giving rise to red dendritic patterns. The author considers these differences to be analogous to the differences observed in the experiments of Oliver Lodge upon the photo-electric loss of charge first observed by Hertz.

VII. "Theorems on the Attraction of Ellipsoids for certain Laws of Force other than the Inverse Square." By E. J. Routh, F.R.S. Received May 11, 1895.

(Abstract.)

The object of the author is to find finite expressions for the potentials of an ellipsoidal shell, and of a solid ellipsoid when the law of force is the inverse κ^{th} power of the distance, κ being positive or negative. It is shown in the beginning of the paper that the two cases in which κ is an even integer and an odd integer require different treatment.

After discussing some special cases, we come to the first general theorem. Supposing that κ is even and that the shell is a thin homogeneous homœoid, the potential is found to assume very different forms according as κ is greater or less than 3, so that the law of the inverse square is just on one side of the boundary. When $\kappa > 3$, the potential can be completely integrated, and an expression is found containing $\frac{1}{2}(\kappa-2)$ terms, and involving only the differentiation of an integral rational function of xyz of $\kappa-4$ dimensions. The general form at an internal point is

$$V = \frac{2\pi\mu}{(\kappa-1)(\kappa-3)}\left(\frac{2}{E}\right)^{\kappa-3}\left\{1+\frac{1}{2^2}\frac{E\Delta}{\kappa-4}+\frac{1}{2^4}\frac{E^2\Delta^2}{1.2(\kappa-4)(\kappa-5)}+\ldots\right\}P,$$

where
$$P = (\alpha^2x^2+\beta^2y^2+\gamma^2z^2)^{\frac{1}{2}(\kappa-4)}$$
$$E = 1-\alpha x^2-\beta y^2-\gamma z^2$$
$$\Delta = \frac{1}{\alpha}\frac{d^2}{dx^2}+\frac{1}{\beta}\frac{d^2}{dy^2}+\frac{1}{\gamma}\frac{d^2}{dz^2}.$$

When κ is <3 the potential takes the form of a single integral

$$V = \frac{2\pi\mu}{\kappa-1}\int_0^\infty u^{-\frac{1}{2}}du\left(u^2\frac{d}{du}\right)^t\frac{abcu^{\frac{3}{2}}}{Q\lfloor t}\left(1-\frac{x^2}{a^2+u}-\frac{y^2}{b^2+u}-\frac{z^2}{c^2+u}\right)^t,$$

where $t = \frac{1}{2}(2-\kappa)$. This reduces to the ordinary well-known form when $t = 0$, *i.e.*, when the law is the inverse square.

Proceeding next to a thin heterogeneous homœoid, the density being $\phi(\xi\eta z)$ where ϕ is a function of i dimensions, different cases

are found to arise according as κ is greater or less than 3 and i greater or less than $\kappa-2$. If $\kappa>3$ and $i<\kappa-2$ the potential can be completely integrated. A finite expression is given containing not more than $\kappa-3$ terms, and involving differential coefficients of an integral rational function of xyz. For an internal point the general form is

$$V_1 = \frac{2\pi\mu}{\kappa-1}\frac{1}{L(\kappa-3)}\Sigma\frac{L(h)}{L(\kappa-4-h)}\left(\frac{2}{E}\right)^{h+1}(\Delta'+\tfrac{1}{2}\Delta)^{\kappa-4-h}(\alpha^2x^2+\beta^2y^2+\gamma^2z^2)^{\frac{\kappa-4}{2}}\phi,$$

where Δ and Δ' are two differential operators, and Σ implies summation from $h=0$ to $\kappa-4$. The potential is also found for an external point. If $i>\kappa-2$ or $\kappa<3$ the potential contains a single integral which reduces to a known form when we can put $\kappa=2$. There are two standard expressions, one of which is of the form

$$V_2 = \frac{2\pi\mu}{\kappa-1}\frac{\underline{|t}}{\underline{|2t}}\int_0^\infty \frac{abc\,du}{Q}M\psi\left(\frac{ax}{a^2+u},\frac{by}{b^2+u},\frac{cz}{c^2+u}\right),$$

where $$M = \frac{D^t}{\underline{|t}}+\frac{uRD^{t+1}}{\underline{|t+1}\,2^2}+\ldots,\ t=\frac{\kappa-2}{2}.$$

D is a differential operator and R a quadratic function of xyz, which are explained in the paper.

