

III. "Results derived from the Natality Table of Kőrösi by employing the Method of Contours or Isogens." By FRANCIS GALTON, F.R.S. Received January 12, 1894.

There are three variables in the statistics of natality. The age of the father is one, that of the mother is another, and the percental offspring of parents of those ages is the third. These three variables may be coordinated in the same way as that which is daily followed at meteorological offices in dealing with (1) the longitudes of the various stations, (2) their latitudes, and (3) the barometric height at each. After these data have been entered on a chart in their proper places, contours, known by the name of isobars, are drawn to show the lines of equal barometric pressure. In natality tables, the ages of the father and the mother take the place of the longitudes and latitudes in weather charts, and lines of similar birth rates, or as I would call them, "isogens," take the place of isobars.

Table I contains the means of each set of four adjacent entries as shown by the arrangement below, the left-hand diagram showing the four entries, and the right-hand one showing their mean. The

	Father's Age.				
	38	39	40		39
Mother's Age.	28	21	23		
	29	21	32	24·25	29
	30				

entries themselves were copied to the nearest integer from Kőrösi's tables. The means are recorded in Table I to the nearest integer only, subject to an allowance of correction not exceeding 0·30 for the sake of slight smoothing; thus 24·25, which would otherwise have been entered as 24, might be treated as if it were  $24\cdot25 + 0\cdot30 = 24\cdot55$  and be entered as 25. Similarly 24·75 might be entered either as 25 or as 24. It will be seen by the right-hand diagram that the position of the mean corresponds to the first moment of the years shown at the side and top; therefore the interval to which the annual birth rate corresponds is made up of the half year before and after that epoch.

The means that are enclosed in brackets are those in which one or

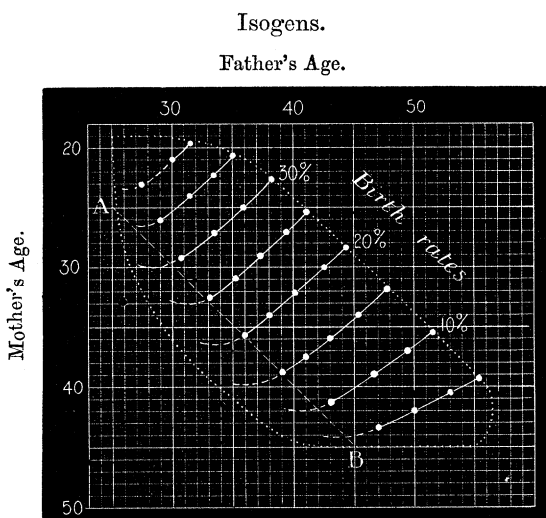
Table I.—Annual Percentage of Births according to the Ages of the Father and Mother, derived from Kőrösi's Table of Natality at Budapest.

The tabular values refer to the half-year before and after the beginning of the year entered at the top and side.

Age of the mother.	Age of the father (the even years are omitted).														
	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53
19	(49)	46	(42)												
21	44	44	42	33	34	(36)									
23	43	42	41	35	35	32	30	(26)							
25	32	36	38	32	31	30	29	28	(25)						
27	31	32	36	33	31	26	27	25	19	(21)					
29		34	35	31	32	27	27	24	21	23	(17)				
31		24	26	22	24	26	24	21	20	18	17	16			
33			28	25	22	22	23	22	18	17	16	15	(12)		
35			24	19	21	19	21	29	16	15	13	10	12	(9)	
37				(13)	18	21	17	17	18	16	14	13	12	12	(13)
39					(17)	(16)	16	15	15	15	14	10	9	9	(5)
41						(11)	(12)	12	10	11	10	10	6	6	(3)
43							(6)	(7)	6	6	5	5	4	4	(3)
45									3	(4)	3	2	3	2	(2)

more of the four squares from which they were derived was blank. They are, of course, less trustworthy than the rest; moreover, they may depend on less than 100 families.

The ages of married couples are distributed over only about one-half of the squares of Table I, as there are too few examples of other ages to be statistically available. This partial distribution is well seen in the diagram of isogens, where a dotted outline encloses all the material that can be used with safety. The broken line AB corresponds to the instances in which both parents are of the same age. The chart is practically limited to marriages in which the wife is less than five years older, and less than seventeen years younger, than her husband.



It will be noticed that the isogens run in nearly straight, diagonal, and equidistant lines across the greater part of the chart. If we omit six squares in the upper left-hand corner where there is no room for an isogen, we shall find these diagonal lines to cross 89 of the total number of 118 entries, or between eight and nine tenths of them. This indicates the existence of a very curious and unexpected law of natality, which is well brought out by Table II, which shows the values measured from the dots marked on the isogens. They have been taken at convenient places to serve as examples, one at the beginning, one at the end of the straight portion of each, and at some other intervening places.

In Table II are given the ages of the father and mother that correspond to each of these dots.

As a consequence of the straightness of the isogens, the *sums of the*

Table II. Values of the Isogens at the Dots.

