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"Electrical Interference Phenomena somewhat analogous to Newton's Rings, but exhibited by Waves along Wires."

By EDWIN H. BARTON, B.Sc., late "1851 Exhibition" Science Scholar. Communicated by Professor ARTHUR W. RÜCKER, M.A., F.R.S. Received February 20,—Read April 19,—Abbreviated* July, 1894.

INTRODUCTION.

The preliminary paper on this subject† gave the results of a single experiment, and approximately accounted for them by a mathematical theory of the reflexion and interference phenomena involved.

Since the publication of that paper the question of disturbances has been investigated, the experimental conditions improved, and various results obtained in confirmation of the original conclusions.

These matters form the subject of the present communication.

The apparatus employed for observing the interference phenomena is diagrammatically represented in fig. 1 and explained in the notes accompanying it.

In Experiment VII the electrometer was placed at HH′ instead of EE′, its usual position. In Experiments VIII and IX two electrometer needles were used, their respective attachments being at FF′ and HH′.

The lengths of SAM and MD, and the special construction of the abnormal part will be specified as required in the course of the paper when reference is made to the figure.

In the former paper‡ it was shown that in the case of a train of electrical waves advancing along the secondary from AA′ towards DD′ fig. 1, we have the following phenomena:—

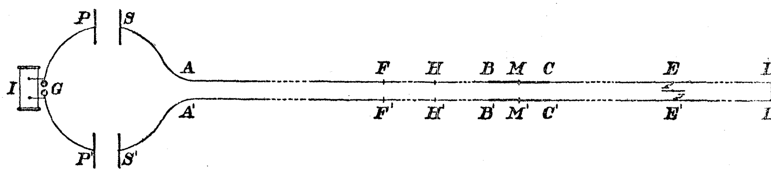
(1.) A partial reflexion at BB′, the beginning of the abnormal part;
(2.) A partial reflexion at CC′, the end of the abnormal part; and
(3.) Interferences between the two sets of waves proceeding from BB′ and CC′ respectively.

* The references given in the abstract of this paper ('Proceedings,' vol. 55, p. 350) are to the full manuscript, preserved in the archives of the Royal Society.

† 'Proc. Roy. Soc.,' vol. 54, pp. 85—96.

‡ *Loc. cit.*, p. 87.

FIG. 1.—Diagram of Apparatus.



EXPLANATION OF FIG. 1.

I. Induction coil worked by two secondary cells. This coil could give regular sparks up to 2 cm. long; their frequency was about 25 per second.

G. Spark gap, usually 2 mm.

PGP′ was 204 cm. measured along the semicircle. The wires PG, GP′ were 2 mm. diameter.

PP′. Condenser plates of zinc 40 cm. diameter to form the ends of the Hertzian primary oscillator.

SS′. Similar plates at a distance of 30 cm. from P and P′, and forming the beginning of the long secondary which consists of copper wires about 1 mm. diameter. Those parts shown dotted in the figure were set up in the corridors adjoining the laboratory. With the above dimensions waves 9 mm. long were produced, their frequency being about 33 millions per second.

AA′ = BB′ = CC′ = DD′ = 8 cm. This distance was maintained throughout the secondary by pieces of dry wood at intervals of about 2 or 3 metres.

BCB′C′. The abnormal (or *altered*) part of the secondary used to produce the reflexion and interference phenomena.

MM′. The middle of the abnormal part.

EE′. The electrometer. The needle is uncharged: it therefore turns in the same direction whenever there is any potential difference between E and E′, whatever the sign of that difference. The needle was suspended by a quartz-fibre, and gave (though uncharged) a deflexion of about 35 scale divisions when the electrometer was connected to a battery of 10 Daniell's cells. In using the electrometer for the electric waves first throws, not steady deflexions, were always read.

DD′. Wire bridge across the main wires.

FH = HB = ED = a quarter of a wave-length.

Hence, as the length of the abnormal part is gradually increased, the total energy of the reflected waves is thereby periodically increased and decreased. It was thus seen that the length of the abnormal part of the secondary corresponds to the thickness of the air film in the optical phenomena known as Newton's Rings, and that the reflexions of the electrical waves at the beginning and end of the abnormal part correspond respectively to the reflexions of light at the first and second faces of the air film.

For the experiments now to be described the results of the mathematical theory previously developed* are as follows :—

* *Loc. cit.*, equations (13), p. 90, and equations (18), p. 92.

(1.) For the *single reflexion* at the beginning of the abnormal part.

Let the amplitude of the original wave-train be a, and that of the reflected one be ab, then

$$b = -\frac{r-1}{r+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1),$$

where r is the ratio of the electrostatic capacity of unit length of the abnormal part of the secondary to that of its normal parts.

(2.) For the *interference* of the wave-trains proceeding from the beginning and end of the abnormal part of the secondary.

Let the incident wave-train be

$$\phi = ae^{-\alpha t+\alpha_1 x} \sin(\beta t - \beta_1 x) \quad \ldots\ldots\ldots\ldots \quad (2),$$

where ϕ denotes electrostatic potential, $\alpha/\alpha_1 = \beta/\beta_1 = v_1$, the velocity of propagation of the waves, and a, α and β are constants depending respectively upon the amplitude, the primary damping and the frequency.

