February 28, 1895.

Sir JOHN EVANS, K.C.B., D.C.L., LL.D., Vice-President and Treasurer, in the Chair.

Professor Alexander Agassiz, who was elected a Foreign Member in 1891, was admitted into the Society.

A List of the Presents received was laid on the table, and thanks ordered for them.

This Meeting having been appointed by Council as a Meeting for Discussion, the following papers were taken as the subject of the discussion :—

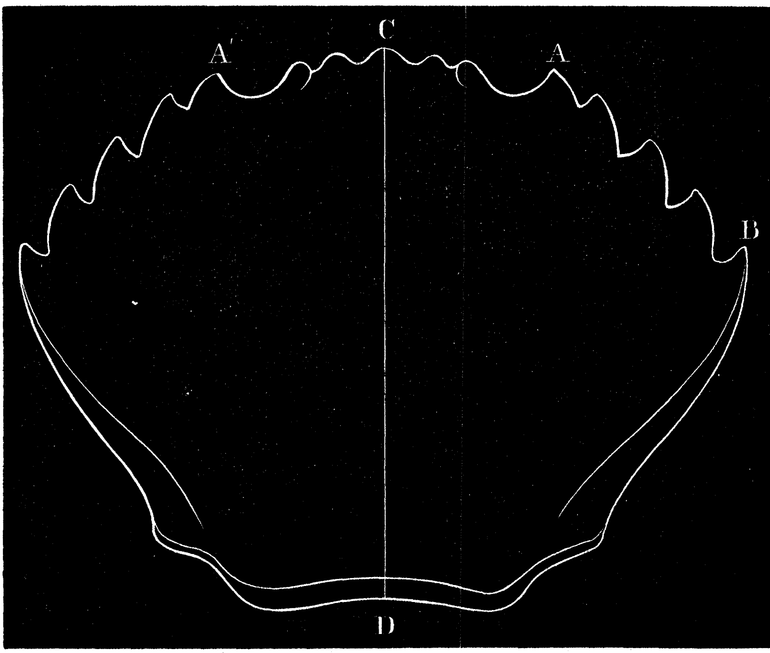
I. Report of the Committee, consisting of Mr. Galton (Chairman), Mr. F. Darwin, Professor Macalister, Professor Meldola, Professor Poulton, and Professor Weldon, "for Conducting Statistical Inquiries into the Measurable Characteristics of Plants and Animals." Part I. "An Attempt to Measure the Death-rate due to the Selective Destruction of *Carcinus Mænas* with respect to a Particular Dimension."—Drawn up for the Committee by Professor WELDON, F.R.S. Received November 20, 1894.

Among the material available for the purposes of the Committee was a sample of *Carcinus mænas*, from Plymouth Sound, including a fairly large number of young females. The distribution of abnormalities in certain dimensions had already been determined for adult females from the same locality ('Roy. Soc. Proc.,' vol. 54, pp. 318—329); and it seemed worth while to compare the frequency of abnormalities in young individuals at various stages of growth with the frequency of the same abnormalities in adult life, so as to determine whether any evidence of selective destruction during growth could be discovered or not.

About 7000 females, varying in length from 7·00 to 13·95 mm., were chosen (at random, except as regards their size), and two dimensions were measured in each. The results were then compared with those of the corresponding measurements, made upon a sample of 1000 adult females from the same locality, which are recorded in the paper just referred to.

The dimensions chosen were:—(1) the "*frontal breadth*"—the distance in a straight line between the tips of the extra-orbital teeth of the carapace (from the point A, fig. 1, to the corresponding point on the opposite side); and (2) the "*right dentary margin*," measured in a straight line from the apex of the first to that of the last lateral tooth (from A to B, fig. 1). The "length" of each crab was taken as the length of the carapace, from the tip of the middle inter-orbital tooth to the posterior margin (from C to D, fig. 1). This is, of course, not the total length of the body; but the curvature and flexibility of the abdomen render an exact determination of the real body length very difficult.

FIG. 1.



In order to compare the variability of a dimension in crabs whose carapace is only 7 mm. long with that of the corresponding dimension in adult crabs, whose carapace length is from 40—50 mm. or more, it is evidently necessary to adopt some method of picturing the crabs as of one standard size; and accordingly the measures obtained have always been expressed in terms of the carapace-length of the crab to which they belong, taken as 1000. The measurements were made by means of a screw, of 1 mm. pitch, carrying the object across

the field of a microscope, and by means of graduations on the head of the screw the observations were recorded to the nearest hundredth of a millimetre. It is believed that the probable error of any observation is not much more than one hundredth of a millimetre. In order to minimise the effect of errors of observation, the results, after being expressed as fractions of the carapace-length, were sorted into groups, such that the measures in each group did not differ by more than 0·004 of the carapace-length, and all measures in the same group were treated as identical. The unit employed in tabulating the results was therefore 0·004 of the carapace-length; but in what follows the results are expressed, for the greater convenience of the reader, in thousandths of the carapace-length. It will be noticed that the principal effect of this alteration upon the results is to diminish their apparent regularity—an aberration of one unit of measurement appearing as four units in the tables below.

1. *Variation in Frontal Breadth.*

An initial difficulty in determining the error of distribution of frontal breadths about their mean in young crabs, arises from the great rapidity with which the mean itself changes during growth. The mean frontal breadth in the smallest specimens was found to be 853·14 thousandths of the carapace-length, while at maturity it is only 604·94 thousandths. The rate at which this change occurs can be gathered from the following table of the crabs measured, and the same result is graphically shown in fig. 2.

From this table it appears that the mean frontal breadth changes at such a rate that when the carapace-length has increased 0·2 mm., the frontal breadth has almost always diminished by less than four thousandths, that is to say, by less than one of the units of measurement here employed. For the purpose of the present investigation the mean was therefore considered stationary during every period of increase in size of not more than 0·2 mm., and the young crabs were accordingly sorted into groups, the individuals of each group differing by less than 0·2 mm. in respect of their carapace-length. The distribution of frontal breadths about the mean was then examined in each group separately.

As the difference in size between the largest and the smallest of the growing crabs was 7 mm., it follows that the material was divided into thirty-five groups. This subdivision of the material had great disadvantages, because, instead of a single group of over 7000 individuals, varying about the same mean, from which a fairly reliable indication of the law governing frequency of deviation might have been expected, the average number of individuals in any one of the available groups was only 200; and from so small a number of obser-

Table I.—Mean Frontal Breadth (F) expressed in thousandths of the Carapace-length, corresponding to various Carapace-lengths (C), together with the Number of Individuals on which each Determination is based.

C.	F.	Number.	C.	F.	Number.
7·1	853·14	159	10·7	798·01	225
7·3	852·43	186	10·9	794·96	162
7·5	850·89	172	11·1	792·14	222
7·7	844·27	142	11·3	789·26	218
7·9	844·22	132	11·5	789·26	230
8·1	837·13	224	11·7	786·07	211
8·3	835·41	219	11·9	784·53	225
8·5	830·08	214	12·1	782·42	224
8·7	826·80	207	12·3	780·92	226
8·9	823·75	214	12·5	778·39	219
9·1	821·26	191	12·7	772·76	183
9·3	818·33	205	12·9	771·62	233
9·5	815·89	214	13·1	770·36	131
9·7	811·60	195	13·3	769·86	162
9·9	809·95	226	13·5	767·70	158
10·1	809·27	245	13·7	762·51	201
10·3	803·21	253	13·9	763·47	211
10·5	800·53	232	(Adult)	(604·94)	(998)

[*Note.*—The carapace-length given in the table is the mean of all lengths included in each group. For example, the entry 7·1 includes all crabs measured in which the carapace-length was between 7·00 mm. and 7·19 mm., and so on.]

vations no satisfactory demonstration of the law of variation at any given moment of growth could be obtained. Nevertheless it was necessary, before proceeding further, to ascertain with some certainty what the law of variation through the whole series really was. The belief in which the work was undertaken was, that the law of variation would be found throughout to be that of the ordinary probability equation ; and this belief was tested in the following way:—In each of the thirty-five groups, the arithmetic mean of the frontal breadths, and the mean of all the deviations from it, were determined; and from the "mean error" found in this way the modulus of the probability function was calculated. Then, by calling the mean of each group zero, and expressing the deviations from the mean in terms of the modulus, a number of curves were obtained, in each of which the modulus was unity and the mean zero; a similar curve of adults was constructed, and the corresponding ordinates of all the thirty-six curves so obtained were added together. It is evident that, if the chance function really expresses the law of variation throughout the series, then the curve resulting from the treatment described will be a symmetrical probability curve of unit modulus. The actual

