General Conclusions.

The final result of the discussion of the spectra of stars of the δ Cephei class is to show that they must be placed on the ascending arm of the temperature curve, at a stage higher than stars like α Tauri, in which the mean temperature is not very different from that of the sun. Stars of equal temperature on the descending side of the curve, of which Castor may be taken as a type, show precisely the same lines, the enhanced and cool lines having the same relative intensities, but with inverted intensities of the hydrogen and metallic lines, and with somewhat less continuous absorption in the ultra-violet. The difference between stars like δ Cephei and those of the sun is therefore partly due to a difference of temperature and partly due to a difference of physical condition such as is demanded by the meteoritic hypothesis. This result enables us to understand why some members of the δ Cephei class should show such a very special kind of variability.

α Cygni also finds a natural place on the ascending arm of the temperature curve, at a stage higher than δ Cephei, and all the difficulties met with in attempting to classify it on Vogel's view of decreasing temperature alone are removed.

"On Lunar and Solar Periodicities of Earthquakes." By ARTHUR SCHUSTER, F.R.S. Received May 18,—Read June 17, 1897.

1. In a paper recently communicated to the Royal Society "On Lunar Periodicity in Earthquake Frequency," Mr. C. G. Knott gave some results, from which he argued that a real connexion between tidal effects and earthquakes probably existed. These results are based on a method which has frequently been employed. The records of earthquakes are grouped together and expressed by means of a Fourier series, and conclusions are based on the greater or smaller values of the coefficients of this series. In order to decide what value is to be attached to such investigations, it seems necessary in the first instance to discuss what would be the order of magnitude of the coefficients, on the supposition that the events have happened perfectly at random without any connecting law. It is the object of this paper to solve this question, and to apply the solution to the periodicities which are supposed to exist in the frequency of earthquakes.

2. If it is required to investigate a possible period of p intervals of time in a series of members, t_1, t_2, t_3, &c., it is usual to arrange the numbers according to the following scheme, where t_1' stands for t_{p+1}, t_1'' for t_{2p+1}, &c.:—

$$\begin{array}{cccc} t_1 & t_2 & t_3 \dots\dots & t_p \\ t_1' & t_2' & t_3' \dots\dots & t_p' \\ \vdots & \vdots & \vdots & \\ t_1^{s-1} & t_2^{s-1} & \dots\dots & t_p^{s-1} \\ \hline \mathrm{T}_1 & \mathrm{T}_2 & \mathrm{T}_3 \dots\dots & \mathrm{T}_p \end{array}$$

T_1, T_2, &c., represent the sums of the vertical columns. In the case discussed in the first part of Mr. Knott's paper the intervals are hours, and p is taken to be 25, *i.e.*, approximately equal to the interval between two successive meridian passages of the moon. T_1 would therefore represent the number of earthquakes which have happened within an hour after the meridian passage of the moon.

The numbers T may be expressed by a periodic series of the form

$$\begin{aligned} \mathrm{S} = a_0 + a_1 \cos\theta + a_2 \cos 2\theta + \dots\dots + a_p \cos p\theta \\ + b_1 \sin\theta + b_2 \sin 2\theta + \dots\dots + b_{p-1} \sin(p+1)\theta \quad \dots\dots \quad (1), \end{aligned}$$

where $S = T_1$ if we substitute $\theta = 2\pi/p$, and generally S becomes T_q by the substitution $\theta = 2\pi q/p$.

The coefficients are determined by a well-known process, which gives

$$\left.\begin{aligned} pa_0 &= \mathrm{T}_1 + \mathrm{T}_2 + \dots\dots\dots\dots\dots + \mathrm{T}_p \\ \tfrac{1}{2}pa_1 &= \mathrm{T}_1 \cos\theta + \mathrm{T}_2 \cos 2\theta + \dots\dots + \mathrm{T}_p \cos p\theta \\ \tfrac{1}{2}pb_1 &= \mathrm{T}_1 \sin\theta + \mathrm{T}_2 \sin 2\theta + \dots\dots + \mathrm{T}_p \sin p\theta \end{aligned}\right\} \theta = 2\pi/p \quad \dots\dots \quad (2).$$

The amplitude of the first periodic term would be $r_1 = \sqrt{a_1^2 + b_1^2}$.