Let the energy of the wave-train, when incident upon the beginning of the abnormal part (BB′ fig. 1) be ι, let $\iota\tau$ be the total energy of all waves transmitted through the abnormal part, and let $\iota\rho$ be that of all the waves returned along BA after one or more reflexions at C. Then

$$\left.\begin{aligned} \tau &= \frac{1-b^2}{1+b^2} + \mathrm{U}\,; \\ \rho &= \frac{2b^2}{1+b^2} - \mathrm{U}\,; \\ \text{where} \quad \mathrm{U} &= \frac{1-b^2}{1+b^2} \cdot 2b^2 e^{-\alpha t_2} \frac{(\alpha/\beta)\sin\beta t_2 + \cos\beta t_2 - b^2 e^{-\alpha t_2}}{1 - 2b^2 e^{-\alpha t} \cos\beta t_2 + b^4 e^{-2\alpha t_2}} \end{aligned}\right\} (3).$$

and t_2 denotes the time occupied by the waves in twice traversing the length, BC, of the abnormal part of the secondary.

Approximate Theory of Disturbances.

Thus far, then, we have dealt only with a *single* incidence of the wave-train upon the abnormal part of the secondary, and have found expressions for the two trains thereby produced. But in order to represent more completely what occurs with the apparatus arranged as in fig. 1, it is further necessary to trace the history of the two wave-trains to which the original one thus gives rise.

If the reflected wave-train were quickly extinguished and never again reached the plates, SS′ fig 1, and if the transmitted wave-train after passing the electrometer, and thus giving the deflexion by which it is measured, were also quickly extinguished without again reach-

ing the abnormal part CB, then, and then only, would the theory hitherto developed suffice. Clearly, however, these conditions are not fulfilled.

On the contrary, the waves go to and fro along the secondary, suffering—

(α) Total reflexion with reversal of electrification at the short circuited end DD′ (fig. 1),

(β) Total reflexion without reversal of electrification at SS′, and

(γ) Partial reflexion and partial transmission at each incidence on the abnormal part BC.

We have, thus, a case of binary fission at each incidence of a train on the abnormal part, each such fractional train returning to the abnormal part to be further, in like manner, subdivided. Theoretically, this process continues *ad infinitum.*

From these considerations it may easily be inferred that, to avoid hopeless confusion, the distance from the abnormal part to either end of the secondary must exceed half the length of the train of electrical waves, or in symbols :—

$$\text{SB} > \tfrac{1}{2}\text{X} \text{ and } \text{CE} > \tfrac{1}{2}\text{X} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(A)},$$

where X is the effective length of the wave-train, and the other letters refer to fig. 1.

It was also found necessary to avoid placing the abnormal part midway between the two ends, S and D, of the secondary. For, in that case, the two sets of waves respectively reflected at and transmitted through the abnormal part would, after travelling to opposite ends of the secondary, again meet at the middle and interfere with each other. This disturbance was sufficiently obviated by fulfilling the condition—

$$(\text{SM} \sim \text{MD}) > \tfrac{1}{2}\text{X} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(B)}.$$

But when conditions (A) and (B) are both fulfilled, there is still a residual disturbance. For, although the electrometer is placed to receive the systems transmitted through the abnormal part, it, in consequence of their repeating coursings to and fro, actually receives also feebler systems of reflected waves.

Thus, let the fraction of incident wave-energy transmitted by the abnormal part be τ, and let the ratio of the electrometer readings with and without the abnormal part be τ', then the author has shown that τ' is approximately given by the equation

$$\tau' = \tau \frac{1 - s_1 s_2}{(1-s_1)(1-s_2) + \tau(s_1 + s_2 - 2s_1 s_2)} \quad \ldots\ldots\ldots \quad (4).$$

where s_1 and s_2 express the attenuation of energy suffered by the

waves in passing along lengths of the secondary equal respectively to twice SM and twice MD (fig. 1).

τ' is always a little greater than τ except in the limiting cases where $\tau = 0$ and $\tau = 1$. Thus, the experimentally-determined ratio needs a negative correction. This is easily applied by graphical methods.

Now, since τ' is a function both of τ and of the attenuation (or *secondary* damping) it becomes necessary, in order to utilise Equation (4), to estimate the value of this damping.

The theory of one method devised for this purpose is as follows :—

Suppose the arrangement of apparatus shown in fig. 1 to be modified thus. Let the 1-mm. diameter copper wires be continued beyond DD′ for some distance, and after that let the wires for a further length be of iron of 0·1 mm. diameter. Also let the bridge shown at DD′ be movable and capable of being placed at pleasure anywhere beyond the electrometer.

And consider, first, the changes in the electrometer throws as the bridge is moved from the electrometer, but still always upon the copper wires. Let the electrometer throws be plotted as ordinates, and the distances of the bridge from the electrometer as abscissæ. Then, for positions of the bridge immediately beyond the electrometer the curve so obtained is conspicuously wavy, and continues sensibly so for a distance equal to half the appreciable length of the wave-train. This is the part corresponding to the curve shown by V. Bjerknes ('Wied. Ann.,' vol. 44, p. 522, 1891), and is due to the interferences between the wave-trains advancing towards and reflected from the bridge DD′. Beyond this part, whatever the rate of decay of the waves, we have a continuous droop in the curve. The exact form of the curve depends, in part, upon this decay, and the ordinate of the asymptote to the curve may be shown to be a simple function of σ(where $e^{-\sigma x}$ is the law of decay of the amplitude of the waves).

Let the distance SE (fig. 1) be L, and let y_x be the ordinate of the curve at the point whose abscissa is x, the ordinate at the origin being unity. Then we have for the equation of the curve—

$$y_x = \frac{1 - e^{-4\sigma L}}{2} \cdot \frac{1 + e^{-4\sigma x}}{1 - e^{-4\sigma(L+x)}} \quad \ldots\ldots\ldots\ldots\ldots \quad (5).$$

and for that of its asymptote—

$$y_\infty = \tfrac{1}{2}(1 - e^{-4\sigma L}) \quad \ldots\ldots\ldots\ldots\ldots \quad (6).$$

The ratio y_∞ is difficult to determine if the copper wires only are used, but with the thin iron wires the electrometer throws rapidly fall off to their minimum value, and thus y_∞ is readily obtained as desired.