Percentage of births in the year.  A	Examples of the corre- sponding ages of the		B + C.	Accepted mean of B + C.	A + B + C.
	Mother. B	Father. C			
40	23	27½	50½	51	91
	21	30	51		
	19½	31½	51		
35	26	29	55	55½	90½
	24	31½	55½		
	22	33½	55½		
	20½	35	55½		
30	29½	30½	60	60½	90½
	27	33½	60½		
	25	35½	60½		
	22½	38	60½		
25	32½	33	65½	66½	91½
	31	35¼	66¼		
	29	37¼	66¼		
	27	39½	66½		
	25½	41	66½		
20	35¾	35¾	71½	72	92
	34	38	72		
	32	40	72		
	30	42½	72½		
	28¼	44¼	72½		
15	39	39	78	79	94
	37½	41	78½		
	36	43	79		
	34	45¼	79¼		
	31¾	47¾	79½		
10	41¼	43	84¼	86	96
	39	46½	85½		
	37	49½	86½		
	35½	51½	87		
5	43½	47	90½	93	98
	42	50	92		
	40½	53	93½		
	39½	55½	95		

ages of the parents to which each point in the straight portion of the same isogen refers are *constant*. The difference between their ages is of no account whatever in eight or nine tenths of the total number of marriages; it is only when the wife is older than the husband or when she approaches the limit of the child-bearing age, that this curious

law ceases to hold true. The connexion between it and the straightness of the isobar is easily understood from the equation to a straight line of  $x + y = \text{constant}$ , for if  $x$  represent the age of the father,  $f$ , and if  $y$  represent that of the mother,  $m$ , then  $f + m = \text{constant}$ . That this is a fact is conspicuously evident from the columns headed B + C in Table II. This is the first curious law.

Again, through a coincidence between the increasing age of either parent and the decrease of fertility, it happens that the sum of the three elements of (1) father's age, (2) mother's age, (3) percental birth-rate in a year has a value that is itself approximately constant, as is seen in the column headed A + B + C. Its lowest limit is  $90\frac{1}{2}$  and its highest up to the isogen of 10 per cent. is 96, but it has increased to 98 at the isogen of 5 per cent. If we accept for it a constant value of 93 or 94 we shall never be far wrong in the larger part of the chart.

From this follows the second curious law that if we wish to calculate the percental birth-rate per annum for a married couple within the limits of the chart where the isogens run straight and parallel, we have only to add the ages of the father and mother and subtract the total from 93 or 94, in order to obtain it with considerable precision. The approximate limits within which this law obtains are: (1) the wife is not to be older than her husband; (2) she is not to be less than twenty-three years of age, nor (3) more than forty.

*Example.*—In any large number of husbands and wives living under like conditions to the inhabitants of Budapest, whose respective ages at their nearest birthdays, to 21st June, 1892, were: that of the father, thirty-five, that of the mother, twenty-seven; then the number of children born to them during the year 1892 would be at the rate of  $93 - (35 + 27)$  per cent. = 31 per cent.; the isogen makes it about 32 per cent.\*

I shall not now enter into the other salient peculiarities of the isogens further than to allude to the curious change in their course which occurs when the wife is older than the husband. When she is from thirty to thirty-eight she certainly seems to be appreciably more fertile with a husband of her own age or somewhat older than she is with one who is younger. I should hesitate to ascribe this to physiological causes without corroborative evidence derived from breeders of stock. It is very possible that indifference on the part of young husbands to ageing wives may have something to do with it.

It is almost needless to say that if it be desired to obtain the observed birth-rates for a mother of any specified age and for fathers of

\* A rough mechanical arrangement was exhibited by which isogens may be drawn. It consists of three sliding pieces connected by a string. A coloured patch is pasted on the back board to show the limits within which the isogens drawn by it are trustworthy.

various ages, the corresponding line of Table I will give the information, while if the smoothed values are wanted, a similar line in the chart of isogens will give them after being smoothed, not in one dimension only *but in two dimensions*. Similarly, as regards the birth-rates for a father of any specified age and for mothers of various ages, by following the vertical columns instead of the horizontal lines.

In conclusion, I would remark that, though the method of isogens applied to Kőrösi's tables fully discusses the distribution of mean birth-rates, those tables do not enable us to determine the second postulate of paramount importance, namely, the degree of conformity of individual cases to the means of many cases. We know nothing thus far about the facility of error at the various positions in the chart, whether or no it conforms to the normal law of frequency; still less, what is its modulus, or whether the modulus is constant throughout the chart or varies in accordance with some definite law.

The answer to these questions admits of being obtained by a moderate amount of work on the original observations, selecting at first a few squares for exploratory purposes, such as are (1) distributed evenly about the chart, and (2) contain each of them not less than some 300 observations, and (3) whose means accord with the smoothed isogens that pass over the squares, thereby affording satisfactory centres of reference.

IV. "Appendix to a Communication entitled 'The Mechanical Equivalent of Heat.'"\* By E. H. GRIFFITHS, M.A. Communicated by R. T. GLAZEBROOK, F.R.S. Received December 7, 1893.

*Section I.*

In a communication which I had the honour of making to the Royal Society in the spring of this year, the following statement occurs (p. 420):—"We are (with the help of Mr. Callendar) now entering on a careful direct comparison of thermometer  $E_m$  with a new form of air thermometer, which, there is every reason to believe, will give very accurate results, but we are unable to assign any definite limit to the time that this investigation may take."

A great number of comparisons have been made during the summer of this year by Mr. Callendar and myself between the mercury thermometer  $E_m$  used by me for determining the temperature of the calorimeter, the Tonnelot thermometer, No. 11,048, described in the above paper (pp. 426—433), the platinum thermometer N, by which the mercury thermometer  $E_m$  had been previously standardised,

\* 'Phil. Trans.,' vol. 184 (1893), A, pp. 361—504.

# Isogens.

Father's Age.

Mother's Age.