It is seen that a_0 is equal to the mean value of all the quantities T or to s times the mean values of all the quantities t.

From the above equations it follows that—

$$\frac{r_1^2}{4a_0^2} = \frac{(\mathrm{T}_1 \cos\theta + \mathrm{T}_2 \cos 2\theta + \dots + \mathrm{T}_p \cos p\theta)^2 + (\mathrm{T}_1 \sin\theta + \mathrm{T}_2 \sin 2\theta + \dots + \mathrm{T}_p \sin p\theta)^2}{(\mathrm{T}_1 + \mathrm{T}_2 + \dots \mathrm{T}_p)^2} \quad \dots\dots \quad (3).$$

We may take the quantity $\rho = r_1/a_0$ as a measure of the periodicity corresponding to p intervals. Our problem now is this: "*What is the probability that ρ should lie between any two assigned values ρ_1 and ρ_2, on the supposition that the events are all distributed at random.*" The problem may be put into a more general form. The events, like the earthquakes in Mr. Knott's paper, have all been put into the same compartment if they happened within certain interval of time (an hour in this case), no matter whether they happened at the beginning or at the end of that interval. This simplification is introduced only for purposes of more easy arithmetical calculation. Theoretically,

and if we had a sufficient quantity of material, it would be better to reduce the lengths of the intervals and to increase their number p. If this process is carried sufficiently far equations (2) become

$$pa_0 = n,$$
$$\tfrac{1}{2}pa_1 = \cos\kappa t_1 + \cos\kappa t_2 + \ldots + \cos\kappa t_n,$$
$$\tfrac{1}{2}pb_1 = \sin\kappa t_1 + \sin\kappa t_2 + \ldots + \sin\kappa t_n \quad \ldots\ldots\ldots \quad (4),$$

where n is the total number of events and κ stands for $2\pi/\mathrm{T}$, T being the whole length of the period, and t_1, t_2, &c., the times of occurrence of successive events.

Equation (3) will become

$$\frac{n}{2}\frac{r_1}{a_0} = \{(\cos\kappa t_1 + \cos\kappa t_2 + \ldots \cos\kappa t_n)^2 + (\sin\kappa t_1 + \sin\kappa t_2 + \ldots + \sin\kappa t_n)^2\}^{\frac{1}{2}} \quad \ldots\ldots \quad (5).$$

The meaning of the expression on the right-hand side is best illustrated by means of a diagram. On a circle with centre at O and unit radius, take points P_1, P_2 such that the angles between the lines OP_1, OP_2 and a fixed direction are kt_1, kt_2, &c. If OP_1, OP_2 represent forces of equal intensity but different directions, the right-hand side of (5) gives the magnitude of the resultant force. As, according to hypothesis, the events may happen with equal probability at any time, every position on the circle is equally probable for every point P. Under these circumstances it has been shown by Lord Rayleigh* in a paper "On the Resultant of a large number of Vibrations of the Same Pitch and Arbitrary Phase," that the probability of the resultant having a value lying between s and $s+ds$ is

$$\frac{2}{n}e^{-s^2/n}\,sds \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6),$$

n being the total number of vectors combined. It is a simple matter to pass from this result to the solution of our problem.

From (5) and (6) it follows that the probability for the value of $nr_1/2a_0$ lying between $\frac{1}{2}n\rho$ and $\frac{1}{2}n(\rho+d\rho)$ is

$$\frac{n}{2}\rho e^{-\frac{1}{4}n\rho^2}\,d\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7),$$

and this is therefore also the probability that r_1/a_0 has a value intermediate between ρ and $\rho+d\rho$.

The expectancy for r_1/a_0 is

$$\frac{n}{2}\int_0^\infty \rho^2 e^{-\frac{1}{4}n\rho^2}\,d\rho = \sqrt{\frac{\pi}{n}} = \frac{1{\cdot}77}{\sqrt{n}} \quad \ldots\ldots\ldots\ldots \quad (8).$$

* 'Phil. Mag.,' vol. 10, p. 73 (1880, II).

The probability that the value of r_1/a_0 exceeds ρ is

$$\frac{n}{2}\int_{\rho}^{\infty} \rho \; e^{-\frac{1}{4}n\rho^2}\, d\rho = e^{-\frac{1}{4}n\rho^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (9).$$

If the higher coefficients $r_2 = \sqrt{a_2^2+b_2^2}$, &c., are treated in the same manner, the same expression is found to hold.