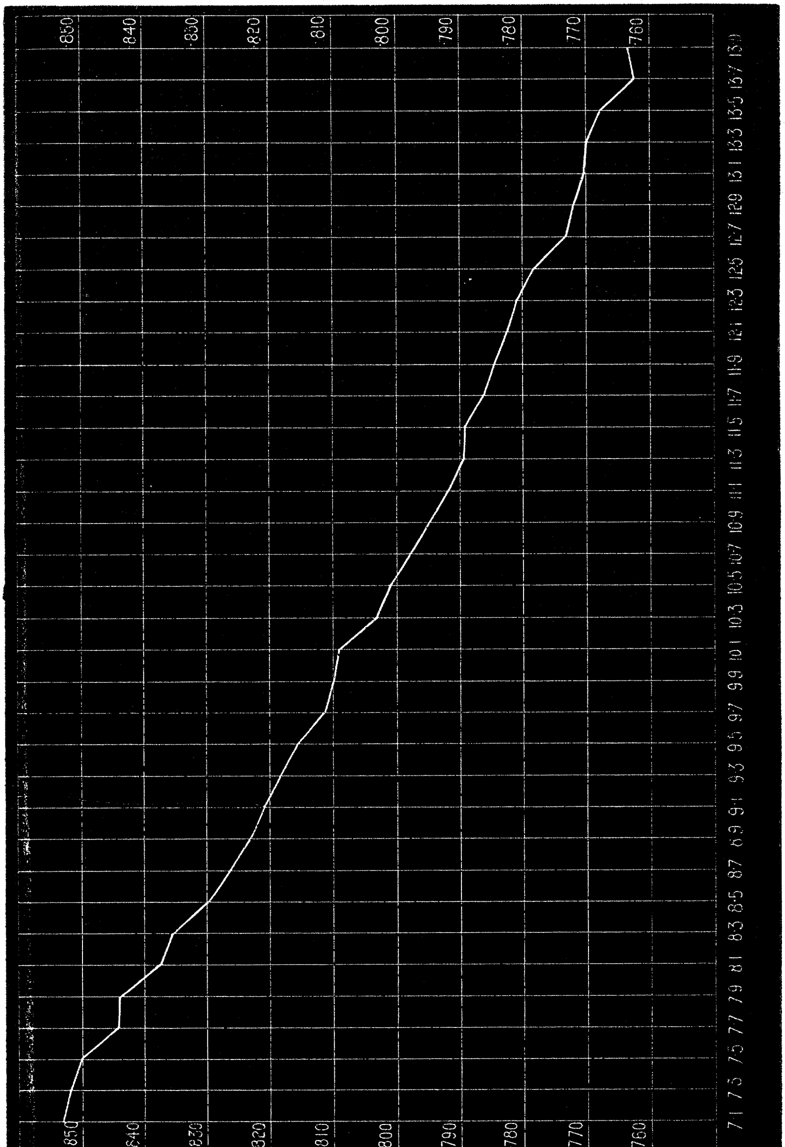


FIG. 2.—Diagram to illustrate the Change in the Mean Value of the Frontal Breadth with Growth in Carapace-length. Ordinates represent fractions of the Carapace-length. Abscissæ represent Carapace-length in millimetres.

curve obtained is plotted in Fig. 3, and the frequency of occurrence of every observed deviation is compared with that indicated by the tables of the probability integral in Table II. In spite of some discrepancies, the general agreement between the observed frequency of

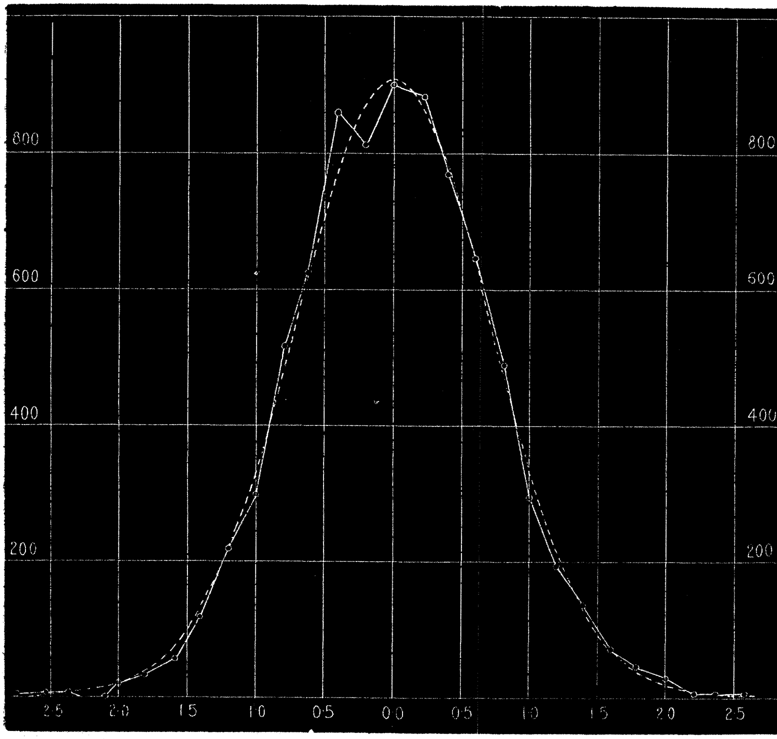


FIG. 3.—Distribution of Frontal Breadths in 8069 Female Crabs from Plymouth Sound, old and young. Deviations expressed in terms of the Modulus. The three cases of deviation greater than three times the Modulus are omitted.

deviations and that indicated by the probability integral is fairly close. The mean error of the observed curve is 0·5621, whereas it should be $1/\sqrt{\pi} = 0{\cdot}5642$, the difference between the two figures being less than 0·5 per cent. The error of mean square is 0·7123, instead of 0·7071, a difference of less than 1 per cent. The sum of the squares of the positive deviations is 2115. The sum of the negative deviations is 1992. The total number of individuals of deviation more than 0·1 is 3593 on the positive, 3574 on the negative side, a difference of about one-half per cent.

On the whole it may be said that the result agrees with that given by the theory of probability as well as could be expected from the number of observations, and that the law of frequency of variation throughout the series may, as was hoped, be assumed to agree with the ordinary law of chance.

From the result so far obtained it followed that a determination of the quartile deviation, or any other of the constants of the pro-

Table II.—Frequency of all Observed Deviations from the Mean Frontal Breadth in 8069 Female Crabs, young and adult, from Plymouth. The Deviations expressed in terms of the Modulus.

Limits of deviations.	Mean deviation.	Observed frequency.	Theoretical frequency.
Over +3·29	+4·790	2	
From +3·10 to 3·29	+3·280	1	
" +2·90 " 3·09	—	0	
" +2·70 " 2·88	+2·880	1	12
" +2·50 " 2·69	+2·575	4	
" +2·30 " 2·49	+2·355	4	
" +2·10 " 2·29	+2·207	4	
" +1·90 " 2·09	+2·003	28	17
" +1·70 " 1·89	+1·788	46	36
" +1·50 " 1·69	+1·598	71	71
" +1·30 " 1·49	+1·395	137	129
" +1·10 " 1·29	+1·205	194	217
" +0·90 " 1·09	+1·006	295	336
" +0·70 " 0·89	+0·805	492	481
" +0·50 " 0·69	+0·589	649	635
" +0·30 " 0·49	+0·391	769	774
" +0·10 " 0·29	+0·191	896	872
" +0·09 " −0·09	−0·011	902	907
" −0·10 " 0·29	−0·213	814	872
" −0·30 " 0·49	−0·404	862	774
" −0·50 " 0·69	−0·611	626	635
" −0·70 " 0·89	−0·806	517	481
" −0·90 " 1·09	−1·000	299	336
" −1·10 " 1·29	−1·194	219	217
" −1·30 " 1·49	−1·403	118	129
" −1·50 " 1·69	−1·589	58	71
" −1·70 " 1·89	−1·808	33	36
" −1·90 " 2·09	−2·016	18	17
" −2·10 " 2·29	−2·210	1	
" −2·30 " 2·49	−2·368	4	
" −2·50 " 2·69	−2·520	3	12
" −2·70 " 2·89	−2·750	1	
—	—	—	
Over	−4·450	1	

bability equation, would be a trustworthy guide to the frequency of abnormalities at various periods of growth. But just as the individual groups were too small to allow of a determination of the law of abnormality in each, so they were too small to give trustworthy values of the quartile. The quartile deviation changes so slowly with growth, that it may without serious error be assumed to be constant during the period represented by 1 mm. of growth in carapace-length: that is, through the period covered by five of the groups into which the growing crabs were sorted. Therefore, after the quartile deviation had been determined in every group, the results were arranged in fives, and the mean of every consecutive five was

taken as the quartile deviation through 1 mm. of growth. The results are shown in Table III.

Table III.—Quartile Deviation of Frontal Breadths (Q) for various Magnitudes of Carapace-length (C).