EXPERIMENTS.

The chief experiments made will now be dealt with in the following order :—

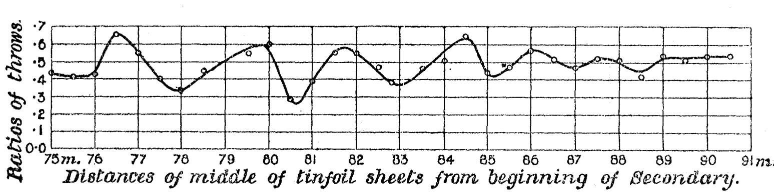
(1.) Undesired interferences of separate wave-trains. Experiments I—III.
(2.) Estimate of rate of decay of waves. Experiment IV.
(3.) Analogy to Newton's rings by transmission under improved conditions. Experiments V and VI.
(4) Analogy to Newton's rings by reflexion. Experiment VII.
(5) Abnormal parts which produce no reflexion. Experiments VIII and IX.

Experiment I.—Undesired Interferences of Distinct Wave-Trains simultaneously reflected at and transmitted through the Abnormal Part of the Secondary.

To investigate this question the arrangement of apparatus shown in fig. 1 was adopted. The abnormal part consisted of a single pair of tinfoil sheets each 50 cm. long and 32 cm. deep, the two being hung immediately opposite each other, one on each wire of the secondary. The total length, SAD, of the secondary was 162 m., the distance SAM was varied from 75 m. to 90·5 m. by steps of 0·5 m. each. Electrometer readings with and without the sheets were taken alternately in order to eliminate the possible errors due to irregular working of the primary sparks. The observations taken are shown in Table I.

The result is graphically exhibited in the curve, fig. 2.

FIG. 2.—Curve showing Undesired Interferences.



Experiment II.—Interferences as in Experiment I, but with Thin Iron Wires at the end of the Secondary.

Various experiments were made with the secondary lengthened about 70 mm. by the addition of thin iron wires, in the hope that they would absorb all the incident waves and thus do away with their repeated coursings and consequent interferences. The result was an

Table I.

Electrometer throws.			Distances SAM, fig. 1.	Ratios of throws, *i.e.*, quotients of cols. 3 and 2.
Without abnormal part.		With abnormal part.		
Actual observations.	Interpolated means.			
27·7	25·8	11·2	75 m.	0·43
23·8	26·8	11·1	75·5	0·41
29·7	27·6	12·0	76	0·43
25·6	26·0	17·1	76·5	0·66
26·4	29·2	16·0	77	0·55
32·0	32·2	12·8	77 5	0·40
32·5	32 2	10·8	78	0·34
32·0	31·8	14·4	78·5	0·45
31·6	28·8	15·8	79·5	0·55
26·1	27·7	16·6	80	0·60
29·4	31·5	9·0	80·5	0·29
33·6	33·6	13·2	81	0·39
33·6	34·4	19·0	81·5	0·55
35·2	35·0	19·4	82	0·55
34·7	34·0	16·3	82·5	0·48
33·1	32·7	12·8	82·8	0·39
32·3	33·6	15·6	83·5	0·46
35·0	33·4	18·0	84	0·51
31·8	31·4	20·5	84·5	0·65
31·0	32·5	14·3	85	0·44
34·0	33·1	15·7	85·5	0·47
32·2	33·1	18·4	86	0·56
34·0	33·4	17·3	86·5	0·52
32·8	31·1	14·8	87	0·47
29·4				
32·5	30·6	15·9	87·5	0·52
28·7	27·2	13·9	88	0·51
25·7	20·6	8·7	88·5	0·42
15·5				
30·3	30·4	16·2	89	0·53
30·6	29·6	15·1	89·5	0·51
28·6	27·4	14·5	90	0·53
26·3	25·8	13·7	90 5	0·53
25·3				

undulating curve whose ordinates lay between the limits 0·51 and 0·36. Thus the extreme values of the ordinates differed from their mean value by about $17\frac{1}{2}$ per cent. of the latter. The electrometer throws, however, were now reduced to about one-third of their former value. This was owing to the continuation of the wires so far beyond the electrometer instead of their termination by a bridge distant only a quarter wave-length from it. It seemed, therefore, that the iron wires did more harm than good, and they were accordingly abandoned in favour of the arrangement described in the next experiment.

Experiment III.—Search for Interferences as in Experiment I, but with the Abnormal Part away from the middle of the Secondary.

As the outcome of Experiment II the theory resulting in condition (B), p. 71, was developed, and the present experiment tried in accordance therewith. The effective length of the wave-train was experimentally found to be about 70 m. Hence, to fulfil (B) it was necessary to make (SAM ∽ MD) > 35 m.

The apparatus shown in fig. 1 was then arranged thus: SAD = 164 m. SAM varied from 95·25 m. to 105·75 m., and electrometer throws were taken alternately with and without a pair of tinfoil sheets as in Experiments I and II.

This experiment was performed twice and the mean result, when plotted as in Experiment I, gave a curve undulating between the limiting ordinates 0·55 and 0·46. Thus the extreme values of the ordinates differed from their mean value by about 9 per cent. of the latter.