Our final result may now be expressed as follows:—

If a number n of disconnected events occur within an interval of time T, all times being equally probable for each event, and if the frequency of occurrence of these events is expressed in a series of the form

$$a\left(1+\rho_1 \cos 2\pi \frac{t-t_1}{T} + \rho_2 \cos 4\pi \frac{t-t_2}{T} + \ldots + \rho_n \cos 2p\pi \frac{t-t_n}{T}\right),$$

the probability that any of the quantities ρ has a value lying between ρ and $\rho+d\rho$ is

$$\frac{n}{2}\rho e^{-n\rho^2/4} d\rho,$$

and the expectancy for ρ is

$$\sqrt{\pi/n}.$$

In proving this proposition it was assumed that the number of intervals into which the period T is subdivided is very large, but this condition is not essential. To suit accurately the process employed by Mr. Knott, we should have to consider the vectors OP_1, OP_2, &c., to be confined to fixed directions forming angles $2\pi/p$ with each other. But it follows directly from the method employed by Lord Rayleigh that his results must apply to this case also, if p is a multiple of four. It is further not necessary to enquire whether (7) holds in the most general case, when p, for instance, is an odd number, because the process employed by Mr. Knott and others is justified only on the assumption that the number of intervals into which the period is subdivided is so large that a further increase of it would not alter the relative value of the coefficients of the Fourier series.

3. Unfortunately Mr. Knott has adopted a common but in my opinion mischievous practice, which renders some further reductions necessary before we can apply expressions (7) and (8). Instead of basing his calculations on the number of earthquakes which took place in any particular lunar hour, he first takes overlapping means of the numbers put down for five consecutive hours. This practice has its legitimate use, when it is desired to make periodicities apparent to the eye by plotting down a series of numbers which by themselves may be too irregular to bring out the peculiarities which it is intended to show. The averaging of a

certain number of successive figures eliminates to some extent the shorter periods, and therefore emphasises the longer ones. But this elimination is done in a much better and more complete manner by Fourier's analysis, and when it is therefore intended to submit the figures to that process of calculation, the smoothing down of the original numbers, represents a waste of arithmetical work, uncompensated, as far as I can see, by any advantage.

To judge of Mr. Knott's results it is necessary therefore to trace the effect of the process of calculation employed by him. The problem to be solved for that purpose may be stated thus :—

A number, p, of figures, $y_1, y_2 \ldots y_p$, is expressed in terms of the periodic series (1). A second series is then formed by taking the sum of m successive numbers, the first two members of the series being $y_1+y_2+\ldots y_m$ and $y_2+y_3+\ldots y_{m+1}$. The original series is supposed to repeat itself, so that the last member of the derived series becomes $y_p+y_1+y_2+\ldots+y_{m-1}$. It is required to find the relation between the coefficients of the Fourier expansion for the two series.

It will be sufficient to find the solution of this problem for the case that the original series y_1, y_2 represent equidistant ordinates of the curve $y = \cos \kappa x$, so that

$$y_1 = \cos \kappa x_1, \quad y_2 = \cos(\kappa x_1+\delta), \quad y_3 = \cos(\kappa x_1+2\delta) \ldots$$

$$y_m = \cos(\kappa x_1+(m-1)\delta).$$

The solution of this case really includes the general one, for the original series y is supposed to be represented by a number of terms, each of which is simply periodic, and to each of which the result of the special case may be separately applied.

The first term of the derived series may be obtained by a well known process, for

$$y' = y_1+y_2+\ldots y_m = \frac{\sin \frac{1}{2} m\delta}{\sin \frac{1}{2}\delta} \cos\left(\kappa x_1+\frac{m-1}{2}\delta\right).$$

The subsequent terms are obtained simply by altering the value of x_1.

The derived series, y', y'', &c., represents therefore the equidistant ordinates of the curve

$$y = \frac{\sin \frac{1}{2} m\delta}{\sin \frac{1}{2}\delta} \cos\left(\kappa x+\frac{(m+1)\delta}{2}\right).$$

It is a curve having the same period as the original one, but having an amplitude reduced in the ratio $\sin \frac{1}{2} m\delta / \sin \frac{1}{2}\delta$. Applying this result to the separate terms of the series (1) and remembering that δ represents the difference in phase between two successive

terms of the original series, it is seen that the coefficients a_q and b_q are both reduced in the ratio $\sin \frac{\pi m q}{p} \Big/ \sin \frac{\pi q}{p}$. This result has to be divided by m to fit it to the case that the derived series is not formed by taking the *sum* of m successive numbers, but by taking their *mean* value.