C.	Mean Q.
7·5	9·42
8·5	9·83*
9·5	9·51
10·5	9·58
11·5	10·25
12·5	10·79
13·5	10·09
(Adult)	(9·96)

The values here given are probably not very reliable, but they show that in the youngest individuals the quartile deviation is distinctly less than at maturity; that it increases with increase of size, until a time arrives when it is distinctly greater than in adult life; and that finally it diminishes again.

The initial features of this result,—the smallness of the quartile error at a young age, indicating relative infrequency of deviations, and the increase during growth, have been observed by Bowditch in the case of human stature. The result obtained by Dr. Bowditch and that here described are both simply confirmations of Darwin's statement, that many variations appear at a late period of development.

The initial increase in the quartile error may be attributed to the fact that average young produce upon the whole average adults, while animals which exhibit a deviation of known amount in the young state, exhibit on the whole a greater deviation with advancing age. If this view be the true one (and it is hoped that next year it may be possible to test it by observation of living crabs, which can be measured at various periods of growth), then, in a Plymouth crab, which is of unit deviation when its carapace is 7 mm. long, the most probable deviation when it has grown to be 12·5 mm. in length will be 10·79/9·42 = about 1·15 units. The probable error of this expectation is the expression of irregularities in the rate of growth, which cannot at present, for want of knowledge, be adequately discussed.

From the age represented by a length of 12·5 mm., the quartile

* Of the four very abnormal values shown in the table, three occurred in this group. They have been omitted in the determination of the quartile deviation, which would otherwise become 9·92.

error diminishes, and the parallel between the behaviour of the frontal breadth in Plymouth crabs and that seen by Bowditch in the stature of civilised human beings ceases to hold. The obvious suggestion by which to account for this seems to be that in the United States, where Bowditch made his observations, human beings are under conditions of such civilisation that there is considerable protection of the physically unfit; and that here, as in other civilised countries, any influences which might in a savage race produce selective destruction are reduced to a minimum, whereas in the case of the crabs such selective influences are active.

It is, of course, possible that the deviation of "abnormal" young may in each individual case first attain a maximum and then diminish with advancing age; if this is the case, we cannot know without experiment. In the absence of such experiment, the hypothesis may be provisionally adopted that the diminution in the frequency of individuals of given deviation is due to a selective destruction, and the consequences of this hypothesis will be examined.

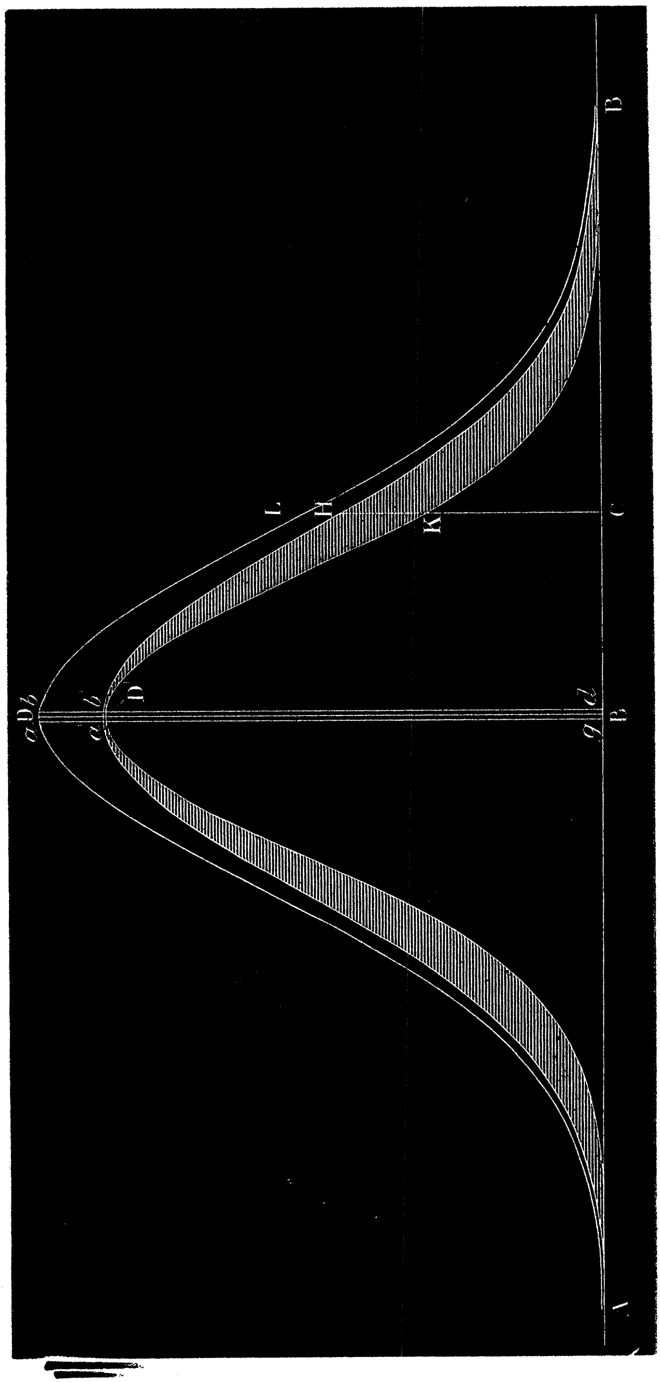
Consider a population of crabs, measured at the time of their maximum variability, and suppose the distribution of deviations among the population to be accurately represented, for a particular organ, by a probability equation of modulus c_1. Now, let the number of individuals of deviation lying between $\pm a$ be represented by the area *abgd* (fig. 4); then, if $gd = 2a$ be small, compared with the observed range of variation, and $k_1 = \frac{\text{area } abgd}{2a}$, in other words, if k_1 be the height of the median ordinate BD of the generalised curve, then the whole number of individuals in the population will be $k_1 c_1 \sqrt{\pi}$.

Now, suppose any destruction, which acts unselectively with regard to the organ in question, to reduce the number of individuals whose deviation lies between $\pm a$, to *cdef*, and let the $\frac{\text{area } cdef}{2a} = k_2$, or the height of the median ordinate BD^1. Since this destruction is unselective, it will destroy an equal percentage of animals of every deviation, and will therefore not alter the modulus. The population will therefore be reduced to $k_2 c_1 \sqrt{\pi}$ in number. This unselective destruction cannot be directly measured.

The selective destruction is most simply conceived as follows:—

In fig. 4 let AD^1HE represent a curve of modulus c_1, and let $BD^1 = k_2$, so that the area of the whole curve $AD^1HE = k_2 c_1 \sqrt{\pi}$ represents the population left after unselective destruction has occurred. Then suppose the modulus to be reduced during growth to c_2, where c_2 is less than c_1, and let AD^1KE be a curve of modulus c_2. The minimum number of individuals which it is necessary to destroy, in order to affect this reduction in the modulus, is evidently represented by the

FIG. 4.



shaded area of the figure. The population after such destruction is $k_2 c_2 \sqrt{\pi}$ in number, and the shaded area represents a number of individuals equal to $k_2 \sqrt{\pi}(c_1 - c_2)$, so that the ratio of animals selectively destroyed to animals which survive all unselective destruction is $\frac{c_1 - c_2}{c_1}$, a quantity which can be experimentally determined.

From the data given in Table III, this ratio, for frontal breadths of Plymouth crabs, becomes $\frac{10{\cdot}79 - 9{\cdot}96}{10{\cdot}79}$ or about 0·077, so that the hypothesis of selective destruction involves a death-rate of about 77 per thousand between the age corresponding to 12·5 mm. in carapace-length and maturity, as a consequence of deviation in frontal breadths, and in the group of structures, whatever these may be, which are directly correlated with it.

This total death-rate does not affect individuals of all deviations alike; an inspection of the figure will show that the death-rate is a function of the deviation, and that function is quite simply determined. Consider any ordinate Eg of the curve ABEC, and let its abscissa, DG, be of magnitude x; then the length of Eg is $k_2 e^{-x^2/c_1^2}$ and the number of individuals of deviation between x and $x + dx$ is $k_2 e^{-x^2/c^2}.\, dx$. In the same way, the height of the ordinate Fg is

$$k_2 e^{-x^2/c_2^2},$$

and the number of individuals of abnormality within unit distance of x after selection is

$$k_2 e^{-x^2/c_2^2} \,.\, dx.$$

The ratio between the number of animals of abnormality x which survive the unselective destruction and those which are selectively destroyed is therefore

$$\frac{\mathrm{E}g - \mathrm{EF}}{\mathrm{E}g} = \frac{e^{-x^2/c_1^2} - e^{-x^2/c_2^2}}{e^{-x^2/c_1^2}} = 1 - e^{x^2(c_2^2 - c_1^2)/c_1^2 c_2^2}$$

So that if g is the selective death-rate among animals of abnormality x, then that death-rate increases as x increases according to the equation

$$g = 1 - e^{-hx^2},$$

where h is the numerical value of $c_1^2 - c_2^2/c_1^2 c_2^2$, c_1 and c_2 being the values of the modulus at the time of its maximum value and at maturity respectively.

