curve during the day hours, from 8 A.M. to 8 P.M., hardly differs from

a curve of sines, the first component being more than ten times as large as any of the others, which therefore influence the temperature, relatively, very little.

The relation of the epoch of the first maximum of the component of the third order to the time of sunrise is decidedly marked, the former occurring, on the average, about 12°, or 48 m. after sunrise; the mean deviation of the interval from that amount being only 7°, or 28 m.

The periodical variation in the position of the maximum leads, during the winter months, to a *positive* maximum of this component about 1 P.M., which is combined with *negative* maxima four hours earlier and later, which correspond to the *reduced* temperature in the mornings and afternoons of the *shorter* days. In like manner, in the summer months, when this component has a negative *maximum* about 1 P.M., instead of a negative *minimum*, as in winter, there will be two *positive* maxima, one four hours earlier, the other four hours later, corresponding to the *higher* temperature in the mornings and afternoons of the *longer* days.

It will be seen that these positions of the midsummer and midwinter maximum phases correspond respectively to days of 16 hours with nights of 8 hours, or days of 8 hours and nights of 16 hours, and that at these seasons, when the variations of temperature, due to these differences, are greatest, the amplitudes of this component are also the greatest. At the equinoxes, with 12-hour days and nights, the component becomes a minimum; and at this season the change in the position of the maximum takes place as already noticed.

It might be supposed that an analogous relation between the fourth component and the occurrence of days of 18 hours, combined with nights of 6 hours, and *vice versâ*, is likely to arise. But the data are not forthcoming to test this.

Although the several components of the temperature curve cannot be regarded as indications of specific physical efficient causes, the examination of the graphical representations of the various curves presents points to which attention may usefully be drawn. The chief of these are the following:—

In the summer months the time of mean temperature is nearly where the first component becomes zero, the second and third components then balancing one another.

In the winter the time of morning mean temperature is later than in summer, and occurs when a positive value of the first component is equal to a negative value of the second.

The time of afternoon mean temperature throughout the year is somewhat either before or after 7 P.M., and almost exactly coincides with the time when the first and second components are equal, with opposite signs.

In the summer the time of absolute minimum is between the hours of 3 A.M. and 6 A.M., during which the whole of the components are negative.

Sunrise in December is about an hour and a half before the time of mean temperature; while in June it is more than four hours earlier.

Sunset in December is rather more than three hours *before* the time of mean temperature; in June it is about half an hour *after* that time.

The *rationale* of some of the empirical rules for obtaining the mean daily temperature from a limited number of observations is supplied by reference to the harmonic expressions for the hourly deviations of temperature from the mean value; it being borne in mind that the relative magnitude of the fourth component is very small.

In the first place, it will be seen that by adding together the harmonic expressions for any two hours twelve hours apart, the whole of the *odd* components disappear, and that the sum is twice the mean value, added to twice the sum of the *even* components of the selected hours, which are equal. Disregarding the components above the fourth order, if the selected hours are such that the component of the second order is zero, which will be the case at hours corresponding to $\mu_2+45°$ or $\mu_2+135°$, then half the sum of the temperatures at the selected hours will be the true daily mean added to the fourth component for the selected hour, which at English stations will never amount to $\frac{1}{2}°$, and on the average is less than $\frac{1}{5}°$.

At Greenwich the mean between the observations at $4\frac{1}{2}$ A.M. or $10\frac{1}{2}$ A.M. and the corresponding afternoon hours in January, will differ by less than $\frac{1}{10}°$ from the true value, and similar results will be obtained for June by the mean of observations made at 3 A.M. or 9 A.M. and the corresponding hours in the afternoon.

By taking the mean of observations at any four hours, at intervals of six hours, both the odd components and those of the second order will disappear, and the result will only differ from the true mean by the amount of the fourth component for the selected hours.

As this component disappears when $\mu_4 \pm 22\frac{1}{2}° = 0°$ or $180°$, the hours at Greenwich that will give the best result are 2, 8, 14, and 20, or 5, 11, 17, and 23.

So, if the mean of any three hours at equal intervals of eight hours be taken, the sums of the first, second, and fourth components will disappear, and the result will only differ from the true mean by the amount of the third component for the selected hours, which in no case can be so much as $\frac{3}{4}°$.