The effect of smoothing down the irregularities of the observed numbers has therefore the effect of reducing the qth coefficients of the Fourier series in the ratio

$$\sin \frac{\pi m q}{p} \Big/ m \sin \frac{\pi q}{p}.$$

In the case under discussion, $m = 5, p = 25$, so that the amplitudes of the first four coefficients of the series are reduced respectively in the ratios

$$0{\cdot}938, \qquad 0{\cdot}765, \qquad 0{\cdot}517, \qquad 0{\cdot}244.$$

The number of earthquakes taken into account was 7427, so that the expectancy for the amplitude of any one of the coefficients, irrespective of the smoothing process, would be

$$\sqrt{\pi/7427} = 0{\cdot}0206.$$

If this number is multiplied by the above fractions, and then by 1000 in order to make the units agree with those of Mr. Knott's paper, we obtain the numbers placed in the first row of Table I; the second row gives by comparison the coefficients actually found by Mr. Knott.

Table I.

Coefficients.	C_1.	C_2.	C_3.	C_4.
Expectancy for the coefficients by the theory of probability	19·3	15·7	10·6	5·02
Coefficients found by Mr. Knott........	10·3	17·9	10·9	3·97

The numbers in this table do not support Mr. Knott's contention, but seem to me rather to be a striking confirmation of the theory of probability. It must be remembered that the "expectancy" only gives the average value of a great many cases, the individuals of which may differ considerably from that average. Thus it may be calculated with the help of expression (9) that in about one case out of every four the coefficient C_1 would come out still smaller than the number found by Mr. Knott, while the coefficient C_2 would be larger

than Mr. Knott's number in two out of every five cases. It will, I think, be admitted that, until further evidence is brought forward, the lunar day cannot be considered to affect earthquakes.

4. In the second part of his paper, Mr. Knott treats of the monthly and fortnightly periodicities. He expresses himself very guardedly as to the results of this part of the investigation, chiefly on the ground that the amplitudes found for the period of the "nodical" month are quite as great as those found for the "tropical" or "synodic" months. As there is no conceivable reason why the nodical month should affect earthquakes, the conclusion will reasonably be drawn from this, that any period chosen at random would give similar results. I think we must take it, in default of further evidence, that this argument is valid and outweighs the not very conclusive considerations which seem to Mr. Knott to favour the reality of these lunar periods, in spite of the weakness of the evidence to which he himself draws attention. The number of earthquakes taken into consideration are about 4730, and this would give an expectancy for each of the coefficients equal to 0·0258. A reducing factor has to be applied which differs for the different coefficients as has been explained in the case of the periods depending on lunar day. I obtain in this way the numbers shown in Table II. For the sake of completeness, I give the amplitudes found by Mr. Knott for the various months, and also the mean amplitudes of the different monthly periods.

Table II.

"Month."	C_1.	C_2.	C_3.	C_4.
(1) Anomalistic	46·2	47·8	12·9	16·5
(2) Tropical	54·7	40·7	23·1	17·2
(3) Nodical	49·5	55·2	28·3	17·6
(4) Synodic	11·0	52·1	24·5	4·7
Mean of (1), (2), (3), (4)	40·4	49·0	22·2	14·0
Expectancy by theory of probability	24·5	20·9	15·5	9·3

The striking feature of this table consists in the first place in the roughly equal amplitudes for the different kinds of months, and secondly in the fact that the expectancy is in all cases smaller than the mean of the coefficients found, and in nearly equal proportions at any rate for the monthly, third-monthly, and quarter-monthly periods.

5. This points not so much to the reality of the period found, but to some common cause which has led to too low an evaluation of the expectancy. The latter quantity has been calculated under the

assumption that the earthquakes are altogether independent of each other and take place at random. Any regularity would be contrary to this assumption and might affect the expectancy. Thus, for instance, it is known that earthquakes take place in groups, a large earthquake being generally followed by some minor shocks.