For the frontal breadth of Plymouth crabs, the value of h is about 0·015; so that of the whole number of animals which attain the size 12·5 mm., having an abnormality x of their frontal breadth, the

fraction destroyed as a consequence of this abnormality before reaching maturity is

$$1-e^{-0 \cdot 015x^2}.$$

It will, of course, be understood that little trust can be placed in the absolute numerical results which are here put forward; the point which seems worthy of confidence, and which if it be indeed a reality is of very great importance, is the form of the result. For by purely statistical methods, without making any assumption as to the functional importance of the frontal breadth, the time of life at which natural selection must be assumed to act, if it acts at all, has been determined, and the selective death-rate has been exhibited as a function of the abnormality, while a numerical estimate which is at least of the same order as the amount of the selective destruction has been obtained.

The method by which the result described has been arrived at is likely to be capable of application to a very considerable number of cases. Mathematically considered, the conditions which were at first assumed and then proved to obtain in the organ discussed are by no means general. It is necessary for the employment of this method that the variations should be distributed on each side of the mean with sensible symmetry, and that the position of minimum selective destruction should be sensibly coincident with the mean of the whole system. Such statistical information as is at present available leads to the belief that these conditions may be expected to hold for a large number of species, which are sensibly in equilibrium with their present surroundings, so that their mean character is sensibly the best, and the change of mean from generation to generation is at least very small. They cannot be expected to hold in cases of rapid change such as those induced artificially by selection under domestication, or naturally by rapid migration or other phenomena resulting in a rapid change of environment.

For the investigation of such rapid change, it would be necessary to treat the more general case, in which the chances of deviations of opposite sign are not sensibly symmetrical, and in which the mean is not necessarily the position of minimum destruction. The treatment of this case requires the help of a professional mathematician.

It will be well to mention here a curious indirect confirmation of the result just described, based on evidence derived from a quite different source.

An attempt has been made to show that physiological accidents of a kind leading to change in the length of a portion of the carapace affect a crab at a rate measured by the value of the quantity $1-e^{-hx^2}$. The symmetrical distribution of variations from the mean which has been shown, especially by Mr. Galton, to occur in dimensions of weight, length, muscular strength, and other characters of various

organs in men, moths, sweet peas, and other things at various periods of life, made it seem probable that if selective destruction could be shown to occur in these cases, the expression for intensity of destruction, in terms of the deviation, would in all these cases be of the same form as that already arrived at. That is to say, the expression for the effect of physiological accidents of a number of different kinds, affecting a number of organisms in no way specially alike, is probably always of the same form. The question at once occurred, whether this expression might not be of general application, as a measure of the effect of physiological accidents upon the animal body.

The most convenient case in which to look for an answer to this question is the case of muscular tissue, in which the effect of accidents of stimulus can be readily measured. The recent paper of Cyboulski and Zanietowski (Pflüger's 'Archiv f. Physiologie,' Bd. 56, p. 45) gives an excellent series of data for determining the relation between energy of stimulus applied to a nerve, and effect upon the muscle, as measured by energy of external work performed in contraction. These observers give a large series of tables, in which the energy of stimulus, applied by discharging a condenser of known electrical capacity through a nerve, is given in one column, and in another is the work done by the muscle stimulated, measured by the height through which a known mass is lifted.

As is well known, the application of stimuli of less than a certain magnitude produces no muscular contraction; but if the maximum stimulus which can be applied without causing a contraction be reckoned as zero, the subsequent relation between stimulus and contraction does, in fact, agree very closely with that indicated by successive values of the quantity $1-e^{-hx^2}$.

In spite of the evident care and skill with which Cyboulski and Zanietowski have performed their experiments, their curves are slightly irregular. In order to minimise the effect of these slight irregularities, three of their results were treated in the following way:—In each system of observations the maximum subliminal stimulus was subtracted from the magnitude of the applied stimulus in each case; the three numbers representing the height of the muscle contraction for unit stimulus beyond this point in the three cases were added together; and so on throughout. The result is plotted in fig. 5, the height of the sum of three contractions being indicated by the ordinates of the points $\odot$; the intensity of the corresponding stimulus, *minus* the subliminal stimulus, being measured along the abscissa.

The dotted curve is given by

$$g = 1-e^{-x^2/(8\cdot 15)_2},$$

8·15 being the "modulus" of the system of ordinates, determined from their moments about the axis of g. The coincidence between the two, rough as it is, is surely more than accidental!

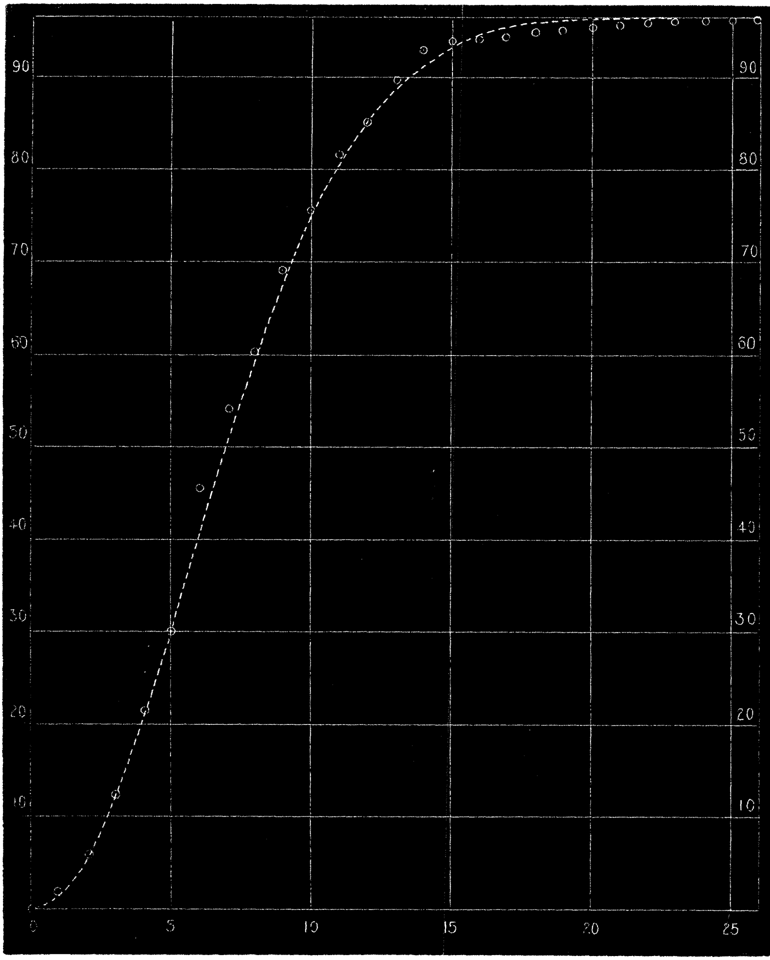


FIG. 5.—Sum of three Muscle-curves from Cyboulski and Zanietowski's paper. Each ordinate represents the sum of three muscle-contractions in millimetres: abscissæ represent stimulus applied to the nerve, expressed in ergs $\times 10^{-7}$, and reckoned from the close of the subliminal period.

The most interesting relation to be investigated in the light of this result would undoubtedly be the relation between sensation and stimulus in man; but existing data seem too imperfect to give any

trustworthy result. It may be remarked that a few years ago measurements of the relation between muscle contraction and nerve stimulus, made with an imperfection comparable with that which characterises attempts to measure sensation, were held to obey a logarithmic law closely similar to the formula of Fechner and Weber.

II. *Variation in the Right Dentary Margin.*

The mean size of the right dentary margin was found to change, with increase of carapace-length, at such a rate as to render necessary the same subdivision of the material as that adopted in the case of frontal breadths. The change of mean will be gathered from Table IV, where it is seen that the change is slightly slower and less regular than in the frontal breadths, while its direction is reversed—the right dentary margin becoming larger, the frontal breadth smaller with increase of size.

Table IV.—Mean Length of Right Dentary Margin (D) expressed in thousandths of the Carapace-length (C) corresponding to various observed Carapace-lengths.

C.	D.	C.	D.
7·1	380·30	10·7	416·00
7·3	379·73	10·9	419·01
7·5	383·06	11·1	419·13
7·7	385·89	11·3	421·26
7·9	388·76	11·5	423·34
8·1	390·60	11·7	423·21
8·3	391·97	11·9	425·49
8·5	396·70	12·1	424·67
8·7	396·58	12·3	426·25
8·9	397·04	12·5	428·55
9·1	400·32	12·9	429·35
9·3	403·71	12·9	429·54
9·5	404·50	13·1	432·17
9·7	408·63	13·3	434·87
9·9	409·66	13·5	429·16
10·1	411·66	13·7	435·18
10·3	412·79	13·9	436·87
10·5	413·81	(Adult)	(495·14)

These observations were treated in the same way as those of frontal breadths; and the result of expressing the deviations from the mean in terms of the modulus of every group, and then summing deviations of corresponding magnitude, is shown in Table V, and graphically in fig. 6.