By adopting hours when $\mu_3 \pm 30 = 0°$ or $180°$, the third component disappears, and this result will be obtained at Greenwich by combining observations at 3, 11, and 19 hours, or 7, 15, and 23.

2. *Temperature at the Seven Observatories.*

The examination of the tables will show that in their main characteristics the results closely resemble those for Greenwich, and it will not be necessary to discuss them in any detail.

The amplitude of the component of the first order is, however, in all cases less than that observed at Greenwich, the lowest values being those for Valencia and Falmouth, no doubt due to their position on the sea coast, for which stations the means for the years are $2°·28$ and $2°·35$ compared with $5°·10$ at Greenwich.

The Kew values most resemble those at Greenwich, but the mean maximum at Kew is more than $1°$ less, and the mean for the year $\frac{1}{2}°$ less.

The mean values of μ_1 for the seven observatories lie between $205°$ and $220°$, that for Greenwich being $214°$. The means of the summer values are about $3°$ or $4°$ less than the mean of the year, and of the winter values as much above it, as in the case of Greenwich.

The amplitudes of the first components conform approximately, but not so closely as at Greenwich, with the sine of the sun's meridian altitude, but with a flattening of the curve in the summer months, and a tendency at some of the stations to a maximum value in May.

The components of the second and third orders, beyond which the analysis is not carried for these observatories, conform in all important respects to those for Greenwich, the numerical values of the latter being, however, in all cases somewhat higher. The epochs of maximum follow the same laws, with an increased divergence of the summer epoch from that of the winter at the more northern stations.

Making allowance for their smaller amplitude, the empirical formulæ expressing the mean values of the P_2 and P_3 components differ little from those obtained for Greenwich.

In order to test, and in some degree throw light on the character and significance of the harmonic components of temperature that have been under discussion, and bearing in mind that they cannot be considered to represent separate effects of physical forces operating at the assumed periods of the components, I have, at the suggestion of Professor G. Darwin, calculated the harmonic components that would produce a curve representing an intermittent heating action such as that of the sun, continued only during a portion of the day, and commencing and ending abruptly at sunrise and sunset.

Such a procedure disregards all cooling effects, and only deals with the sun's direct heating action, which I have assumed to be proportional to the sine of his altitude. Also with a view of obtain-

ing figures in some degree comparable with those derived from actual observation, I have assumed (following the empirical formula before given) the power of a vertical sun to be 10. Having calculated the sun's altitude for each hour of the day, for midwinter, the equinox, and midsummer, for certain selected latitudes, the corresponding heating effects have been computed, to which the usual method of analysis having been applied, the following results are obtained (pp. 74 and 75).

These figures represent the values of the components at midnight. The signs indicate that the maximum of the first component is in all cases at 180°, or noon; of the second component the maximum in all cases it at 0°, or midnight; of the third component in all latitudes the maximum in the winter is at 60°, or 4 A.M., in the summer at 0°, or midnight, the change taking place at the equinox, when the component becomes zero; of the fourth component in the lower latitudes up to 40°, the maximum is at 45°, or 3 A.M., at all seasons; in the higher latitudes the maximum is at 0°, or midnight, in winter and summer, and at 45°, or 3 A.M., at the equinox.

For comparison, the following results from actual observations at latitudes specified are also given in similar form.

The close correspondence of the main features of these two tables is obvious.

The conclusion is unavoidable, that, although both in the actual and hypothetical cases the harmonic components when combined are truly representative of the peculiar forms of the curves from which they were derived, this affords no evidence of the existence of recurring cycles of action corresponding to the different components, but that the results are, to a great extent, due to the form of the analysis.

The diurnal curve of temperature is not symmetrical in relation to the mean value, the maximum day temperature being much more in excess than the minimum night temperature is in defect. To adjust the first component, which is symmetrical about its mean value, to the actual unsymmetrical curve, it must be modified by the other components. That of the second order which has one of its maxima not far removed from the minimum of the first order supplies the chief portion of the compensation due to this cause.

