

establishing the existence of a fluid interior, as supposed by M. Folie, rather affords an additional reason for discarding this hypothesis.

II. "On the Abelian System of Differential Equations, and their Rational and Integral Algebraic Integrals, with a Discussion of the Periodicity of Abelian Functions." By Rev. W. R. WESTROPP ROBERTS. Communicated by Rev. G. SALMON, D.D., F.R.S. Received January 17, 1895.

(Abstract.)

Before entering on the discussion of the Abelian system of differential equations, I treat of some general algebraic theorems having reference to the differences of various sets of "facients," and give a wider definition to the term "source," hitherto used to signify the source of a covariant, and treat of two operators,  $\delta$  and  $\Delta$ .

I then show how, by forming what I call a "square-matrix," all the conditions can be obtained which are fulfilled when a polynomial  $f(z)$  of the degree  $2n$  in  $z$  is a perfect square. With regard to these conditions, I remark that any one of them being given all the others can be found by successive operations of the operator  $\delta$ .

I next treat of the system of differential equations termed "Abelian," in which there are  $m$  quantities and  $m-1$  equations, comprehended in the typical form

$$\Sigma \frac{z^i dz}{\sqrt{f(z)}} = 0,$$

where  $\Sigma$  relates to the  $m$  quantities  $z_1, z_2, \dots, z_m$ , and  $i$  may have any integer value from  $i = 0$  to  $i = m-2$ , it being understood that  $f(z)$  is a polynomial of the degree  $2m$  in  $z$ ; and I show that, if  $f(z) \equiv z^{2m} + P_1 z^{2m-1} + P_2 z^{2m-2} + \dots + P_{2m}$ , be reduced to the degree  $2m-2$  in  $z$  in the following manner—

$$f(z) + \{\phi(z)\}^2 - 2\phi(z) \cdot L(z) \equiv F(z),$$

where

$$\phi(z) \equiv (z-z_1)(z-z_2) \dots (z-z_m) \equiv z^m + p_1 z^{m-1} + \dots + p_m,$$

and 
$$L(z) = z^m + \frac{P_1}{2} z^{m-1} + \lambda_2 z^{m-2} + \lambda_3 z^{m-3} + \dots + \lambda_m,$$

$\lambda_2, \lambda_3, \dots, \lambda_m$  being  $m-1$  arbitrary constants, all the rational and integral algebraic integrals of the Abelian system

$$\Sigma \frac{z^i dz}{\sqrt{f(z)}} = 0$$

z 2

are immediately found by forming the "square-matrix" for  $F(z)$  and so obtaining the conditions that  $F(z)$  should be a perfect square. The various relations so found connecting the quantities  $p_1, p_2, \dots, p_m$ , and  $m-1$  arbitrary quantities  $\lambda_1, \dots, \lambda_m$ , are algebraic integrals of the above system of differential equations, and are all *rational* and *integral*. I then apply the general theorem to the case  $m=2$ , or the case of elliptic integrals, and easily deduce the result given by Cayley in his work entitled an 'Elementary Treatise on Elliptic Functions' (p. 340).

I next apply the theory to the case of  $m=3$ , and deduce two algebraic integrals, and show how the remaining relations may be found, and lastly to the case  $m=4$ .

The next subject treated of is the source of  $F(z)$ , from which we derive a differential equation which I call the *fundamental equation* in the theory of Abelian integrals and functions, as its integral leads us to a form which, when operated on by  $\delta$ , leads us to a new algebraic equation, which again leads to another by a second application of the operator. By this method I obtain a number of interesting results, many of which are now given for the first time, as far as I am aware.

I then define Abelian functions and, by a method of treatment depending on what precedes, show that they are periodic functions and determine their periods.

We have at first sight  $2m-1$  independent periods, and I reduce them to  $2m-2$  by an easy application of the foregoing theory.

The above is a short abstract of what my paper contains, the most important portions of it being (a) the determination of the algebraic integrals in a *rational* and *integral* form; (b) the easy proof of the periodicity of Abelian functions.

I omit from this paper a discussion of the case in which the number of variables exceeds  $m$ , as likely to make my communication too lengthy.

### III. "On the Application of the Kinetic Theory to Dense Gases" By S. H. BURBURY, F.R.S. Received January 12, 1895.

(Abstract.)

#### 1. Start with Clausius' equation

$$\frac{3}{2}pV = T_r + \frac{1}{2}\Sigma\Sigma Rr,$$

in which  $p$  denotes pressure per unit of area,  $V$  volume, and  $T_r$  kinetic energy of relative motion. Also  $R$  is the repulsive force,  $r$  the distance between the centres of two spheres, and the summation includes all pairs.