Table V.—Frequency of all observed Deviations from the Mean Length of the Right Dentary Margin in 8020 Female Crabs, Young and Adult, from Plymouth. Deviations in terms of the Modulus.

Limits of deviations.	Mean deviation.	Observed frequency.	Theoretical frequency.
	+5·530	1	
Over +3·20	+3·760	1	
	+3·340	1	
From +3·00 to 3·19	+3·120	1	2·76
„ +2·80 „ 2·99	+2·945	2	
„ +2·60 „ 2·79	+2·677	3	
„ +2·40 „ 2·59	+2·493	3	
„ +2·20 „ 2·39	+2·253	12	4·71
„ +2·00 „ 2·19	+2·050	9	11·29
„ +1·80 „ 1·99	+1·888	16	24·99
„ +1·60 „ 1·79	+1·705	45	51·14
„ +1·40 „ 1·59	+1·492	132	96·49
„ +1·20 „ 1·39	+1·276	155	168·30
„ +1·00 „ 1·19	+1·108	238	271·13
„ +0·80 „ 0·99	+0·897	375	403·41
„ +0·60 „ 0·79	+0·700	587	554·36
„ +0·40 „ 0·59	+0·493	735	703·61
„ +0·20 „ 0·39	+0·305	775	824·82
„ +0·00 „ 0·19	+0·112	939	893·04
„ −0·00 „ 0·19	−0·088	871	893·04
„ −0·20 „ 0·39	−0·270	833	824·82
„ −0·40 „ 0·59	−0·495	698	703·61
„ −0·60 „ 0·79	−0·739	553	554·36
„ −0·80 „ 0·99	−0·896	446	403·41
„ −1·00 „ 1·19	−1·093	240	271·13
„ −1·20 „ 1·39	−1·279	162	168·30
„ −1·40 „ 1·59	−1·479	89	96·49
„ −1·60 „ 1·79	−1·691	45	51·14
„ −1·80 „ 1·99	−1·932	18	24·99
„ −2·00 „ 2·19	−2·108	15	11·29
„ −2·20 „ 2·39	−2·295	6	4·71
„ −2·90 „ 2·59	−2·487	3	
„ −2·60 „ 2·79	−2·710	2	
„ −2·80 „ 2·99	−2·903	3	
	−3·330	2	2·76
	−4·180	1	
Over 3·00 ..	−4·830	1	
	−5·960	1	
	−7·030	1	

The symmetry of these results is fairly good, the number of positively abnormal individuals being 4030, the number of negatively abnormal 3990. The sum of the squares of the negative deviations is 2145·5; the sum of the squares of the positive deviations being 2099·9—a difference of about 2 per cent. This difference is greater than in the case of the frontal breadths; but a reference to the table will show that the removal of a single individual, namely, the indi-

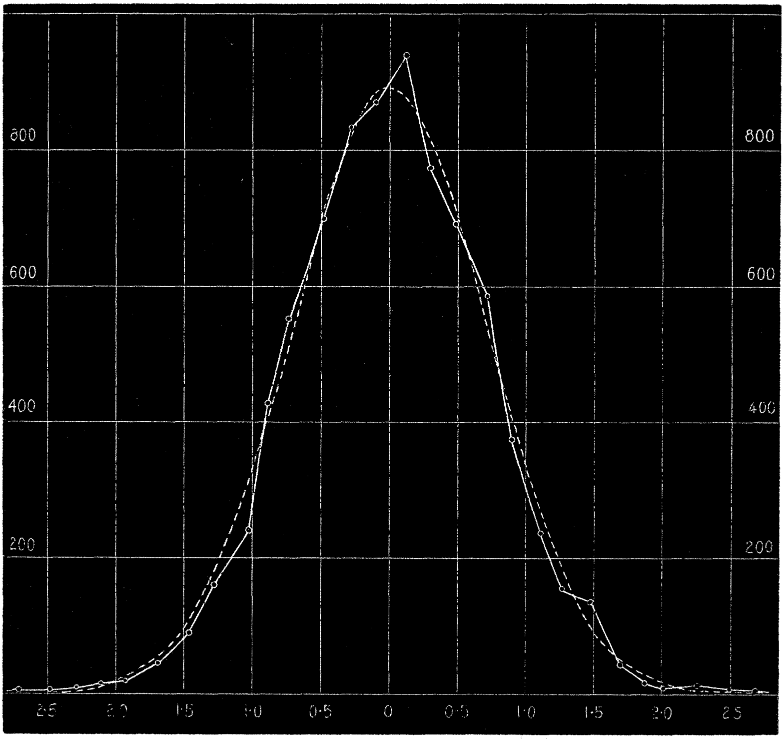


FIG. 6.—Deviations of 8020 measures of Right Dentary Margin in Female Crabs, old and young, from Plymouth Sound, expressed in terms of the Modulus. Eleven individuals of deviation greater than three times the Modulus are omitted.

vidual of deviation equal to −7 times the modulus, would make the sum of the positive and negative squares almost exactly equal.

The mean error of the whole system is 0·5688 instead of 0·5642, or nearly 1 per cent. too great. The error of mean squares is 0·7276 instead of 0·7077, or 2·8 per cent. too great.

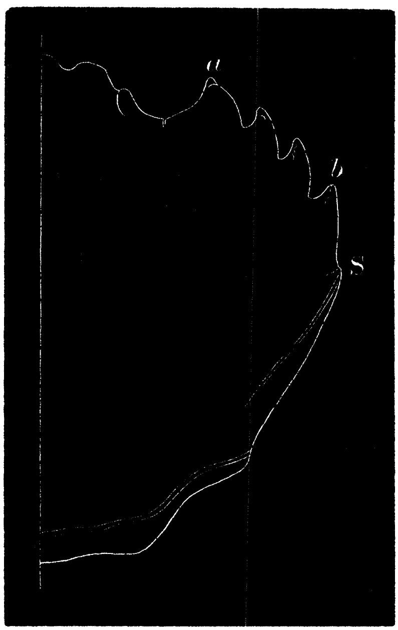
From these values of the mean error and error of mean square, as well as from the presence of a deviation so great as seven times the modulus, it is evident that some cause has been at work, producing large abnormalities with a frequency greater than that indicated by the theory of chance. Reference to the table shows that deviations of more that 2·5 times the modulus do in fact occur twenty times, instead of five or six times, as they should do So that deviations of this magnitude occur about three and a half times too often.

The sporadic occurrence of considerable deviations, which do not obey the general law of frequency of variation, is a phenomenon

which has been supposed by many naturalists to be of great importance in evolution, and the present case is therefore worthy of discussion.

The following suggestion is offered as an explanation of the large *negative* deviations. As shown in Fig. 1, there are normally five teeth in the dentary margin; but occasionally (in over 1 per cent. of individuals) only four teeth occur. The reduction in teeth may apparently be effected in various ways: sometimes it is impossible to say that one tooth rather than another is missing; and the case then resembles those cases of variation in the segmentation of a vertebral column, for example, recently discussed by Bateson ("Materials for the Study of Variation," *passim*, especially, however, p. 124). In other cases, the reduction appears to be effected by a process resembling the filling up of the interval between two teeth, so that the points of the teeth project only very slightly. A careful outline of a specimen exhibiting this condition is given in fig. 7. It is evident that

FIG. 7.



in this case the little tubercle S which indicates the position of the fifth tooth is the point from which the measurement should be taken; but if the obliteration of the fifth tooth had progressed but a little further, no indication of its presence would remain; and the dentary

margin, measured from the tip of the first to the tip of the last visible tooth, would really have been measured from *a* to *b*. Such a measure would not be homologous with the rest, and ought not to be included in the scheme. But, since reduction in the number of teeth may take place in other ways, and since it is impossible in a given case to distinguish the manner in which it has occurred, the measurements were necessarily recorded.

Another frequent cause of disturbance is the breakage of the last tooth, followed by its regeneration. All cases of obviously recent injury were of course excluded; and for this reason the total number of individuals discussed is reduced from 8069 to 8020. But the selection of material was felt to be so dangerous a proceeding that all cases in which there was any doubt as to the occurrence of an injury were included. The wrongful inclusion of a dozen such cases would account for the excess of positive abnormalities: for it is evident that a breakage of the tip of the last tooth would increase the distance AB in fig. 1.

While, therefore, the observations admit of the interpretation that about once in 500 cases a "sport" of magnitude greater than that provided for by the theory of chance does regularly occur, the considerations which have been submitted make this interpretation at least doubtful.