It is thus seen that, although the straight line hoped for was not obtained, yet this curve is less wavy than either of the two previous ones. And since, also, this arrangement yields the maximum effect at the electrometer, it was adopted in the experiment for the analogy to Newton's rings by transmission hereafter described.

Experiment IV.—Estimate of the Rate of Decay of the Waves in their Advance along the Secondary.

The arrangement of the apparatus adopted for this determination has already been described under the headings of Theory (p. 72). In this experiment the droop of the curve sought was expected to be very slight. Consequently, for the sake of accuracy, each point of the curve was determined by twenty-one electrometer readings. Six sets of observations were made with SAE, fig. 1, = 160 m. They yielded a curve whose asymptote was—

$y_\infty = 0·34$; whence we obtain

$$\sigma = 0·0018 \quad \ldots\ldots\ldots\ldots \quad (7).$$

A second series of observations with SAE = 91·5 m. afforded the values $y_\infty = 0·31$ and

$$\sigma = 0·0029 \quad \ldots\ldots\ldots\ldots \quad (8).$$

Experiment V.—Analogy to Newton's Rings by Transmission under best Conditions.

This experiment was made with exceptional care, as it is the chief one of the series. Most of the others either served to indicate the

conditions adopted in this one or were in other ways subsidiary to it. The construction of the abnormal part was such as to reflect a large fraction of the wave-energy incident upon it. Its position was so chosen as to avoid the interferences of higher order already noticed. Thus the true effect sought was large, the disturbing ones small. The arrangement of the apparatus is shown in fig. 1. The details are as follows.

The abnormal part consisted of tinfoil sheets each 32 cm. deep and 50 cm. long. These were hung opposite each other upon the two wires of the secondary. Lest a shift of the middle point of the sheets along the wires might have a slight disturbing effect on the electrometer throws, the abnormal part was always lengthened or shortened by the same amount at *each end*, thus leaving the middle point M undisturbed. The length SAM was throughout the experiment 101 m., MD being 63 m.* so as to comply with condition (B). The tinfoil sheets were kept properly spaced by very accurately cut wood separators, one for each half metre's length of the sheets. The same separators, but at longer intervals, were used throughout the line. The vertical edges of consecutive sheets were made to overlap about 2 cm. and allowed to hang in simple contact.

Lest this contact should be insufficient and so involve an error, the sheets were built up in this way to a length of 2·25 m. (a quarter of a wave-length), and the ratio determined of the wave-energy transmitted by it to that incident upon it. Then an abnormal part, consisting of zinc plates soldered together was substituted, and the fraction of wave-energy transmitted again determined. In each case, to insure accuracy, twenty-one electrometer readings were taken alternately with and without the abnormal part. The two ratios thus determined were :—†

(1) For the tinfoil sheets .... 0·135 ± 0·007
(2) For the zinc plates soldered .... 0·135 ± 0·002

Hence this apparently imperfect contact seemed sufficient for the case, and was adopted in the main experiments now under consideration. A spark gap of 2 mm. was used throughout both in the experiments just described and in what now follows.

The method of taking the observations so as to eliminate the possible errors due to the irregularities in the sparking of the primary was as follows. Several electrometer throws were taken, at first

* When making the corresponding experiment, described in the former paper p. 96, SAM was taken equal to MD. I had not, up to then, seen that it was desirable to avoid that arrangement.

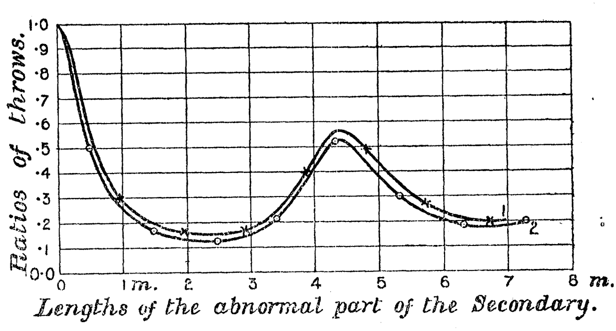
† The probable errors were calculated by the formulæ given in Kohlrausch's 'Leitfaden der Praktischen Physik,' Leipzig, 1892, pp. 1—3, and proved in Dr. B. Weinstein's 'Handbuch der Physikalischen Maassbestimmungen,' vol. i.

without any tinfoil sheets on the wires, then a couple of readings with, say, two sheets on each wire; then two readings with four sheets on each wire, and so forth. After the abnormal part had thus been built up to a considerable length, it was then in like manner shortened, two readings being taken for each length, concluding with several readings without any sheets at all on the wires. The whole process was carried out four times, the lengths used in the various cases being varied so as to obtain eight electrometer readings for each half metre's length of the abnormal part. Thus, in the first and third sets of observations, the lengths of the abnormal part were approximately 0, 1 m., 2 m., 3 m. .... 3 m., 2 m., 1 m., 0; whereas, in the second and fourth sets, the lengths were about 0, 0·5 m., 1·5 m., 2·5 m. .... 2·5 m., 1·5 m., 0·5 m., 0.

A single set of readings up to nearly 7 m. length of abnormal part is given in Table II. During the process of lengthening the abnormal part the ratios are obtained by taking for the divisor the *initial* mean throw without abnormal part, whereas, during the shortening, the like *final* value is used as the divisor. Hence the double values for the ratio corresponding to the length 6·72 m. The last column of the table contains the mean of the ratios during lengthening and shortening.

The kind of agreement obtained by the four sets of observations is shown by the two curves in fig. 3. In these curves the abscissæ

FIG. 3.—Preliminary Curves from Experiment V.