Examples are given throughout to illustrate the mode in which these general formulæ are to be used, and full references to all other writings on the same subject as far as they are known to the author.

Passing on to a solid ellipsoid the potentials at an internal point for both a homogeneous and a heterogeneous ellipsoid are discussed both when $\kappa>$ and <3 and finite expressions are found, which reduce to known forms when $\kappa=2$. When the point is external and the strata are similar ellipsoids with any law of density, expressions for the potential are found for the cases $\kappa=4$, 6, 8, and 10. These are reduced to depend on a single integral. Except when $\kappa=4$, these can be completely integrated when the solid is homogeneous and in some other cases.

When κ is an odd integer, there is a division of cases according as κ is $<$ or >2. In the first of these cases, finite expressions for the potential of a thin homœoid are found (1) when homogeneous, and (2) when heterogeneous. There are corresponding expressions for a solid ellipsoid. As explained in the text, these results differ in form rather than in substance from some already known, but they are treated in a different manner.

In the second case, when $\kappa>2$, the integrations become very long. Finite expressions for the potential of a homogeneous homœoid are, however, found (1) when the force varies as the inverse cube, and (2)

when the force varies as the inverse κ^{th} power, and $\kappa > 3$, the formulæ for the inverse cube being much simpler than for any greater power.

Lastly, the potential of a special elliptic disc is discussed which has the property that the level surfaces are confocals, the force varying as any odd inverse power greater than the square.

In this list only those properties have been mentioned which the author believes to be new.

Presents, June 13, 1895.

Transactions.

Berkeley:—University of California. University of California Studies. Vol. I. No. 2. 8vo. *Berkeley* 1894; Bulletin of the Department of Geology. Vol. I. Nos. 5—9. 8vo. *Berkeley* 1894–95; Report of Work of the Agricultural Experiment Stations. 1892–94. 8vo. *Sacramento* 1894; A Brief Account of the Lick Observatory, by E. S. Holden. 8vo. *Sacramento* 1895; [and 7 other University Publications]. 8vo.
The University.

Berlin:—Gesellschaft für Erdkunde. Zeitschrift. Bd. XXX. No. 2. 8vo. *Berlin* 1895. The Society.

K. Preuss. Akademie der Wissenschaften. Sitzungsberichte. 1895. Nos. 1—25. 8vo. *Berlin.* The Academy.

Bremen:—Naturwissenschaftlicher Verein. Abhandlungen. Bd. XIII. Heft 2. Bd. XV. Heft 1. 8vo. *Bremen* 1895.
The Society.

Cambridge, Mass.:—Museum of Comparative Zoölogy. Bulletin. Vol. XXV. No. 12. 8vo. *Cambridge, Mass.* 1895.
The Museum.

Cracow:—Académie des Sciences. Bulletin International. Avril, 1895. 8vo. *Cracovie.* The Academy.

Edinburgh:—Royal Society of Edinburgh. Proceedings. Vol. XX. Pp. 305—384. 8vo. *Edinburgh* [1894–95]. The Society.

Leipsic:—Königl. Sächsische Gesellschaft der Wissenschaften. Abhandlungen (Math.-phys. Classe). Bd. XXII. No. 1. 8vo. *Leipzig* 1895; Berichte über die Verhandlungen (Math.-phys. Classe). 1895. No. 1. 8vo. *Leipzig.* The Society.

London:—Anthropological Institute. Journal. Vol. XXIV. No. 4. 8vo. *London* 1895. The Society.

British Astronomical Association. Journal. Vol. V. No. 7. 8vo. *London* 1895; Memoirs. Vol. II. Part 6. 8vo. *London* 1895. The Association.

Entomological Society. Proceedings. 1895. Part 2. 8vo. *London.* The Society.