The value of the probable error, as an indication of percentage abnormality, is diminished by the existence of the large deviations discussed; but the values obtained are of considerable interest: they are as follows:—

Table VI.—Mean Value of Quartile Deviation (Q) of Right Dentary Margin for various Lengths of Carapace (C).

C.	Mean Q.
7·5	8·44
8·5	8·08
9·5	9·36
10·5	8·23
11·5	8·16
12·5	8·05
13·5	8·68
(Adult)	(9·28)

It will be seen from this table that the error of distribution at the ages measured is always less than in adult life, except among crabs, whose carapace length is between 9 and 10 mm. Of the fourteen superfluous deviations of great magnitude, three occur in this group, and the result is a quite untrustworthy determination.

Evidently, therefore, in spite of the abnormally great frequency with which large deviations occur, the whole percentage of abnormalities, among crabs between 7 and 14 mm. in length, is less than it is in adult crabs; and there is a rough agreement between the result obtained from these measurements and that obtained by Bowditch from the measurements of human stature already referred to. So that among female crabs in Plymouth Sound, during the period of life to which these observations refer, there is no indication of any destructive agency which acts selectively upon the dentary margin. Whether such selective destruction occurs among males, or among females at a later period of life, is for the present an open question.

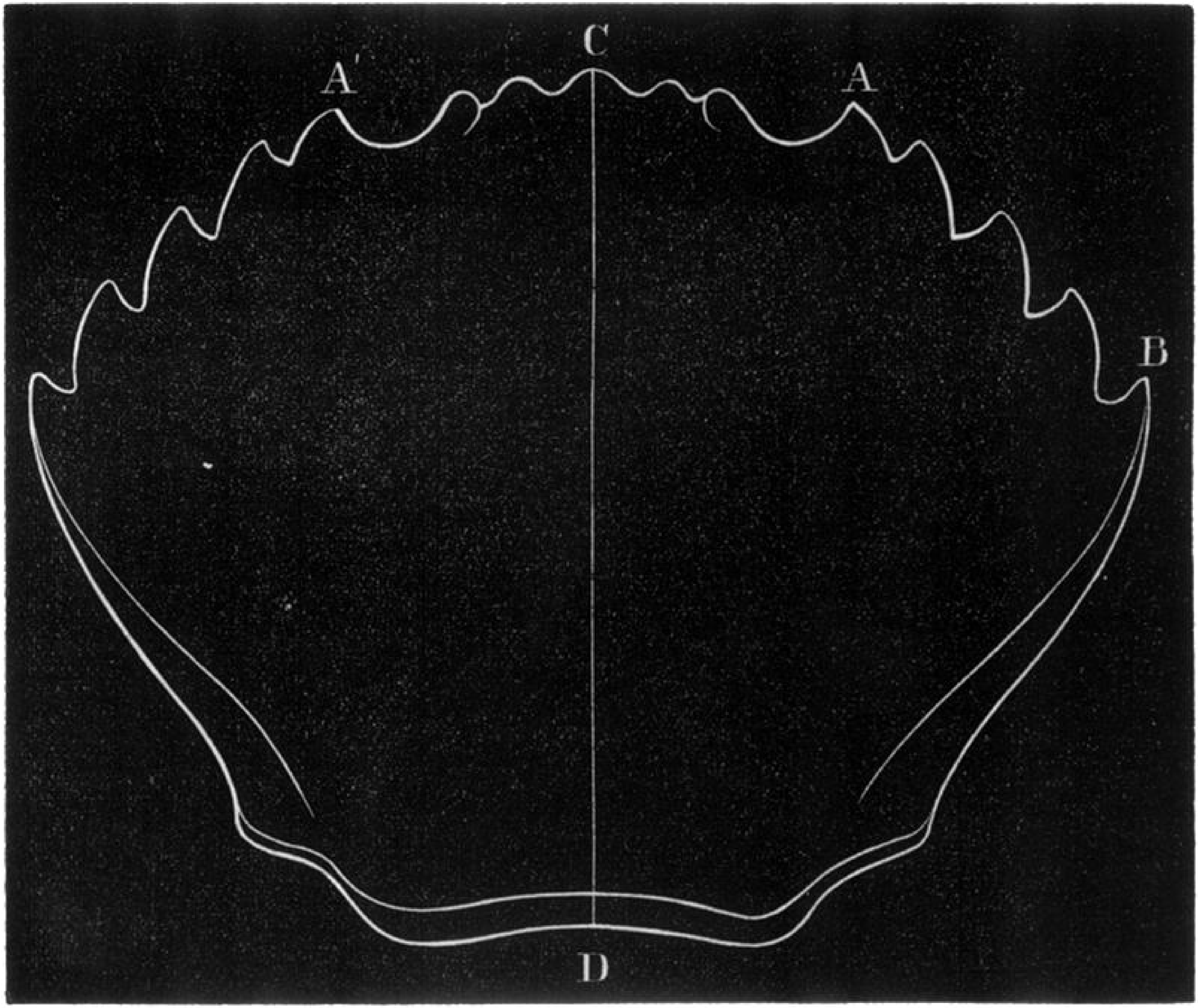
Variation in frontal breadth may, therefore, for the present be considered to be of more importance in the economy of female crabs than variation in the length of the dentary margin—a view which receives confirmation from the dimorphism already shown to exist ('Roy. Soc. Proc.,' vol. 54, p. 324) in the frontal breadth of crabs from Naples, while it is a striking justification of the accepted system of classification, in which the characters of the great groups into which the Brachyura are divided are almost entirely those associated with changes in this dimension.

In conclusion, an important feature of the method employed may be pointed out. The increase of death-rate, associated with a given abnormality of frontal breadth, has here been roughly determined; in the previous paper, already referred to, the effect of abnormality in this dimension upon several other organs of the body was determined; and by the method of that paper it would be possible to determine the effect of parental abnormality upon the offspring. These are all the data which are necessary, in order to determine the direction and rate of evolution; and they may be obtained without introducing any theory of the physiological function of the organs investigated. The advantage of eliminating from the problem of evolution ideas which must often, from the nature of the case, rest chiefly upon guess-work, need hardly be insisted upon.

II. "Remarks on Variation in Animals and Plants. To accompany the first Report of the Committee for conducting Statistical Inquiries into the Measurable Characteristics of Plants and Animals." By Professor W. F. R. WELDON, F.R.S. Received February 19, 1895.

1. The importance of variation as a factor in organic evolution is not seriously disputed; but, if one may judge from the expressions contained in recent essays, naturalists are not agreed as to the

Fig. 1.



