

looked that a displacement is not defined by the direction of axis, and amplitude, of the resultant rotation, together with the magnitude of the component of the corresponding translation along that direction (for in this form the proof is given, the axis being drawn through one end of the common perpendicular to the particular couple in respect of which the theorem is demonstrated), since these elements are common to an infinity of displacements.

This being premised, the laws connecting pairs of axes by successive rotations round which a given displacement of a rigid body in space may be effected are as follows:—

If the first axis (ζ' ξ') is taken arbitrarily, say parallel to a given vector, ζ' , and passing through the term of a second given vector, ξ' , its conjugate is parallel to a vector (ζ), the side common to three quadric cones, the constants of which are functions of ζ' , ξ' , and the vectors defining the displacement.

Each of these cones, whatever the direction of ζ' , passes through one of three fixed vectors.

The directions of the axes being fixed in accordance with the above conditions, the locus of either axis is a plane, the places of the axes in which are so related that the connector of the feet of perpendiculars on them from any fixed point generates a ruled quadric surface.

[The last three paragraphs have been altered (July 15) after a correspondence, since the reading of the note on 15th June, with which Professor W. Burnside, F.R.S. (who, however, is not responsible for any statement herein), favoured me; as the result of which he sent me a geometrical proof that one axis might in all cases be taken arbitrarily both in position and direction. On revising my analysis, I found that what I had taken as an equation of condition was reducible to an identity.]

XI. "On a Graphical Representation of the Twenty-seven Lines on a Cubic Surface." By H. M. TAYLOR, M.A., Fellow of Trinity College, Cambridge. Communicated by A. R. FORSYTH, Sc.D., F.R.S. Received June 13, 1893.

(Abstract.)

The converse of Pascal's well-known theorem may be stated thus: if two triangles be in perspective, their non-corresponding sides intersect in six points lying on a conic. An extension of this theorem to three dimensions may be stated thus: if two tetrahedrons be in perspective, their non-corresponding faces intersect in twelve straight

lines lying on a cubic surface. This theorem may be deduced from the equation

$$xyz u = (x + aT)(y + bT)(z + cT)(u + dT),$$

where $T = \alpha x + \beta y + \gamma z + \delta u$; and $a, b, c, d, \alpha, \beta, \gamma, \delta$ are constants. The equations of twelve lines on the surface are evident.

This paper shows how the remaining fifteen straight lines on the surface may be obtained by means of nothing higher than quadratic equations, and determines which of these lines intersect each other.

The paper then proceeds to give a graphical method of representing all the intersections of the twenty-seven lines on a cubic surface by means of a plane diagram, which admits of many interesting transformations.

By the help of such diagrams some of the known relations of the twenty-seven lines to each other are deduced, and some theorems with respect to the lines, which it is believed are new, are established; for instance, the number of closed quadrilaterals, pentagons, and hexagons on the surface is determined, as well as the number of ways in which nine triple tangent planes can be drawn to pass through all the twenty-seven lines, and the number of ways in which twelve of the lines can be chosen, so that they are the intersection of two tetrahedrons in perspective.

XII. "Further Observations on the Shoulder Girdle and Clavicular Arch in the Ichthyosauria and Sauropterygia." By H. G. SEELEY, F.R.S. Received May 25, 1893.

On January 18, 1892, I communicated to the Royal Society observations on the nature of the shoulder girdle and clavicular arch in Sauropterygia, which were read on February 18, and published in the Proceedings on June 25, 1892. These studies had grown out of the examination of new remains of Anomodont Reptiles, which I obtained in South Africa; and were the result of an endeavour to gain a knowledge of structures in which the shoulder girdle in extinct Reptilia admitted of detailed comparison with those materials. I had made examination of the same region of the skeleton in Plesiosaurs and Ichthyosaurs, and communicated the results to the Geological Society, which were published in the Journal of that Society in November and December, 1874.

In the paper of 1892 I endeavoured to correct, enlarge, or justify interpretations previously given. One aspect of this revision led to a controversial paper, challenging some points of interpretation which occur among the facts in my contribution. It is entitled "On the Shoulder Girdle in Ichthyosauria and Sauropterygia," by J. W. Hulke,