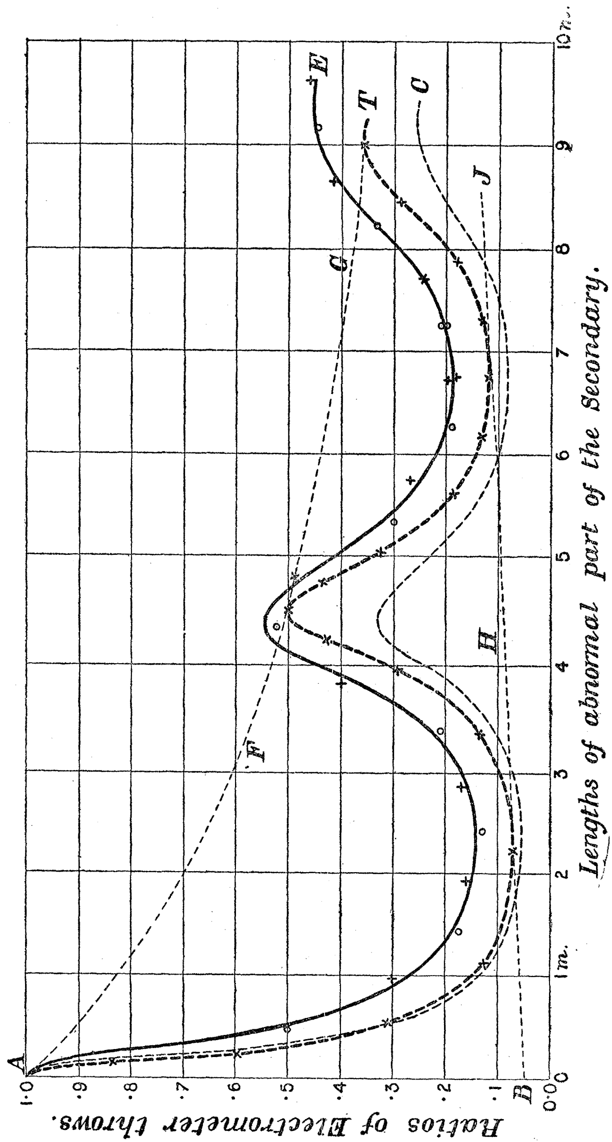
represent the lengths of the abnormal part. In the upper curve the ordinates represent the lengths of the abnormal part. In the upper curve the ordinates represent the mean ratios of the throws obtained in the first and third sets of observations with and without the abnormal part. The ordinates of the lower curve give the values of the same ratios, as obtained from the second and fourth sets of readings.

Table II.

Electrometer throws.	Mean throws.	Length of the abnormal part.	Ratio of throws with & without it.	Mean ratios.
40·8 51·8 36·2 46·2 39·2	42·8	m. 0·00	1·00	1·00
13·9 12·6	13·2	0·97	0·31	0·30
5·7 5·8	5·8	1·93	0·135	0·14
5·8 6·2	6·0	2·89	0·14	0·145
16·9 17·9	17·4	3·85	0·41	0·39
17·9 17·9	17·9	4·80	0·42	0·42
9·6 9·5	9·6	5·76	0·224	0·23
6·7 6·6	6·6	6·72	{0·154 0·174}	0·164
9·0 8·9	9·0	5·76	0·24	0·23
16·2 15·9	16·0	4·80	0·42	0·42
14·0 14·2	14·1	3·85	0·37	0·39
5·6 5·6	5·6	2·89	0·15	0·145
5·7 5·9	5·8	1·93	0·15	0·14
11·8 10 3	11·0	0·97	0·29	0·30
38·4 34·3 40·7 36·2 40·4	38·0	0·00	1·00	1·00

The process was afterwards continued from this point onwards to a length of 9·5 m. The final mean result of the entire experiment is shown in the curve E, fig. 4.

FIG. 4.—Analogy to Newton's Rings by Transmission.



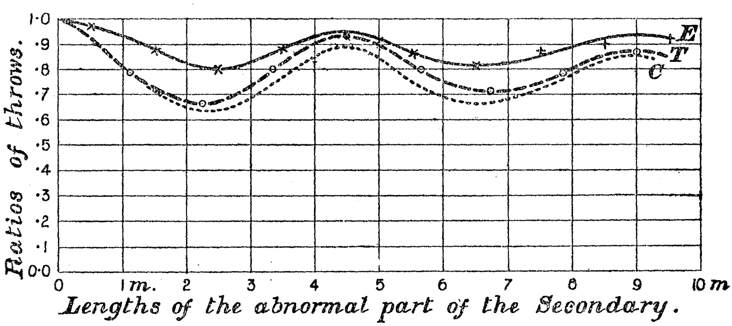
Experiment VI.—Analogy to Newton's Rings, as in Experiment V, but with different Abnormal Part.

In this experiment the lengths of SAD and the position of M were precisely as in Experiment V. The abnormal part, however, consisted simply of the ordinary wires, but put closer together there than elsewhere, namely, 0·68 cm. apart instead of 8 cm.

The electrometer readings were taken thus:—First, with no abnormal part; next, with one of 0·5 m. long; again with no abnormal part, then with one 1·5 m. long, and so forth. The readings with and without the abnormal part were always alternated, and the ratios of the throws taken as in Experiments I—III.

The result of this set of observations is graphically exhibited in the curve E, fig. 5. The abscissæ represent the lengths of the abnormal

FIG. 5.—Second Case of Analogy to Newton's Rings by Transmission.



part, and the ordinates the ratios of the electrometer throws with to those without it.

Experiment VII.—Analogy to Newton's Rings by Reflexion.