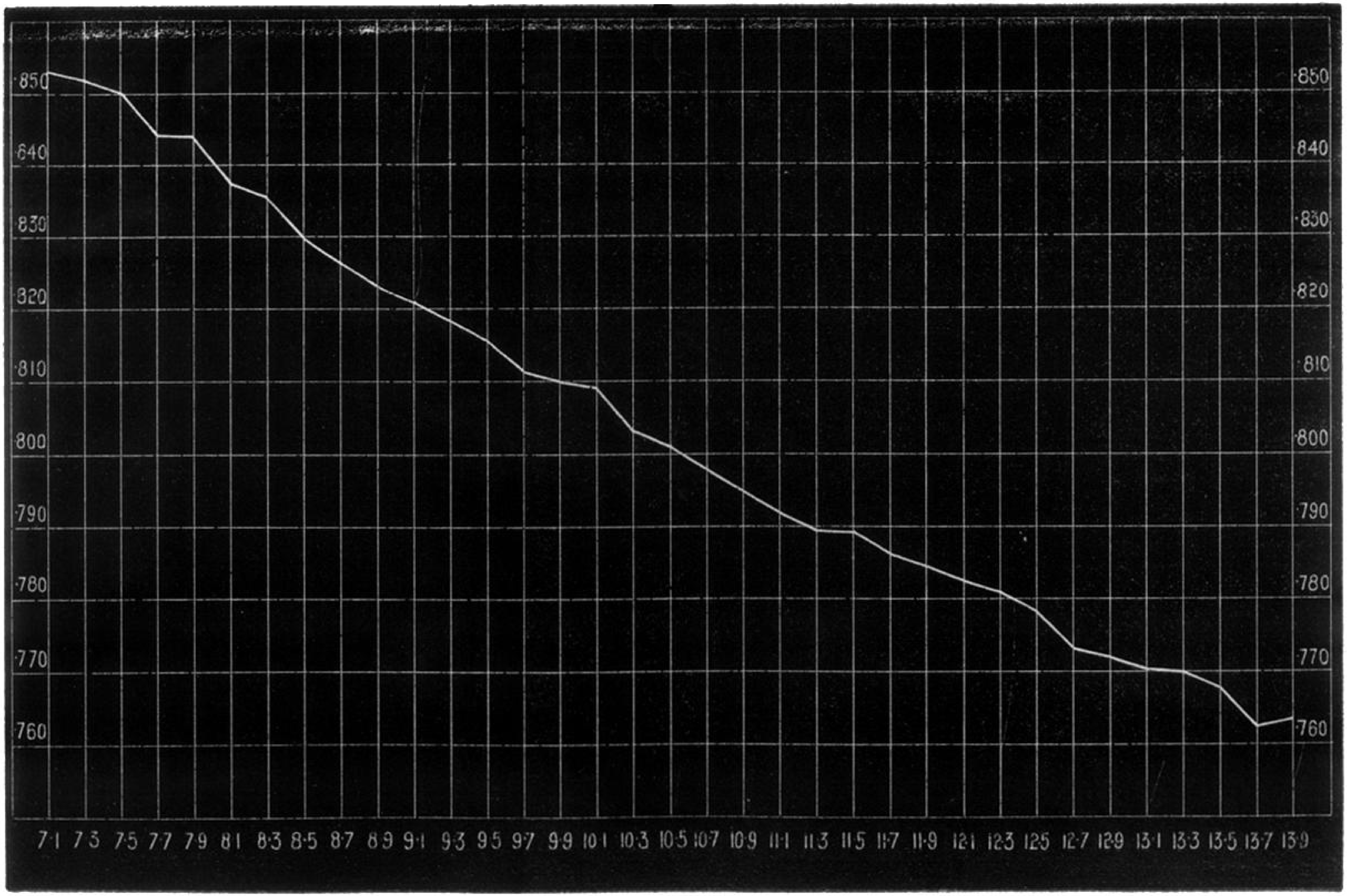


Fig. 2.—Diagram to illustrate the Change in the Mean Value of the Frontal Breadth with Growth in Carapace-length. Ordinates represent fractions of the Carapace-length. Abscissæ represent Carapace-length in millimetres.

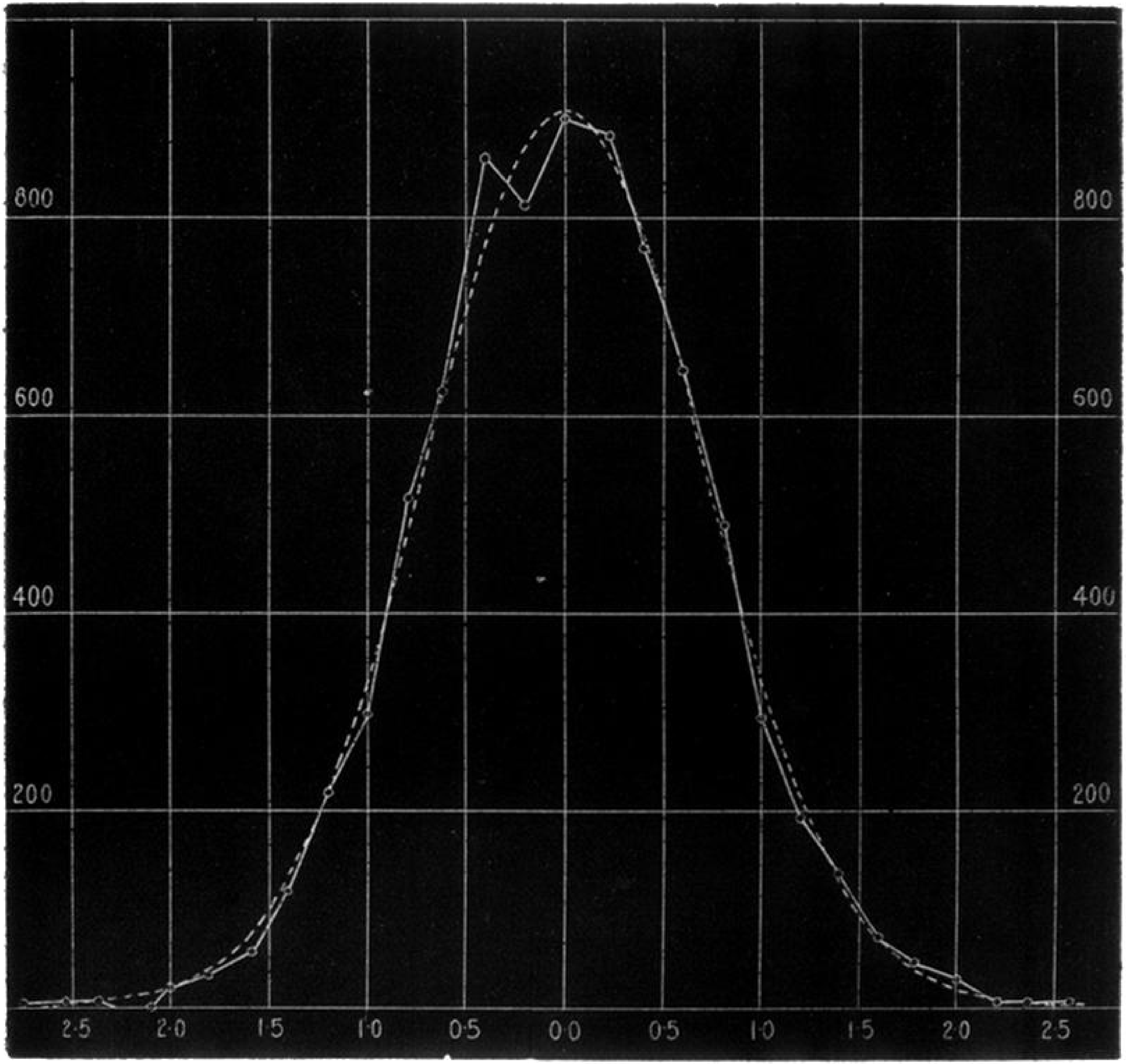
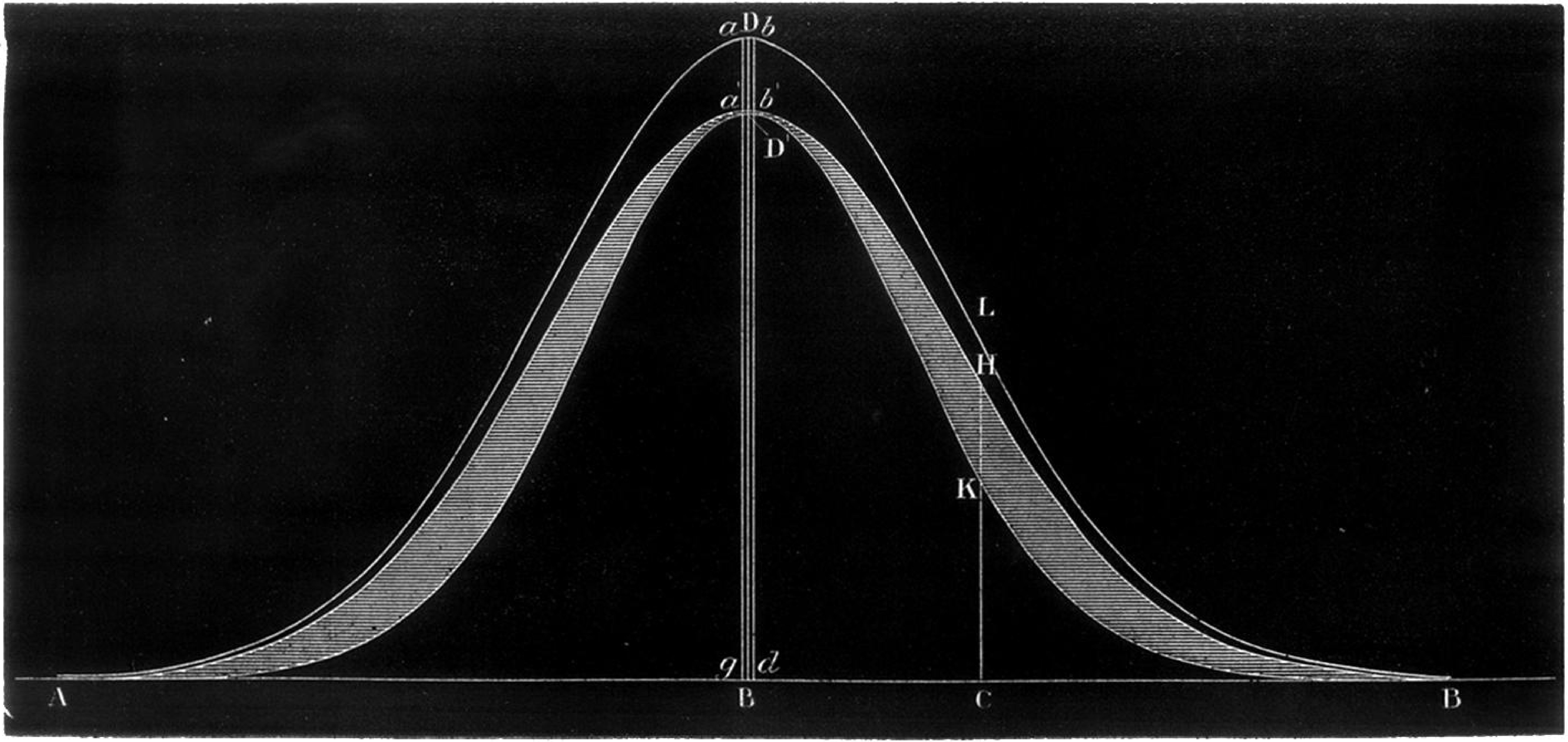


Fig. 3.—Distribution of Frontal Breadths in 8069 Female Crabs from Plymouth Sound, old and young. Deviations expressed in terms of the Modulus. The three cases of deviation greater than three times the Modulus are omitted.