Mr. Knott states that the "obvious" aftershocks were left out of account in his calculation; but there may be aftershocks which are not obvious, and it seems quite likely that every earthquake is followed by a period during which another is more likely to happen than at other times. If there is such a tendency it is easy to see that our calculated numbers for the expectancy will be too low. Take the case, for instance, that all earthquakes happen in groups of two, or, what comes to the same thing, let each earthquake in the investigation of § 2 count as two. The quantities I have called ρ will not be affected by this change, but the total number of earthquakes being doubled, the expectancy calculated according to our formula is now reduced in the proportion of $\sqrt{2}$ to 1, and would therefore be too small in that ratio. It seems to me to be probable that the discrepancy between Mr. Knott's coefficients and the calculated expectancy is explained in this way. Apart from this possible explanation it would not be safe to draw any certain conclusions from any instance in which the calculated amplitude has double the value of the expectancy, for expression (9) shows that the amplitudes will turn out to be even greater than that on the average in one case out of every twenty-three. The matter to be explained is not that any one of Mr. Knott's coefficients is, roughly speaking, twice as great as the expectancy, but that all coefficients show this tendency towards higher values in not very different proportions.

6. It is interesting to discuss, from the point of view of this paper, the periodicities of earthquakes which apparently depend, directly or indirectly, on the position of the sun. We owe to Mr. Davison a very complete discussion of the annual period.* The method employed by him (in determining the amplitudes of the periodic terms), though neither direct nor very accurate, is sufficient for our purpose. Taking as an example the record of 5879 earthquakes in the northern hemisphere, given by Mr. R. Mallet, the results of this paper show that if they were distributed indiscriminately over the whole year the expectancy for the amplitude would be $\sqrt{\pi/5879}$, or 0·023, while Mr. Davison, in § 18 of the paper quoted, gives 0·11 for the amplitude. Similarly, the discussion of 8133 earthquakes, for which the expectancy is 0·020, yields the number 0·29 for the amplitude. Here, then, we have the amplitude in one case equal to five times, and in the other equal to fourteen times, the expectancy.

The probability of the accidental nature of so large an amplitude

* 'Phil. Trans.,' A, vol. 184, p. 1107 (1893).

is in the first case only 1 in 300,000, and in the second almost infinitesimally small. The reality of the period would be thereby established beyond reasonable doubt, unless the peculiarity of earthquakes occurring in groups, as discussed in the previous section, can be shown to raise the expectancy sufficiently. The fact, however, that in each hemisphere the phase of the periodicity found is nearly identical in a great number of cases disposes of all doubt which might remain on that point.

In some of the details Mr. Davison's results would seem to require further confirmation. The evidence, for instance, that the strong and weak shocks follow different laws is not very strong when examined by means of the theory of probability. Mr. Davison classifies the Japanese earthquakes into three groups, according to their intensity, and finds that the maxima of the annual period agree in the two stronger groups and take place in winter, while the maxima for the weakest group occur in summer. The magnitude of the amplitude for the former is 0·17, while the expectancy is 0·074 and 0·133 respectively. Here the excess of amplitude over the expectancy is not sufficiently marked to allow of any certain conclusions being drawn. A similar remark applies to the record of Zante (see § 45 of Mr. Davison's paper).

7. The daily periods of earthquakes have been fully discussed by the same author in a paper published in the 'Philosophical Magazine.' The following table summarises some of the more important results, the twenty-four hours period only being taken into account. The epoch given is that of the maximum, and I have added the expectancy of amplitude calculated on the principles of this paper.

	Number of earthquakes, n.	Expectancy, $\sqrt{\frac{\pi}{n}}$.	Amplitude.	Epoch.
Japan (Tokio), Summer .	543	0·076	0·176	9^h 58^m A.M.
Winter..	661	0·069	0·093	10 39
Year....	1204	0·051	0·130	10 14
Japan, Summer........	597	0·073	0·061	0 2 P.M.
Winter	578	0·074	0·239	11 50 A.M.
Year	1175	0·052	0·147	11 53
Philippine Islands	210	0·122	0·273	10 49
Italy	8177	0·020	0·324	0 25 P.M.

The amplitudes are seen to exceed the expectancy considerably in all cases but one. The reality of the daily period must be considered established, unless the evaluation of the expectancy is faulty, owing

to the fact that tremors occur in groups (§ 5). The good agreement between the phases of the periodicity disposes of that doubt.