In this experiment the arrangement of apparatus shown in fig. 1 was modified by removing the electrometer from EE′ and inserting it at HH′, where HB is a quarter of a wave's length. Throughout the experiment the lengths were as follows:—

$$\text{SAD} = 235 \text{ mm.}, \ \text{SAB} = 164 \text{ mm.}$$

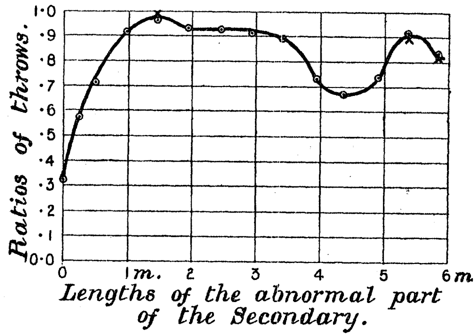
The abnormal part was of tinfoil sheets, 32 cm. deep and 50 cm. long, precisely as in Experiment V, while the end BB′ remained fixed, the length BC was varied from nothing to 6 m., advancing by steps of half a metre.

The electrometer throws were taken, first with a bridge at BB′, then without the bridge but with an abnormal part consisting of a

single pair of tinfoil sheets, next with the bridge at BB′, then again without the bridge but with an abnormal part of two pairs of sheets, and so forth, the readings with and without bridge being always alternated.

The result is plotted in the curve shown in fig. 6, the lengths of

FIG. 6.—Analogy to Newton's Rings by Reflexion.



the abnormal part being taken as abscissæ, and the ratios of electrometer throws as ordinates.

On this curve two apparently anomalous humps may be noticed in the neighbourhood of 1·5 m. and 5·5 m. lengths of the abnormal part. These, however, are not due to errors of observation; for, on carefully repeating the experiment in these regions the first results were confirmed, as shown by the double dots made at those places.

A few readings were also taken with the above general arrangement, but an abnormal part, consisting of the ordinary wires nearer together, just as in Experiment VI. It was thus found that the electrometer throws were about three times as great with an abnormal part a quarter-wave long as with one a half-wave long.

Experiment VIII.—An Abnormal Part which Produces no Reflexion.

Having thus far experimented with the desired interference phenomena, and discussed the various disturbances involved, it now seems of interest to notice particular forms of the abnormal part which reflect no portion of the wave-energy incident upon them. This and the following experiment were tried as tests of the truth of the theory for a single reflexion.

However, to establish the entire absence of reflexion would be very difficult with the single-needle electrometer previously used, because the slightest irregularity of the primary sparking might be mistaken for the effect of reflexion. And no repetition of observations, however extended, would justify the conclusion that the energies transmitted

with and without the abnormal part were absolutely equal. It became expedient, therefore, to use an electrometer which gave deflexions *only* when reflected waves were present. Such an instrument was, fortunately, ready to hand in the differential electrometer used and described by Dr. von Geitler,* and kindly left by him for my use.

In this form of electrometer two needles, rigidly connected with each other and hanging upon the same quartz fibre, are employed. One of these experiences a right-handed torque, owing to attachments at one pair of points on the secondary, whenever these points have a potential difference of either sign, while the second needle, under like conditions, experiences a left-handed torque, owing to attachments at a second pair of points on the secondary, and distant a quarter of a wave's length from the first pair. Thus, when the instrument is properly adjusted, the *mere passing* of a wave-train leaves the needles undisturbed.

If, however, reflexion is by any means produced near the electrometer, so that it is in a region of *standing* waves, it may give a throw.

The arrangement of apparatus adopted may be seen from inspection of fig. 1, which was modified by the removal of the single-needle electrometer from EE′, and the insertion of the differential electrometer at FF′ HH′. The first needle was influenced by attachments to F and F′, and the second by attachments to H and H′. The main wires between the first and second pairs of points were led in a loop, so that they first passed the electrometer at FF′, and a second time at HH′. Thus, all the attachments between the main wires and the electrometer were quite short.

The wires in the normal part of the secondary were 0·116 cm. diameter. The abnormal part consisted also of copper wires, and were in this experiment 0·019 cm. diameter, and 2·25 m. long.

These were tried at two distances apart, namely, 1 cm. and 1·5 cm. The electrometer throws in these cases were $-1·8$ and $+0·6$ scale divisions respectively. Whence, by interpolation, we have 1·37 cm. nearly, as the distance apart at which these wires would yield no reflexion. The electrometer throw with a bridge at BB′ was twenty-four scale divisions.

Experiment IX.—A Second Example of no Reflexion from an Abnormal Part.

In this experiment everything was the same as in the preceding one, except that the abnormal part consisted of wires thicker and wider apart than the rest of the line, instead of being thinner and

* Wiedemann's 'Annalen,' vol. 49, pp. 188—189, 1893.

nearer together. The wires in question were 0·55 cm. diameter, and, when placed at 66 cm. apart, there was practically no reflexion.

Comparison of Theory and Experiment.

It now becomes of interest to compare the results of Experiments V to IX with the theories advanced concerning them.

In Experiment V we have the capacity per unit length of the normal part in electrostatic units approximately given by $1/(4 \log_\epsilon d/w)$, where d is the distance between the centres of the wires and w their radius. For the abnormal part, without any correction for the edges, we have capacity per unit length equals $D/4\pi d$, where D is the width of the tinfoil sheets. Now d was 8 cm., w was 0·05 cm. nearly, and D 32 cm. Thus we obtain for r, the ratio of the capacities, 6·5 nearly. If, however, in estimating the capacity of the abnormal part, a correction for the edges is made, we obtain values of r ranging up to 8 or 9, according to the length of the abnormal part under consideration, and the formula used for the correction.