FIG. 4.



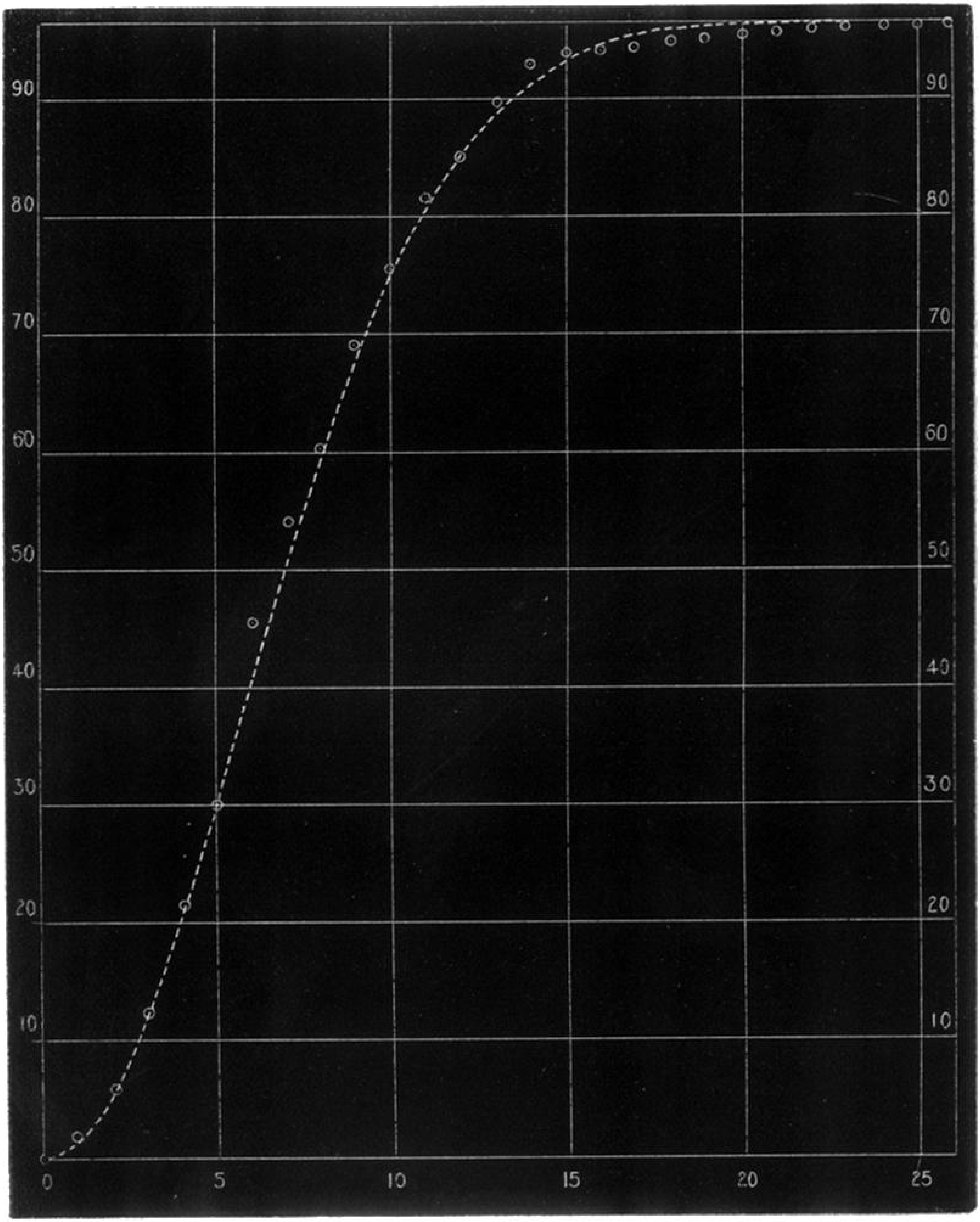


FIG. 5.—Sum of three Muscle-curves from Cyboulski and Zanietowski's paper.

Each ordinate represents the sum of three muscle-contractions in millimetres: abscissæ represent stimulus applied to the nerve, expressed in ergs $\times 10^{-7}$, and reckoned from the close of the subliminal period.

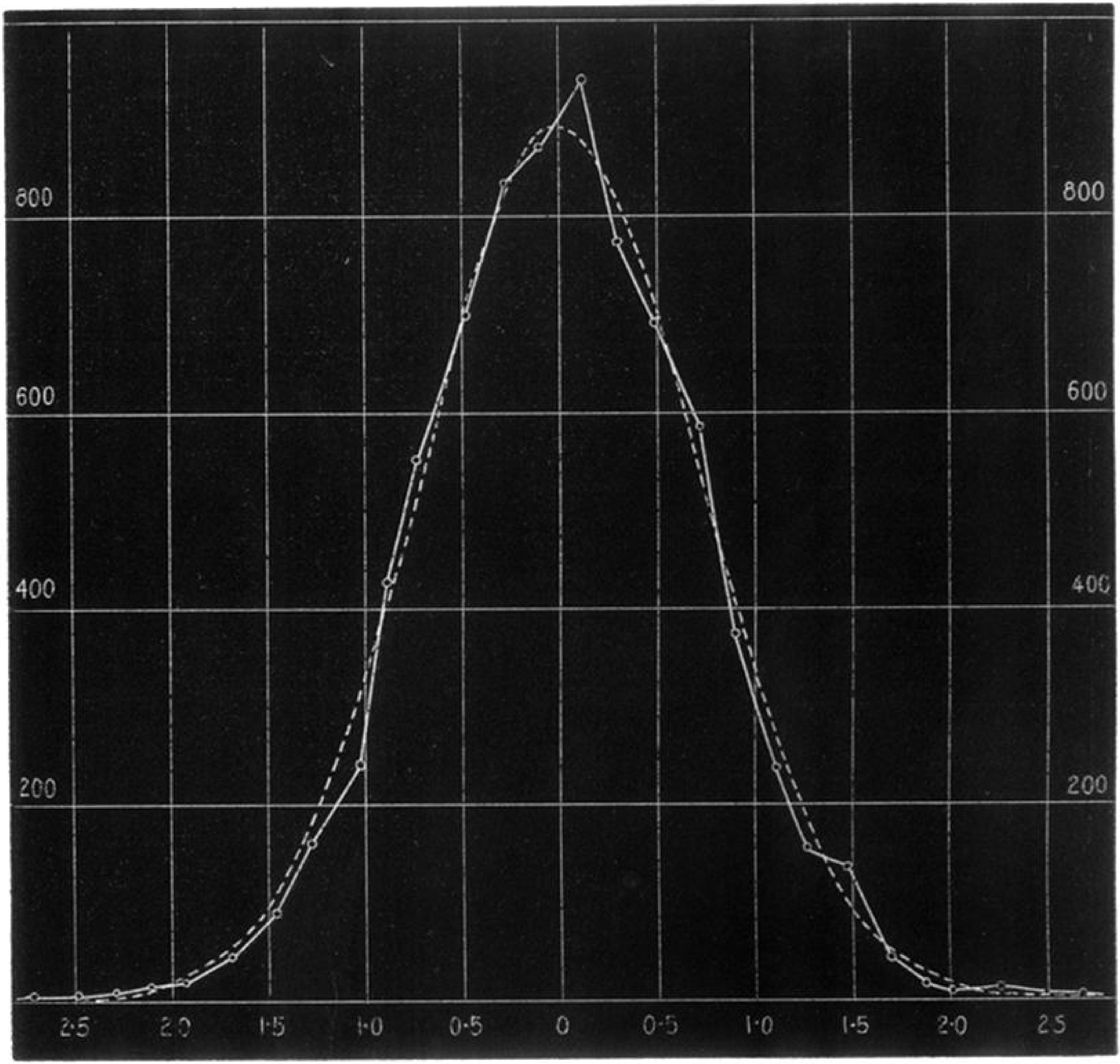


FIG. 6.—Deviations of 8020 measures of Right Dentary Margin in Female Crabs, old and young, from Plymouth Sound, expressed in terms of the Modulus. Eleven individuals of deviation greater than three times the Modulus are omitted.

FIG. 7.