8. The expression for the expectancy deduced in § 2 may with advantage be employed in similar investigations to decide the number of events which it is necessary to take into account in order to establish a periodicity of given amplitude. It follows from the laws of probability that, to be reasonably certain, the amplitudes found should be at least equal to three times the expectancy. Hence, if c be the amplitude looked for,

$$c = 3\sqrt{\pi/n}, \quad \text{or} \quad n = 9\pi/c^2 = 28{\cdot}3/c^2.$$

Thus, for instance, Mr. Knott deduces for his supposed lunar period a range of 6 per cent., or an amplitude of 0·03. In order to establish with certainty such an amplitude, it would be necessary that the number of earthquakes taken into account should at least be equal to 30,000, or over four times the number actually used in the calculations.

9. The difficulty discussed in § 5 would seem to limit in many cases the applicability of the results found. There are, indeed, in nearly every case actually occurring in nature, certain regularities in the manner in which events happen, and this regularity always favours the higher values of the Fourier coefficients. As it will not be possible to estimate in many cases the effect of the expectancy, some other form of treatment will often be called for. The following theorem will, I think, prove useful in these investigations :—

Let y be a function of t, such that its values are regulated by some law of probability, not necessarily the exponential one, but acting in such a manner that if a large number of values of t be chosen at random there will always be a definite fraction of that number depending on t_1 only, which lie between t_1 and $t_1+\mathrm{T}$, where T is any given time interval.

Writing $$\mathrm{A} = \int_{t_1}^{t_1+\mathrm{T}} y \cos \kappa t dt \quad \text{and} \quad \mathrm{B} = \int_{t_1}^{t_1+\mathrm{T}} y \sin \kappa t dt,$$

and forming

$$\mathrm{R} = \sqrt{\mathrm{A}^2+\mathrm{B}^2},$$

the quantity R will, with increasing values of T, fluctuate about some mean value, which increases proportionally to $\sqrt{\mathrm{T}}$, provided T is taken sufficiently large.

If this theorem is taken in conjunction with the two following well-known propositions,

(1) If $y = \cos \kappa t$, R will, apart from periodical terms, increase proportionally to T;

(2) If $y = \cos \lambda t$, λ being different from κ, the quantity R will fluctuate about a constant value;

it is seen that we have means at our disposal to separate any true periodicity of a variable from among its irregular changes, provided we can extend the time limits sufficiently.

The proof of this proposition lies outside the limits of this paper.

The application of the theory of probability to the investigation of what may be called "hidden" periodicities, an instance of which has here been given, may be further extended, and interesting results are obtained when a number of periodicities, such as those supposed to depend on the rotation of the sun about its axis, are critically examined. A full treatment of the subject will shortly appear in 'Terrestrial Magnetism.'

"The Vector Properties of Alternating Currents and other Periodic Quantities." By W. E. SUMPNER, D.Sc. Communicated by O. HENRICI, F.R.S. Received May 28,—Read June 17, 1897.

It has been well known for many years that the variations of a simple harmonic function, such as A $\cos pt$, can be represented by the projection, on a fixed line, of a vector of constant length A supposed to revolve uniformly so as to complete one revolution in a time T given by the relation $p\mathrm{T} = 2\pi$. The angle between the revolving vector and the fixed line is pt at any instant t. In the first edition of Thomson and Tait's 'Natural Philosophy' (vol. 1, p. 38, § 58), it is shown that any two simple harmonic functions of one period can be compounded to a single simple harmonic function of the same period, and that the vector, representing the compounded function, is obtained from those representing the component functions by the ordinary process of vector addition.

This device has proved useful for many purposes, but it has proved especially valuable in connexion with alternate current problems. Its application to such cases was first clearly pointed out by Mr. T. H. Blakesley more than twelve years since. The use of it in alternate current work has gradually developed into what may be described as a vector, or network, method of representing alternate current quantities. In this method the length of the vector denotes the magnitude of the current or voltage, while the angle between any two vectors represents the time which elapses between the instants at which the maximum values of the corresponding quantities occur. In particular, if one line represents the voltage on a conductor, and a second line denotes the strength of current produced in it by this voltage, the power absorbed in the conductor is measured by the product of the length of the two lines into the cosine of the angle between them. It is to this possibility of