Taking as a typical case the length of abnormal part to be 2·25 m., on which special experiments were made, and correcting for the edges by the approximate formula given by Professor Kohlrausch* we obtain $r = 9$, nearly. Whence, from equation (1), $b = -0{\cdot}8$.

It will be seen from the equations (3) that it is further necessary to know the constants α and β. These were determined by experiments similar to those previously devised and carried out by V. Bjerknes.†

In my case, however, the electrometer readings were alternated with the bridge at the variable distance x, and at the quarter wave-length distance beyond the electrometer. A spark gap of 2 mm. was used throughout.

Thus were obtained for the electrical waves advancing along the wires, the wave-length $\lambda_1 = 9$ m.; and for the constant involving the primary damping, we have $\gamma_1 = 2\pi\alpha/\beta = 0{\cdot}524$, or say $\gamma_1 = 0{\cdot}5$ nearly.

Then, assuming that the velocity of propagation of the waves along the wires is practically that of light in air,‡ we obtain, from

* 'Leitfaden der praktischen Physik,' Leipzig, 1892, p. 357.

† Wiedemann's 'Annalen,' vol. 44, pp. 519—522, 1891.

‡ This is known to be the case from various experiments, and also from theoretical considerations. See, for example:—

(1.) Professor Hertz, "Ausbreitung der elektrischen Kraft," Leipzig, 1892, or the English translation by Professor D. E. Jones, B.Sc.

(2.) MM. Sarasin and De la Rive, 'Archives des Sciences Physiques et Naturelles,' vol. 29, No, 5. Genève. 1893.

(3.) Professor J. J. Thomson, F.R.S., "Recent Researches on Electricity and Magnetism," pp. 279 and 451—467.

(4.) Professor Oliver Lodge, F.R.S., 'Phil. Mag.,' August 1888, vol. 26, p. 228; 'Report Brit. Assoc.,' 1888, p. 567; and 'Proc. Roy. Soc.,' vol. 50, pp. 29—39.

$\lambda_1 = 9$ m., the value of the frequency, namely, $\beta/2\pi = 33\frac{1}{3}$ millions per second, nearly.

From these data the curve T, fig. 4, is plotted, the lengths of the abnormal part being abscissæ and the corresponding fractions of incident wave energy transmitted being ordinates.

It is thus noticeable that the theoretical curve, though of similar form, lies wholly below the experimental one. If, however, we apply to the latter, a correction based upon Equation (4) and the results of Experiment IV, we obtain curves which lie wholly below the theoretical one. One such is shown in curve C, fig. 4, in which σ is taken equal to 0·0029.

Thus, while the comparison of theory and experiment has not resulted in the establishment of an exact agreement between them, it is yet so far satisfactory to notice* that the three curves are precisely similar in form, and that the theoretical one lies entirely between the two experimental ones, with and without correction. It is also noteworthy that Mr. Yule, in his recent work† on "The Passage of Electrical Wave-trains through Layers of Electrolyte," found a discrepancy of the same sign and of like amount between the experimental curve and the theoretical one.

The dotted lines above and below the main curve in fig. 4 indicate the limits between which the latter lies. These limiting lines would be horizontal but for the primary damping of the waves emitted by the oscillator.

We thus see that the curve in the neighbourhood of $\lambda_1/4$ affords the best means of determining b when it is nearly unity. For the troughs of the curve are broad, and therefore the curve is at that part for some distance nearly parallel to the axis of the abscissæ. Thus a great deviation from the true length $\lambda_1/4$ would make but a small one in the corresponding ordinate. Secondly, for large values of b, this ordinate is but very slightly affected by the primary damping. Hence a great error in the measurement of α makes but a slight one in the determination of b from this part of the curve. Thus, taking $\gamma_1 = 0{\cdot}5$, and accepting the experimental curve, we obtain $b = -0{\cdot}71$ and $r = 6$, or taking the experimental curve, with correction, as shown by the lower line in fig. 4, we get $b = -0{\cdot}8$ and $r = 9$. And it was between these limits 6 and 9 that r was calculated to lie.

In Experiment VI.—Since the same wires were used for the abnormal part but closer together, namely, at a distance $d' = 0{\cdot}68$ cm. instead of $d = 8$ cm., we have for the ratio of capacities

$$r = \frac{1/4 \log (d'/w)}{1/4 \log (d/w)} = \frac{\log (8/0{\cdot}05)}{\log (0{\cdot}68/0{\cdot}05)} = 2.$$

* Professor Hertz considered the agreement between experiment and theory as close as was to be expected.

† 'Phil. Mag.,' pp. 543—544, Dec. 1893.

Thus, by Equation (1) we obtain $b = -\frac{1}{3}$. From this value of b and those last used for α and β we derive the curve shown by T, fig. 5.

This, as in the case of Experiment V, lies wholly below the experimental curve. But here, again, the correction to the experimental curve applied in accordance with Equation (4), and the higher value of σ from Experiment IV yields the curve C, fig. 5, lying still lower than the theoretical one.