Further, from the character of the analysis, when the diurnal curve is symmetrical on either side of the hour half-way between noon and midnight, that is, when the day and night are equal in length, the third component becomes zero. Any departure from this symmetry introduces a component of the third order, with the result that with a day shorter than 12 hours one maximum will fall in the day between 6 A.M. and 6 P.M., and the other two in the night between 6 P.M. and 6 A.M.; while with a day longer than 12 hours, two maxima will

Latitude.	Winter. Components.				Equinox. Components.				Summer. Components.			
	I.	II.	III.	IV.	I.	II.	III.	IV.	I.	II.	III.	IV.
0°	−4·6	+1·9	0	−0·4	−5·1	+2·1	0	−0·5	−4·6	+1·9	0	−0·4
20	−3·4	+1·7	−0·3	−0·3	−4·7	+2·0	0	−0·4	−5·3	+1·7	+0·3	−0·3
30	−2·8	+1·4	−0·3	−0·3	−4·4	+1·7	0	−0·4	−5·3	+1·6	+0·4	−0·2
40	−1·9	+1·2	−0·4	−0·1	−3·9	+1·6	0	−0·4	−5·2	+1·2	+0·4	−0·1
45	−1·5	+1·0	−0·4	+0·1	−3·5	+1·5	0	−0·4	−5·2	+1·1	+0·4	+0·1
51½	−1·0	+0·8	−0·4	+0·1	−3·2	+1·3	0	−0·3	−4·8	+0·8	+0·4	+0·2
65	0	0	0	0	−2·2	+1·0	0	−0·2	−3·9	0	+0·1	+0·1

Stations.	Winter.				Equinox.				Summer.			
	Components.				Components.				Components.			
	I.	II	III.	IV.	I.	II.	III.	IV.	I.	II.	III.	IV.
Singapore .... (Lat. 1° 15′ N.)	−5·0	+1·8	−0·3	−0·2	−6·5	+2·0	+0·7	−0·5	−4·6	+1·5	+0·3	−0·5
Hong Kong .... (Lat. 22° 18′ N.)	−2·4	+1·0	−0·7	−0·8	−2·0	+0·7	−0·0	−0·1	−1·9	+0·6	+0·1	−0·1
Lyons .... (Lat. 45° 46′ N.)	−2·5	+1·0	−0·4	+0·1	−6 3	+1·5	+0·1	−0·3	−7·6	+0·8	+0·6	+0·2
Greenwich .... (Lat. 51° 30′ N.)	−1·7	+0·8	−0·3	+0·1	−4·7	+1·3	+0·2	−0·2	−7·7	+0·6	+0·6	+0·2
Fort Rae............ (Lat. 62° 40′ N.)	−1·1	+0·7	−0·3	+0·3	−7·7	+1·9	+0·4	−0·5	−6·0	+0·6	+0·2	+0·1

occur in the day and only one in the night. In the former case the negative portions of the component correspond with the reduced morning and afternoon temperatures of the short day, and in the latter the two positive phases correspond with the higher temperature of the mornings and afternoons of the longer day.

These conclusions are in conformity with those previously indicated.

The available data are insufficient to enable us to say whether the corresponding results connected with the fourth component are as fully supported by observation as in the case of the third, but the facts so far as they go confirm this view.

It may also be pointed out that, if instead of reckoning the epochs of maximum from midnight, that nearest to noon had been adopted, it would have been seen that there is a distinct tendency for all these epochs to approach noon, affording evidence, which is perhaps hardly required, that they are all closely dependent on the passage of the sun over the meridian.

For Greenwich the results would be—

	Winter.	Equinox.	Summer.
1st component......	222°	215°	210°
2nd "	200	198	181
3rd "	194	—	(193)
4th "	190	(193)	196

In the case of the third and fourth components, the figures enclosed in brackets are epochs of minimum.

II. "The Effects of Mechanical Stress on the Electrical Resistance of Metals." By James H. Gray, M.A., B.Sc., and James B. Henderson, B.Sc., International Exhibition Scholars, Glasgow University. Communicated by Lord Kelvin, P.R.S. Received February 10, 1893.

(Abstract.)

This investigation was begun for the purpose of obtaining an easily worked method of testing the effect of any mechanical treatment on the density and specific resistance of metals.

For alteration of density, copper, lead, and manganese copper wires were tested. The effect of stretching was always to diminish the density, the alteration being small however: for copper about $\frac{1}{2}$ per cent., and for lead $\frac{4}{5}$ per cent. The effect of drawing through holes in a steel plate was somewhat greater, showing at first an in-