In Experiment VII.—It may be seen from the Equations (3) or from general considerations that the total intensity of the reflected waves must be complementary to that of the transmitted ones, that is $\rho + \tau = 1$. Consequently the theoretical curve showing ρ as ordinates instead of τ would be obtained by inverting the theoretical one for τ. Hence, we see that the general characteristics of the curve for ρ may be thus outlined:—

(1) A damped wavy-formed curve with—
(2) *narrow* troughs at $l = 0, \lambda/2, \lambda$, &c., and
(3) *broad* crests at $l = \lambda/4, 3\lambda/4$, &c.,

where l is the length of the abnormal part. These general characteristics are possessed by the experimental curve obtained. It must, of course, be borne in mind that the ordinates of the curve thus experimentally determined do not represent ρ, but represent approximately ratios proportional to $1 + \rho$, since we have both the reflected and the *incident* waves passing the electrometer. On this account the wavy form of this curve is less strongly marked than in the case of transmission. Probably also other, and undesired, interferences arose between the on-coming and reflected waves in the neighbourhood of the electrometer, thus causing the two anomalous humps at 1·5 m. and 5·5 m.

In Experiment VIII.—For the cases to which Equation (1) applies we see that there is no reflexion when $r = 1$, that is, when no change in the capacity of the secondary occurs. Now the approximate expression for the capacity of two equal parallel cylinders depends not upon their *absolute* but upon their *relative* dimensions only. Hence, if Equation (1) is correct, it must be possible to arrange a part of the wires in a form which appears very abnormal, but yet so as to produce no reflexion. We have simply to introduce, at any part, thinner wires placed proportionately nearer together or thicker ones in like manner further apart. Thus, since in either case, the capacity is unaltered by the change in question, we have $r = 1$, and the fraction of wave-energy reflected disappears.

In the experiment under consideration the readings taken pointed to the distance 1·37 cm., from centre to centre of wires, as being that which, with the wires in use, would give no reflexion. Theory gives as the correct distance 1·32 cm. Thus the discrepancy is not great;

in fact, it may perhaps be within the limits of the errors of observation.

In Experiment IX.—Here theory points to a distance apart of 38 cm. as that which should produce no reflexion. The experiment, however, gave 66 cm. This discrepancy at first sight appears serious. When, however, it is noticed that in the case of cylinders widely separated a very great further increase in their distance is required to produce a small decrease in their capacity, the discrepancy does not seem so great. Indeed, when the distance between the wires in this case is altered from 38 cm. to 66 cm., the capacity is only changed by about 10 per cent. It is conceivable that a discrepancy of that order might be due to the sloping portions of wire which served to connect the normal wires, spaced at 8 cm., with the abnormal ones spaced at 66 cm.

Summary of Chief Results.

The principal conclusions to be drawn from the foregoing theory and experiments taken in conjunction may be stated as follows:—

(1.) In experimenting with electrical waves of high frequency passing along a pair of parallel wires with short-circuited end, and containing a portion which produces partial reflexion, it is necessary to make right choice of the lengths before and after such source of reflexion in order to avoid disturbing interferences.

(2.) A sudden change in the capacity of the secondary produces a partial reflection at that place of change. The ratio of the amplitudes of the reflected and incident wave-trains may be expressed as a simple function of the change in capacity. See Equations (12) and (13) in the previous paper.

(3.) If a sudden change in the capacity of the secondary is succeeded by a sudden reversion to the normal state of the wires, then reflexions occur at each of these places of abrupt change. And, if the distance between these two points is comparable with a wave-length, then the waves proceeding from them will interfere, consequently when the distance in question is increased a series of maxima and minima successively obtain, essentially analogous to those which simultaneously occur in the optical phenomena known as Newton's Rings.

(4.) If the secondary has a part which, though abnormal in appearance, introduces no change in its capacity, then no reflexion is produced by it.

For each of the above statements theoretical grounds and experimental confirmation have been adduced. And, although the two are not in exact quantitative agreement, yet I think it will be admitted

that they are approximately so, and that their accordance in all general respects is such that they support each other and warrant the conclusions drawn from them.

I have again to acknowledge my deep indebtedness to the late lamented Professor Hertz for the very able advice he at all times so readily gave me while I was engaged on the above work under him at the University of Bonn during the session 1892—93.

"On the Leicester Earthquake of August 4, 1893." By CHARLES DAVISON, M.A., F.G.S., Mathematical Master at King Edward's High School, Birmingham. Communicated by Professor J. H. POYNTING, F.R.S. Received February 28,—Read May 10, 1894.

On August 4, 1893, at 6.41 P.M. (G.M.T.), an earthquake shock was felt throughout the whole of Leicestershire and Rutland, and in parts also of the adjoining counties of Lincoln, Nottingham, Derby, Stafford, Warwick and Northampton. The disturbed area, therefore, lies entirely within the land. It is also one over which villages and country houses are for the most part closely scattered, and it has thus been possible to obtain a large number of careful and detailed accounts. I have received altogether 391 records from 298 places where the earthquake was observed, and 103 others from 97 places where, so far as known, no trace of it was perceived.*

My inquiries were carried out on the supposition that tectonic earthquakes are, as a rule, mere incidents in the gradual development of faults, that the shock is caused by the friction which results from one rock-mass slipping slightly but heavily over and against the other, the accompanying sound and tremulous motion being due to the exceedingly small and rapid vibrations which proceed chiefly from the margins of the fault-surface over which the slip takes place.†

The interpretation of the evidence collected rests on the following principles:—

(1.) The direction of the fault is parallel, or nearly so, to that of the longer axis of the disturbed area, or of an isoseismal line. (2.) The intensity of the shock increases in both directions from the fault-line until a maximum is reached, and then decreases, so that

* The expenses of the inquiry were defrayed by part of a grant which I had the honour to receive from the Government Research Fund. I regret that I am unable to acknowledge in detail the valuable and courteous assistance rendered by my numerous correspondents.

† "On the Nature and Origin of Earthquake-sounds," 'Geol. Mag.,' vol. 9, 1892, pp. 206—218.